STRESS IN YARN DURING UNWINDING

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Abstract: *To be able to describe yarn unwinding we had to develop firstly a general equation of motion for the yarn in the balloon. We also had to take into account residual stress of the yarn in the interior of the package. During the unwinding the lift-off point moves up and down the package. The motion is analyzed in the rotating coordinate system and the equation of balloon therefore contains two kinds of tensions. The first one is a well known tension in the yarn at the guide-eye while the second one is less known as it only appears when the conditions are quasi-stationary. We will describe the properties of this tension and its effects on the dynamics of unwinding yarn.*

Key words: *unwinding yarn, packages, yarn-dynamics, stress in yarn*

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1. Introduction

The natural yarns, industrial yarns, carpet yarns, filament and glass yarns, staple yarns and cord, the final product of the spinning, twisting and cabling machines, is wound onto a package at the end of the production process (Barr&Catling, 1976). The cross-wound package is thus the most important intermediate product of the textile industry (Clark et al.,1998). The rapid development of textile machinery, the main purpose of which is to increase the production speed, has resulted in the unwinding of the yarn from the cross-wound package becoming one of the main bottlenecks in high-speed machines in the textile industry. To partially solve this problem, yarn pre-winders have been introduced into the production process. A technological solution that leads to an increase in the cost of the production process is not the right solution. Therefore, unwinding yarn from the packages is still an open field for researchers.

The theory of yarn unwinding from a stationary bobbin in its current form was developed in the last century, first with the fundamental work of D.G. Padfield (Padfield, 1956) in which she modified Mack's equations for the balloon (Mack, 1953) and added a term that describes the systemic Coriolis force. In her work, she obtained a solution for a single balloon in yarn unwinding under stationary conditions. It also obtained a solution for calculating the shapes of multiple balloons with a winding angle greater than zero for cylindrical and conical windings. The theory was re-derived by Kothari and Leaf (Kothari&Leaf, 1979) by including the influence of the gravitational force and the tangential component of the air resistance force. Based on numerical calculations, they found that the influence of these two forces can be neglected. In the following, the development of the theory was significantly influenced by Fraser (Fraser et al.,1992). In his work, he used perturbation theory and proved that time dependence can be excluded (eliminate) from the equations of motion for the balloon and found that with stretchable yarn, the tension in the balloon is less than its diameter. This effect is found to be small for the range of elastic constants encountered in conventional yarns. Kong et al (Kong & Rahn, & Goswami 1999) carried out extensive experimental research in the late 1990s and the results proved that the theoretical findings were correct. This opens the way for Mathematical modeling in the search for optimal yarn movement in textile processes. In our previous works (Praček et al.,2011, Praček et al.,2011), we presented how computer simulation allows us to predict the dynamic properties of the yarn and the mechanical tension in it. Both theories were developed in parallel, the balloon theory as well as the theory of the movement of the yarn sliding over the surface of the package before it lifts from the package and forms a balloon.

In this paper, the consideration of yarn unwinding from packages is further supplemented by developing the equation of the tension of the yarn in the balloon from the general equation of motion of the yarn movement during unwinding. Instead of the air resistance force, we insert into the equation a force that describes the effect of the residual tension on the yarn. The theory stems from the fact that in one period of rotation this part of the yarn describes a surface of revolution that has a form of balloon.

