DYNAMIC OPTIMIZATION MODEL FOR UNWINDING YARN IN TEXTILE PRODUCTION

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Abstract: *Stability of the unwinding process has a direct influence on the efficency of the process and on the quality of the final product. An optimal shape of the package leads to an optimal shape of the balloon and to small and steady tension even at high unwinding velocity. Computer modelling is a valuable tool in search of the optimal package shape. We demonstrate a mathematical model for simulating the unwinding from cylindrical and conic packages. We show how the winding angle and the apex angle influence the angular velocity of the yarn during the unwinding. Since the centrifugal forces on the yarn in the balloon depend on the angular velocity, this velocity has a large influence on the tension that we wish to reduce.*

Key words: *yarn unwinding, balloon theory, simulation, residual tension, oscillations*

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This Publication has to be referred as: Pracek, S[tanislav] (2024). Dynamic Optimization Model for Unwinding Yarn in Textile Production, Chapter 10 in DAAAM International Scientific Book 2024, pp.127-138, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-902734-42-6, ISSN 1726-9687, Vienna, Austria
DOI: 10.2507/daaam.scibook.2024.10

1. Introduction

In the fabric manufacturing process, the yarn is unwound from the cross windings packages in the phase of weaving and introducing the weft into the weft. When unwinding the yarn from the cross winding packages, a certain tension is generated in the yarn. This tension is roughly proportional to the square of the unwinding speed. The development of the current yarn unwinding theory began in the 1950s. (Mack, 1953; Padfield, 1956). The old balloon theory was upgraded with the Coriolis and centrifugal force and the realization that the influence of gravity and the tangential component of the air resistance force can be neglected.

More recently, with the help of perturbation analysis, they excluded time dependence from the equations of motion, and derived the movable boundary conditions that apply to windings with a small winding angle (Barr&Catling, 1976; Kothari&Leaf, 1979; Fraser et al.,1992). They have shown that the intire time dependance can be shifted to moving boundary conditionns. In this manner the innitial-value problem of partial differential equations can be reduced to a boundary problem which is much easier to solve. The latest research in this field confirmed the previous theoretical findings with the help of experimental results and extensive numerical calculations (Clark et al.,1998; Kong & Rahn, & Goswami 1999)

Building on this foundation we have simplified the problem even further. In this work we show how a simple model function that describes the package can be used to estimate the unwinding properties of packages of different geometries and different winding types.