2. The equation of motion for yarn

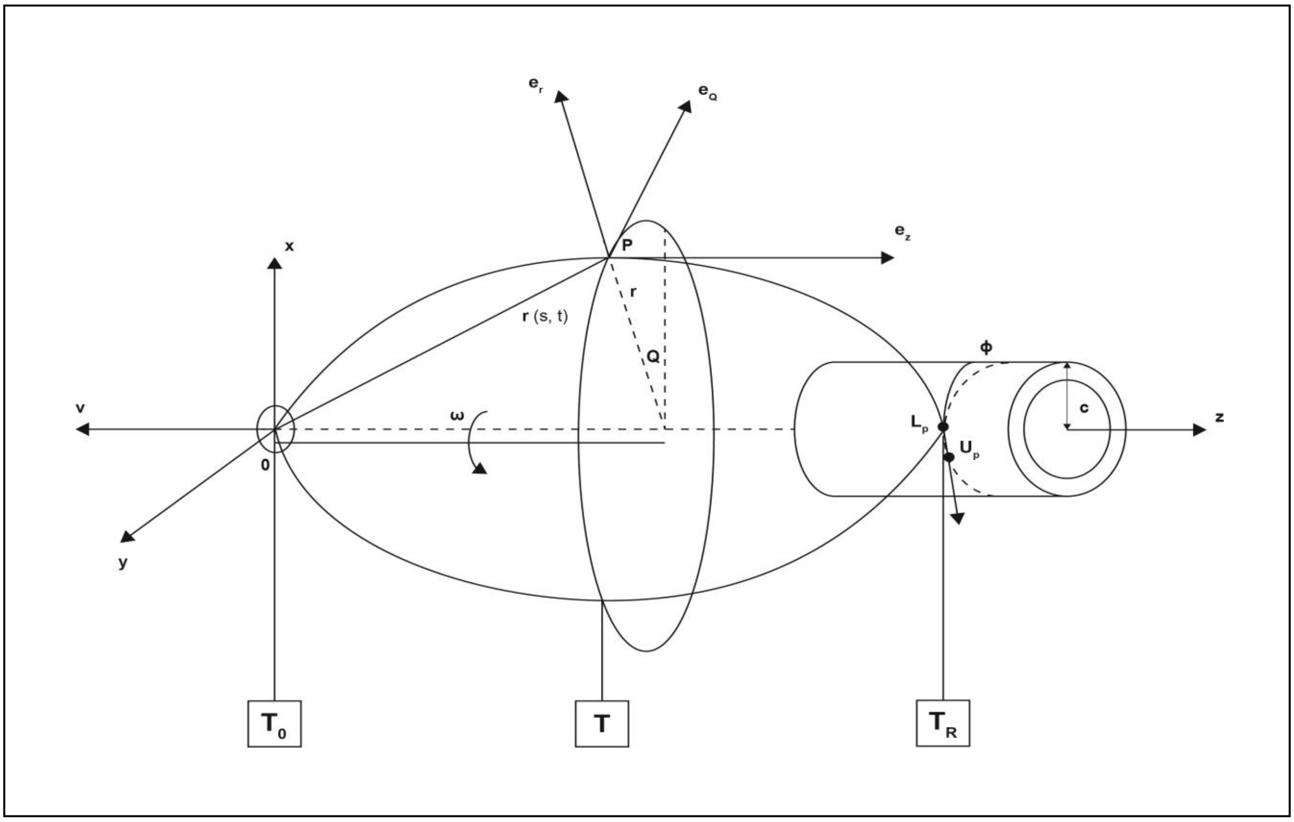

Fig. 1. Mechanical setup in overend yarn unwinding from cylindrical package.

During the yarn unwinding from a stationary package, the yarn slides on the surface of the package before it lifts off to form a balloon. The problem of residual tension defined as the tension of the yarn inside the package during the unwinding can be treated in analogy with the motion of the yarn forming the balloon between the lift-off point (L_P) and the eyelet, through which the yarn is being pulled. We derived the equation of motion of the yarn in one of the previous works (Praček et al.,2011, Praček et al.,2011):

$$\rho\left(D^2\mathbf{r}+2\boldsymbol{\omega}\times D\mathbf{r}+\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})+\dot{\boldsymbol{\omega}}\times\mathbf{r}\right)=\frac{\partial}{\partial s}\left(\mathbf{T}\frac{\partial\mathbf{r}}{\partial s}\right)+\mathbf{f}\,. \tag{1}$$

Where:

f = The air drag for that part of the yarn that forms the balloon (or the force of friction for the part of yarn between unwinding point (U_P) and lift-off point on the package, which is sliding on lower layers of yarn)

T = The yarn tension, and the force is obviously directed along the yarn.