2. Theoretical part

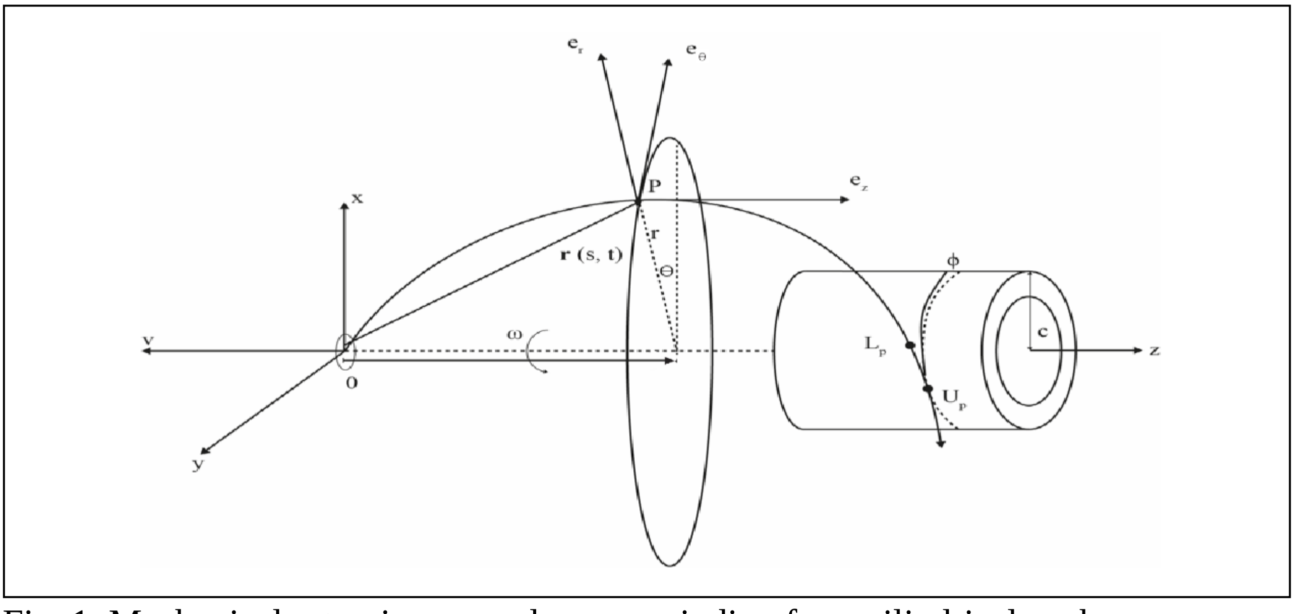

Fig. 1. Mechanical setup in overend yarn unvinding from cilindrical package.

The yarn is unwinding from a fixed cylindrical package in horizontal direction with velocity V through the guide-eye O. The origin of the coordinate system is at the guide-eye.

Point Lp is the lift-off point, i.e., the point where the yarn leaves the surface of the packages to form the balloon. Angle ϕ is the angle of the winding on the package. We are interested in balloon motion, i.e., the time variation of the position radius vector r(s,t) of the yarn in space. The balloon motion is the rotation of the rigid curve around the z axis. It has short characteristic time 2p/w, where w is the angular velocity. In this time one loop of the yarn is unwound. The theory of yarn unwinding off a package and the balloon equations was derived in the previous work (Fraser et al.,1992; Praček et al.,2011):

$$\rho\left(D^2\mathbf{r} + 2\omega \times D\mathbf{r} + \omega \times (\omega \times \mathbf{r}) + \dot{\omega} \times \mathbf{r}\right) = \frac{\partial}{\partial s}\left(T\frac{\partial \mathbf{r}}{\partial s}\right) + \mathbf{f} \tag{1}$$

Before we continue the discussion of the equations of the yarn we introduce dimensionless quantities. This way we minimize the number of three parameters and simplify the analysis of the equations. Moreover, the dimensionless quantities are more convenient for numerical computations.

We will write a bar over the symbol of a quantity to denote that it is a dimensionless quantity. The lengtles will be expressed in multiples of the radius of a package *c*. We get

$$\bar{r} = r =/c, \quad \bar{r} = r/c, \quad \bar{z} = z/c, \quad \bar{s} = s/c. \tag{2}$$

Time will be expressed in multiples of the period of rotation (of the package around its axis) $\tau_0=2\pi/\omega$. It is even better if we omit the factor 2π and express the time in multiples of $\tau=1/\omega$:

$$\bar{\mathrm{t}} = \mathrm{t}/\mathrm{t} = \omega \mathrm{t} \tag{3}$$

Let *V* be the speed with which we pull the yarn. We will express the speed **v** and the speed $\mathbf{v}_n$ as multiples of *V.*

$$\bar{\mathrm{v}} = \mathrm{v}/\mathrm{V}, \bar{\mathrm{v}}_{\mathrm{n}} = \mathrm{v}_{\mathrm{n}}/\mathrm{V} \tag{4}$$

We also work with the force linear densities **f** and **n** and with the tension *T.* The tension is expressed in the same units as force (i.e. N=kgm/s^2) while force linear densities are expressed in units N/m.

$$\left[\rho V^2\right] = \frac{kg}{m}\frac{m^2}{s^2} = \frac{kgm}{s^2} = N \tag{5}$$

$$\left[\frac{\mathrm{rV}^2}{\mathrm{c}}\right] = \frac{\mathrm{N}}{\mathrm{m}} \tag{6}$$

We can now introduce dimensionless quantities

$$\bar{\mathbf{f}} = \frac{\mathbf{f}c}{\rho V^2}$$
$$\bar{\mathbf{n}} = \frac{\mathbf{n}c}{\rho V^2} \tag{7}$$
$$\bar{T} = \frac{T}{\rho V^2}$$

We would also like to express time derivatives of the dimensionless form. Here we use the derivatives of composite functions.

$$\begin{aligned} \frac{\partial}{\partial s} &= \frac{\partial \bar{s}}{\partial s}\frac{\partial}{\partial \bar{s}} = \frac{1}{c}\frac{\partial}{\partial \bar{s}}, \\ \frac{\partial}{\partial t} &= \frac{\partial \bar{t}}{\partial t}\frac{\partial}{\partial \bar{t}} = \omega\frac{\partial}{\partial \bar{t}}. \end{aligned} \tag{8}$$

(14pt)

Therefore we can write the operator *D* as

$$\mathrm{D} = \omega\frac{\partial}{\partial \bar{t}} - \frac{\mathrm{V}}{\mathrm{c}}\frac{\partial}{\partial \bar{s}} = \frac{\mathrm{V}}{\mathrm{c}}\left(\frac{\mathrm{c}\omega}{\mathrm{V}}\frac{\partial}{\partial \bar{t}} - \frac{\partial}{\partial \bar{s}}\right) \tag{9}$$

We also introduce the dimensionless differential operator *D* as.

$$\begin{aligned} \bar{D} &= \Omega\frac{\partial}{\partial \bar{t}} - \frac{\partial}{\partial \bar{s}}, \\ \Omega &= \frac{c\omega}{V}. \end{aligned} \tag{10}$$

the parameter Ω is the dimensionless angular velocity, which will be often used in the sequel. The relation between the operator *D* and $\bar{D}$ is $D = \frac{V}{c}\bar{D}$

Let us now consider the fundamental motion equation (1)

$$\rho\left(D^2\mathbf{r} + 2\omega \times D\mathbf{r} + \omega \times (\omega \times \mathbf{r}) + \dot{\omega} \times \mathbf{r}\right) = \frac{\partial}{\partial s}\left(T\frac{\partial \mathbf{r}}{\partial s}\right) + \mathbf{f} \tag{11}$$

if we replace all quantities with their dimensionless counterparts we get the equation for the left hand side of the equation

$$\rho\left(\frac{V^2}{c^2}\overline{D}^2 c\overline{\mathbf{r}} + 2\omega\times\frac{V}{c}\overline{D}c\overline{\mathbf{r}} + \omega\times(\omega\times c\overline{\mathbf{r}}) + \omega\frac{\partial\omega}{\partial\overline{t}}\times c\overline{\mathbf{r}}\right)$$
$$= \rho c\frac{V^2}{c^2}\left(\overline{D}^2\overline{\mathbf{r}} + 2\frac{c}{V}\omega\times\overline{D}\overline{\mathbf{r}} + \frac{c^2}{V^2}\omega\times(\omega\times\overline{\mathbf{r}}) + \frac{c^2}{V^2}\omega\frac{\partial\omega}{\partial\overline{t}}\times\overline{\mathbf{r}}\right) \tag{12}$$
$$= \rho\frac{V^2}{c}\left(\overline{D}^2\overline{\mathbf{r}} + 2\Omega\times\overline{D}\overline{\mathbf{r}} + \Omega\times(\Omega\times\overline{\mathbf{r}}) + \Omega\frac{\partial\Omega}{\partial\overline{t}}\times\overline{\mathbf{r}}\right).$$

(Here we introduced $\mathbf{\Omega}=\Omega\mathbf{e}_{z,}$, the dimensionless angular velocity vector). Similarly, we get for the right –hand side of the equation

$$\frac{1}{c}\frac{\partial}{\partial\overline{s}}\left(\overline{T}\rho V2\frac{\partial\overline{\mathbf{r}}}{\partial\overline{s}}\right) + \frac{\rho V^2}{c}\overline{\mathbf{f}}$$
$$\rho\frac{V^2}{c}\left(\frac{\partial}{\partial\overline{s}}\left(\overline{T}\frac{\partial\overline{\mathbf{r}}}{\partial\overline{s}}\right) + \overline{\mathbf{f}}\right) \tag{13}$$

Now we cancel the factor $\rho\frac{V^2}{s}$ on both sides of the equation and we obtain the final result:

$$\overline{D}^2\overline{\mathbf{r}} + 2\Omega\times\overline{D}\overline{\mathbf{r}} + \Omega\times(\Omega\times\overline{\mathbf{r}}) + \Omega\frac{\partial\Omega}{\partial\overline{t}}\times\overline{\mathbf{r}} = \frac{\partial}{\partial\overline{s}}\left(\overline{T}\frac{\partial\overline{\mathbf{r}}}{\partial\overline{s}}\right) + \overline{\mathbf{f}} \tag{14}$$

This is the dimensionless form of the motion equation for the yarn.