D = The operator of the total time derivative which follows the motion of the point inside the spinning coordinate system, $D = \partial/\partial t|_{r,\theta,z} - V\partial/\partial s$

$-\rho\, 2\boldsymbol{\omega}\times D\mathbf{r}$ = The Coriolis force

$-\rho\, \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$ = The centrifugal force

$-\rho\dot{\boldsymbol{\omega}}\times\mathbf{r}$ = The Euler force

The equation of motion, can be rewrite with dimensionless quantities (Fraser ,1992) we obtain

$$\overline{D}^2\bar{\mathbf{r}} + 2\Omega\times\overline{D}\bar{\mathbf{r}} + \Omega\times(\Omega\times\bar{\mathbf{r}}) + \Omega\frac{\partial\Omega}{\partial\bar{t}}\times\bar{\mathbf{r}} = \frac{\partial}{\partial\bar{s}}\left(\overline{T}\frac{\partial\bar{\mathbf{r}}}{\partial\bar{s}}\right) + \bar{\mathbf{f}}. \tag{2}$$

The dimensionless angular velocity is

$$\Omega = \frac{c\omega}{V}. \tag{3}$$

Non-inertial reference frames are also useful in studying the yarn unwinding. It may be used, for example, when working within the quasi-stationary approximation to describe the yarn unwinding from packages where layers have a large number of yarn loops (Fraser ,1992).

If we use the quasi- stationary conditions in Equation (2), we get

$$\frac{\partial^2\mathbf{r}}{\partial s^2} - 2\Omega\times\frac{\partial\mathbf{r}}{\partial s} + \Omega\times(\Omega\times\mathbf{r}) = \frac{\partial}{\partial s}\left(T\frac{\partial\mathbf{r}}{\partial s}\right) + \mathbf{f}. \tag{4}$$

The equation for the tension at the point where the yarn rises from the package surface and forms a balloon was justified in one of the previous works (Praček et al.,2011, Praček et al.,2011):

$$T = T_0 - \frac{1}{2}\Omega^2 r^2. \tag{5}$$

Where:

T_0 = the tension in the yarn passing through the guide

3. Numerical simulation

During unwinding yarn, the unwinding point moves slowly along the package surface, so we do not need the Initial Conditions at all. The first boundary condition is that the yarn passes through the guide at the origin, which is written as r (s = 0) = 0, or equivalently *r* (0) = 0, θ (0) = 0, *z* (0) = 0). At the lift-off point, the yarn must be continuous and must not be broken. The following are the conditions for continuity:

$$r(s = s_{Lp}^{+}) = r(s = s_{Lp}^{-})$$
$$r'(s = s_{Lp}^{+}) = r'(s = s_{Lp}^{+}) \qquad (6)$$

The index + or - indicates the point just behind or just before the lifting point. In the lift-off point the yarn is tangential to the package. We can then write:

$$\mathrm{r}'(\mathrm{s} = \mathrm{s}_{\mathrm{Lp}}) = 0 \qquad (7)$$

The nondimensionless boundary condition at the lift-off point becomes.

$$\bar{r}\left(\mathbf{s}_{\mathbf{Lp}}, \mathbf{t}\right) = 1 \qquad (8)$$

Inserting condition (8) into equation (5) the tension at the Lp is then equal to

$$T = T_0 - \frac{1}{2}\Omega^2. \qquad (9)$$

The expression (9) tells us that tension in the yarn T_o passing through the guide in the given point is reduced from the residual yarn tension in the package to the value of tension T in the balloon . If we write T_R for the residual tension in the yarn. We get

$$T = T_0 - T_R. \qquad (10)$$

Where

$$T_R = \frac{\Omega^2}{2} \qquad (11)$$

For packages of general shape is computed by using the relation(Fraser et al.,1992):

$$\omega = \frac{2\pi}{t} = \frac{V\cos\phi}{c\,(1 - \sin\phi)} \qquad (12)$$

The dimensionless angular velocity can obviously be expressed as (Fraser et al.,1992):