Let us now express the air resistance force and the friction force with dimensionless quantities.
For the air resistance force we get

$$\begin{aligned}\overline{\mathbf{f}}_u &= \mathbf{f}\,\frac{c}{\rho V^2}\\ &= -D_n|\mathbf{v}_n|\mathbf{v}_n\,\frac{c}{\rho V^2}\\ &= -D_n|\overline{\mathbf{v}}_n|\overline{\mathbf{v}}_n\,\frac{c}{\rho}\\ &= -\frac{p_0}{16}|\overline{\mathbf{v}}_n|\overline{\mathbf{v}}_n,\end{aligned} \tag{15}$$

Where we introduced the following coefficient of air resistance

$$p_0 = \frac{16cD_n}{\rho}. \tag{16}$$

The number 16 is here for historical reasons, see (Padfield, 1956). This definition is now standard and slows us to compare our results with others.

We also need expressions for $\bar{\mathbf{v}}$ and $\bar{\mathbf{v}}_n$.We get

$$\begin{aligned}
\bar{\mathbf{v}} &= \mathbf{v}/V \\
&= \frac{1}{V}(Dr + \omega \times r) \\
&= \frac{1}{V}\left(\frac{V}{c}\overline{D}c\bar{\mathbf{r}} + \omega \times c\bar{\mathbf{r}}\right) \\
&= \overline{D}\bar{\mathbf{r}} + \frac{c}{V}\omega \times \bar{\mathbf{r}} \\
&= \overline{D}\bar{\mathbf{r}} + \Omega \times \bar{\mathbf{r}},
\end{aligned} \tag{17}$$

for the first one and
(14pt)

$$\begin{aligned}
\bar{\mathbf{v}}_n &= \frac{1}{V}\mathbf{v}n \\
&= \frac{1}{V}\frac{\partial \mathbf{r}}{\partial s} \times \left(\mathbf{v} \times \frac{\partial \mathbf{r}}{\partial s}\right) \\
&= \frac{\partial \bar{\mathbf{r}}}{\partial \bar{s}} \times \left(\bar{\mathbf{v}} \times \frac{\partial \bar{\mathbf{r}}}{\partial \bar{s}}\right).
\end{aligned} \tag{18}$$

for the second one.

$$\bar{\mathbf{f}} = -\mu \bar{n}\frac{\mathbf{v}}{|\mathbf{v}|} + \bar{n}\mathbf{e}_r. \tag{19}$$

The expression for the dimensionless force linear density of the package on yarn is similar the usual (dimensional) expression (Fraser et al.,1992).

It is clear from the form of dimensionless equations that the minimal number of parameters completely characterizing the solution is equal to 3. They are:

- Ω, the dimensionless angular velocity rotating yarn around its axis;
- p_0, the coefficient of air resistance;
- μ, the coefficient of friction.

3. Model of Unwinding Process

Only one parameter remains in equation (14)(without taking into account the external force term). It is the dimensionless angular velocity $\Omega = \omega c/V$. This is the single most important parameter in our model. We will show later on that we can make many important conclusions if we determine how Ω changes with time as the yarn is being unwound. There are two additional dimensionless parameters: μ, the coefficient of friction between yarn and package, and the coefficient of air resistance p_0. These two parameters are approximately constant during the unwinding, so they are less important. The coefficient of friction depends on the quality of the yarn and on the surface properties of the package.

The quality of the surface is comparable for forward and backward unwinding direction, so that the coefficient of friction remains approximately constant.

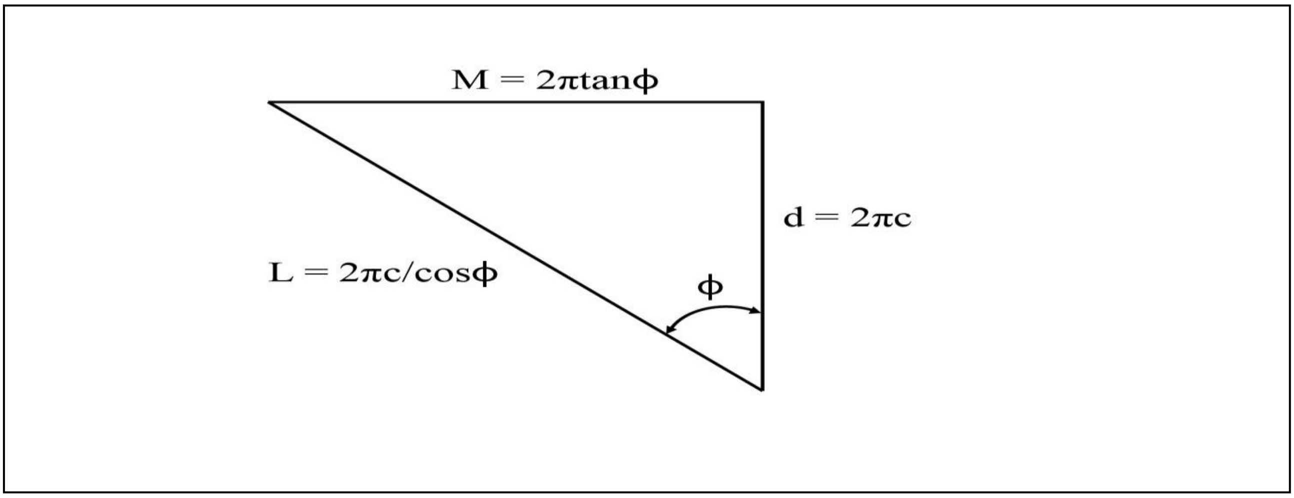

Fig. 2. Cross section of a package along a line parallel to the package axis.

We will show that the winding angle determines the angular velocity ω, if the unwinding velocity V and the package radius c are known. The derivation is applicable in the quasi-stationary approximation which consists of neglecting the variation of system parameters (in particular of the winding angle ϕ) during a single period of the balloon motion around the z axis. The winding angle is positive if the lift-off point is moving towards higher values of the coordinate z during the unwinding (unwinding in the backward direction). The winding angle is negative if the lift-off point is moving towards smaller values of z (unwinding in the forward direction), see figure 1. During a single period $t = 2\pi/\omega$ the balloon revolves once around the axis. During the same time the lift-off point likewise makes a single roundtrip around the package. The length of the yarn which unwinds during a single period is therefore equal to $L = 2\pi c/\cos\phi$. Consequently, the lift-off point moves for $M = 2\pi c\tan\phi$ along the package surface. In the quasi-stationary approximation we can then interpret this change in the following way: the arc-length of the yarn forming the balloon has increased by M, while the yarn segment with the length L – M has been unwound through the eyelet. The yarn velocity is thus $V = (L - M)/t$. The angular velocity that we are interested in is therefore

$$\omega = \frac{2\pi}{t} = \frac{2\pi V}{L - M} = \frac{V\cos\phi}{c\,(1 - \sin\phi)} \tag{20}$$

Introducing the dimensionless angular velocity $\Omega = \omega c/V$, this formula can also be expressed as

$$\Omega = \frac{\cos\phi}{1 - \sin\phi} \tag{21}$$

In conical packages the relation is only slightly modified:

$$\omega = \frac{V\cos\phi}{c(1-\cos\alpha\sin\phi)} \tag{22}$$

The dimensionless angular velocity can be defined as before, we obtain

$$\Omega = \frac{\cos\phi}{1-\cos\alpha\sin\phi} \tag{23}$$

Here α is the apex angle of the conical package. During unwinding the lift-off point moves up and down the package. We can presume that the winding angle is approximately constant in the middle of the package and it changes at the edges of the package where its sign is reversed. To describe the time dependence of the winding angle we must look for a periodic function, because motion of the point is periodic to a good approximation. The most known periodic functions are trigonometric function, such as sine function. This function should be modified so that it will change only slightly when the point moves up or down the packages. We can achieve this by raising the sine to a low fractional power, say 1/40 (we have to be careful about the signs, so we take absolute value of sine function and restore the sign using the signum function:

$$\mathrm{f}(t) = sign(\sin t)\left|\sin t\right|^{\frac{1}{40}} \tag{24}$$

The plot of such a function is shown below:

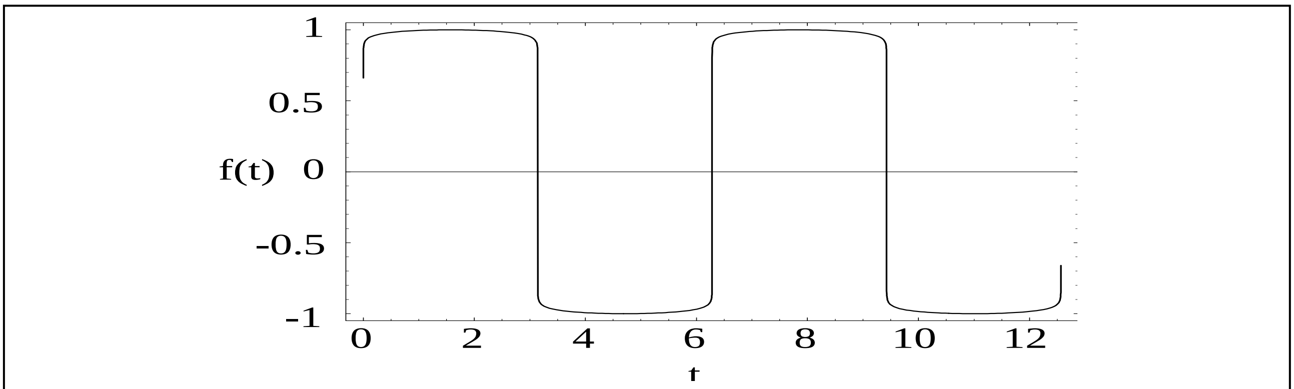

Fig. 3. Function f(t) which describes the time variation of the winding angle $\phi(t) = \phi_0 f(t)$ during the unwinding from a cylindrical package. The time is express in units of the period (i.e., 2πcorresponds to one full period).

Now we assume that the winding angle depends out as

$$\phi(t) = \phi_0 \mathrm{f}(t), \tag{25}$$

where ϕ_0 is the maximal angle of wind. The function f(t) has the required properties. So it is a good model for the description of packages. By this we mean the positive value of the winding angle, i.c, the charateristics of a given package.

With this expression for $\phi(t)$ we can compute how the dimension-less angular velocity depends out. Equation (24) is mathematical model, which describes unwinding a yarn from packages. Equation (25) represent angle of unwinding a yarn on packages.

Dimensionless angular velocity $\Omega = \Omega(\phi(t))$ for cylindrical packages, see equation (21) we can write it like

$$\Omega(\phi(t)) = \frac{\cos(\phi(t))}{1-\sin(\phi(t))} \tag{26}$$

We can write dimensionless angular velocity $\Omega = \Omega(\phi(t))$ for conical packages, see equation (23)

$$\Omega(\phi(t)) = \frac{\cos(\phi(t))}{1-\cos(\alpha)\sin(\phi(t))} \tag{27}$$

4. Results and discussion

We will introduce the dependence of the dimensionless angular velocity Ω of cylindrical and conical packages with simulation solution. The unit of time has been chosen such that the period of unwinding is 2π. This means that from $t = 0$ to $t = \pi$ yarn unwinds backwards (and so the dimensionless angular velocity is larger) while from $t = \pi$ to $t = 2\pi$ it unwinds forwards (the dimensionless angular velocity is smaller).