$$\Omega = \frac{\cos\phi}{1 - \sin\phi} \qquad (13)$$

In a recent paper (Praček et al.,2011) we developed a function, $f(\mathrm{t}) = \mathrm{sign}(\sin \mathrm{t})|\sin \mathrm{t}|^{1/40}$, which would permit to simulate the process of unwinding. In our simulation, we calculate the winding angle ϕ_0 using this function, $\phi(\mathrm{t}) = \phi_0 f(\mathrm{t})$. We get

$$\Omega(\phi(t)) = \frac{\cos(\phi(t))}{1-\sin(\phi(t))} \tag{14}$$

and finally we obtain the simulation model of residual tension in the yarn using equation (10) we can write it like

$$T_R(\phi(t)) = \frac{\Omega^2(\phi(t))}{2} \tag{15}$$

4. Simulation results

We presents the results of changing residual tension in the yarn as we unwind yarn from a cylindrical packages. In our calculations we considered unwinding for two consecutive layers of yarn, so that the package radius remains approximately constant during this time.

Angle ϕ_0	T_R at unwinding backward	Angle ϕ_0	T_R at unwinding forward	Angle ϕ_0	T_R at unwinding backward	Angle ϕ_0	T_R at unwinding forward
1°	0.5178	1°	0.4828	16°	0.8805	16°	0.2839
2°	0.5362	2°	0.4663	17°	0.9132	17°	0.2738
3°	0.5552	3°	0.4503	18°	0.9472	18°	0.2639
4°	0.5750	4°	0.4348	19°	0.9827	19°	0.2544
5°	0.5955	5°	0.4198	20°	1.0198	20°	0.2451
6°	0.6167	6°	0.4054	21°	1.0585	21°	0.2362
7°	0.6388	7°	0.3914	22°	1.0990	22°	0.2275
8°	0.6617	8°	0.3778	23°	1.1413	23°	0.2190
9°	0.6854	9°	0.3647	24°	1.1856	24°	0.2109
10°	0.7101	10°	0.3520	25°	1.2320	25°	0.2029
11°	0.7358	11°	0.3398	26°	1.2805	26°	0.1952
12°	0.7625	12°	0.3279	27°	1.3315	27°	0.1878
13°	0.7902	13°	0.3164	28°	1.3849	28°	0.1805
14°	0.8191	14°	0.3052	29°	1.4410	29°	0.1735
15°	0.8492	15°	0.2944	30°	1.5000	30°	0.1667

Tab. 1. Residual tension as a function of the winding angle

5. Results analysis

In Fig. 2. we show the results for the oscillations of residual tension for a range of ten winding angles from $\phi_0 \sim 0°$ to $\phi_0 = 10°$. We notice that for such packages the tension remains very small.

According to our model the residual tension in the yarn thus only depends on the winding angle which will change with time because this angle is different for layers that are unwinding from front towards rear edge and those that are unwinding as the unwinding point moves from the rear towards front edge. The dependence of residual tension on the winding angle is shown in Tab 1.
Note: The time on the horizontal axis is expressed in units of t_0. The quantity t is equal to the duration of one period, i.e., one cycle of the movement of the lift-off point from the front edge to the back edge and back, divided by 2π. Thus, the time t is the phase of the motion of the lift-off point on the package surface. The time of 2π namely correspond to precisely one cycle.

In Figs. 3 and 4 we show the results for the yarn tension for unwinding from the same design of the package as in Fig. 2, but this time at a higher winding angles from $\phi_0 = 10°$ to $\phi_0 = 40°$. We observe that the oscillations are large. In the moment when the unwinding direction changes the tension undergoes an almost discontinuous change. Such abrupt changes strain the yarn, therefore it is conceivable that the yarn undergoes some damage in the process. In the worst case, the yarn can even break.
The oscillations of residual tension are related to the variation of the dimensionless angular velocity of yarn rotation around the package axis. The amplitude of the dimensionless angular velocity oscillations can be calculated as

$$\begin{aligned}
\Delta\Omega = \Omega_{\max} - \Omega_{\min} &= \frac{\cos\phi_0}{1-\sin\phi_0} - \frac{\cos(-\phi_0)}{1-\sin(-\phi_0)} \\
&= \frac{\cos\phi_0}{1-\sin\phi_0} - \frac{\cos(+\phi_0)}{1+\sin(+\phi_0)} \\
&= \frac{\cos\phi_0 + \cos\phi_0\sin\phi_0 - \cos\phi_0 + \cos\phi_0\sin\phi_0}{(1-\sin\phi_0)(1+\sin\phi_0)} \\
&= \frac{2\cos\phi_0\sin\phi_0}{1^2 - \sin^2\phi_0} \\
&= \frac{2\cos\phi_0\sin\phi_0}{\cos^2\phi_0} \\
&= 2\frac{\sin\phi_0}{\cos\phi_0} \\
&= 2\tan\phi_0
\end{aligned} \tag{16}$$

In the region of interest, i.e. for $\phi < 25°$, we can approximate $\tan\phi \sim \phi$, so we have

$$\Delta\Omega \approx 2\phi_0 \tag{17}$$

Inserting amplitude (17) into equation (11) the amplitude of residual tension in yarn is then equal to

$$\Delta T_R = 2\phi_0^2 \tag{18}$$

According to this expression, the amplitude of residual tension oscillations is approximately proportional to the quadratic winding angle.

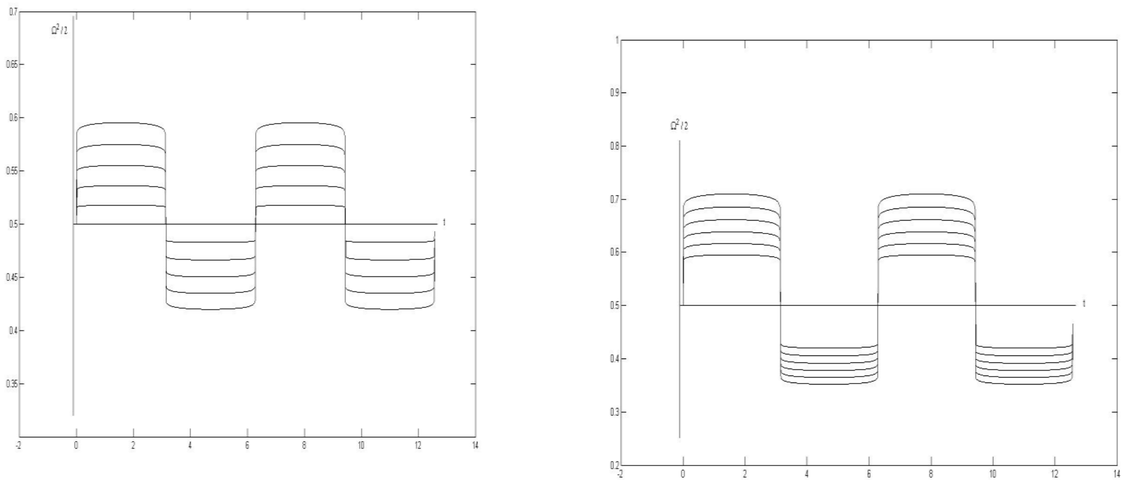

Fig. 2. Oscillations of residual tension during the unwinding yarn from cylindrical packages, where $\phi_0 = 0° - 10°$.