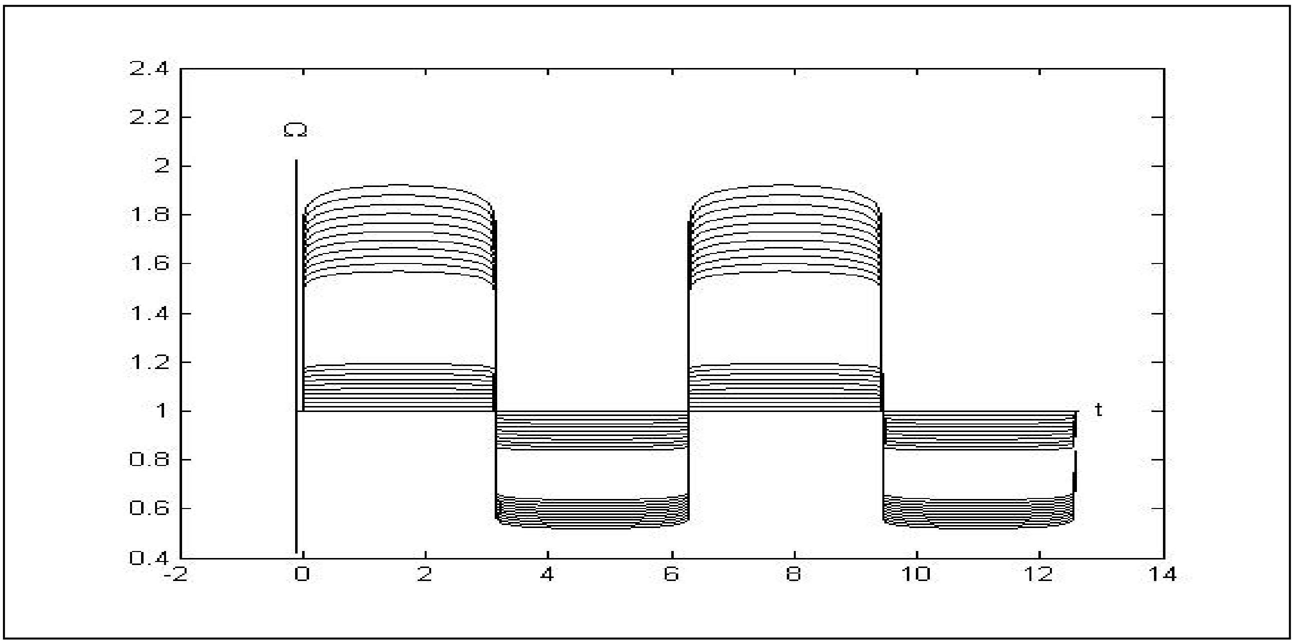

Fig. 4. Vibrations dimensionless angular velocity Ω during the unwinding yarn from cylindrical packages, where φ = 0° to 10° and 25° to 35°.

For cylindrical parallel packages (i.e., when winding angle is close to zero) we get the straight line Ω =1. We investigated the effects of diferent winding angles ϕ and for conical packages, the apex angle α.