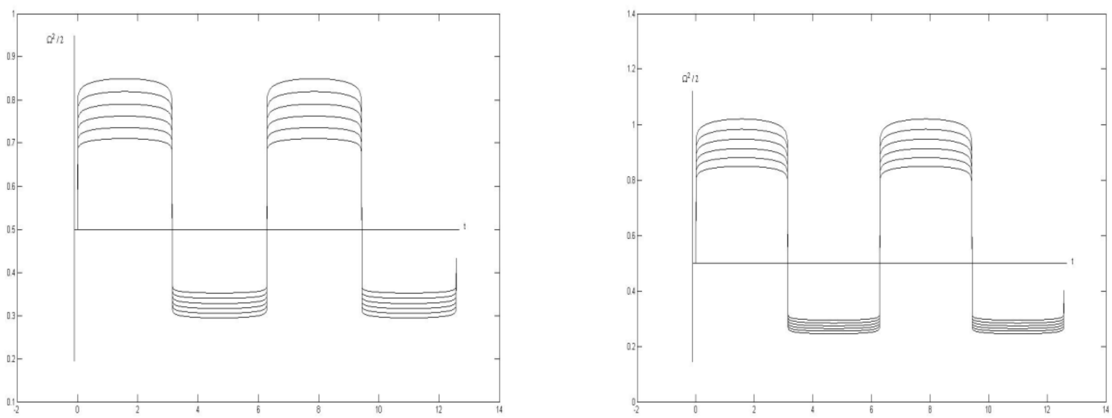

Fig. 3. Oscillations of residual tension during the unwinding yarn from cylindrical packages, where $\phi_0 = 10° - 20°$.

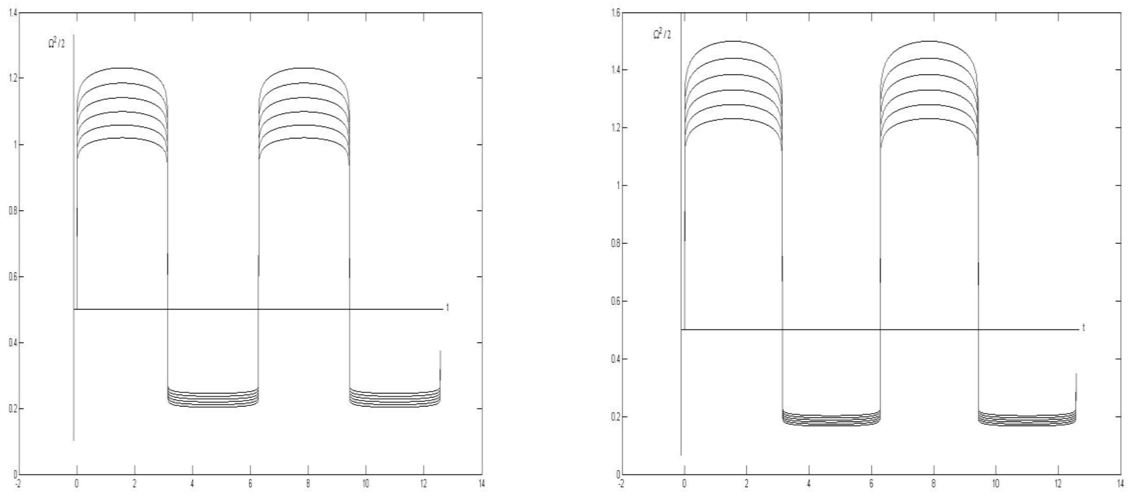

Fig. 4. Oscillations of residual tension during the unwinding yarn from cylindrical packages, where ϕ_0 = 20° - 30°.

6. Conclusion

We saw how the equation for yarn tension in the balloon can be obtained from the general equation of yarn by replacing the usual perturbation theory approach with quasi-stationary approximation. The tension T of the yarn in the balloon consists of two terms from the tension T_0 in the yarn at the guide-eye and from the residual tension T_R in the yarn. Analytical solutions enable a better understanding of interdependencies between various quantities. In cross-wound package with a very small ϕ the residual tension is constant, since in such packages the angular velocity doesn’t change with time. In cross-wound packages with higher winding angle ϕ, the residual tension does change with time. This happens because the angular velocity is higher when the unwinding point moves backward as it is when the unwinding point moves forward. The residual tension is a function of angular velocity, so it is oscillating.

This suggest the possibility to make a compromise: if it is known that the yarn is damaged at some given amplitude of residual tension oscillations, then the possible choices of winding angle ϕ lie of a straight line. One can thus use small winding angle with small dimensionless angular velocity or large winding angle with correspondingly higher dimensionless angular velocity. It is also apparent that during unwinding from packages with a winding angle ϕ=15º, it is possible to unwind at all dimensionless angular velocity shown with a possible exception of those near the maximum values of amplitude.

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