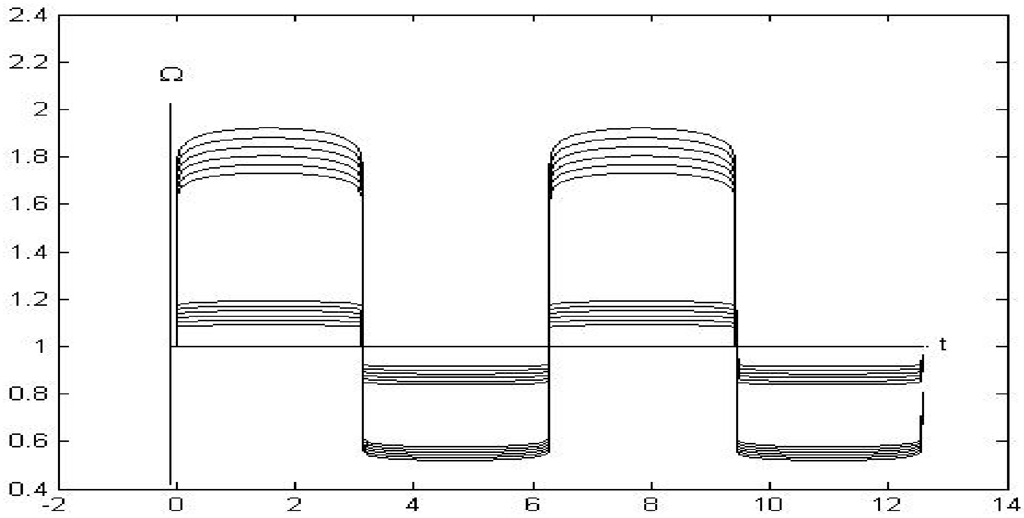

Fig. 5. Vibrations dimensionless angular velocity Ω during the unwinding yarn from cylindrical packages, where φ = 5° to 10° and 30° to 35°.

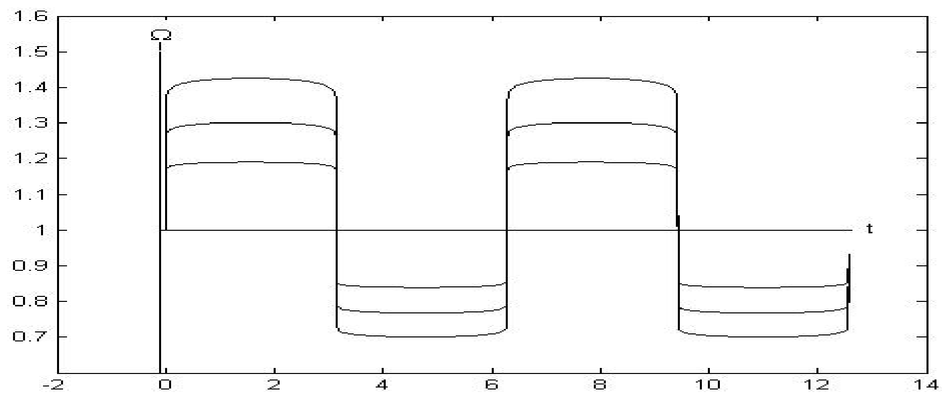

Fig. 6. Vibrations dimensionless angular velocity Ω during the unwinding yarn from conical packages, where φ = 10° to 20° and apex angle α = 5°

In fig 4 we show the results for the vibrations of Ω for a range of different winding angles from 0° to 25°. In parallelly wound package (ϕ=0) the ω is constant, since in such packages the Ω doesn't change with time. In package with higher winding angle ϕ the Ω does change with time. This happens because the angular velocity is higher when the unwinding point moves backwards as it is when the unwinding point moves forewards. The tension is a function of Ω, in agreement with Eg. (14) so it is vibrations. We observe that the vibrations are large.

For winding angle ϕ=25° then Ω varies from Ω=0.6371to Ω=1.5697. In the moment when the unwinding direction changes the Ω undergoes an almost discontinuous change. Such sudden jumps lead to strong strain in the yarn, the yarn can be demaged or even broken in two parts.

In Fig.5 we show the results for the yarn vibrations of Ω for unwinding from the same design of the package as in Fig. 4, but this time at a higher winding angles. The vibrations of Ω in the yarn that we observe in this case rise to very high values, from 1.7321 to 1.9210. At this vibration the unwinding from package with winding angles from 30° to 35° would not be possible.

If we opt, instead, for packages with small winding angles, the characteristics become quite acceptable. In Fig. 5 we also compare the yarn vibrations of Ω from cross-wound packages with winding angles from 5° to 10°. At ϕ=10°, the yarn vibrations of Ω never exceeds1.1918.. We notice that for such packages the yarn vibrations of Ω remains very small, from 0.8391 to 1.1918.

Figure 4 also present the time dependence of the Ω for six winding angles 0°, 1°, 2°, 3°, 4° and 5°. Figure demonstrate the transition from the regime of small winding angles 0°, 1°, which are characteristic for parallel-wound packages and where the yarn vibrations of Ω tend to be negligible, to the regime of larger angles 2°, 3°, 4°, 5° where one readily observes higher vibration of Ω that characterize cross-wound packages. We obtain small vibrations of Ω for all six winding angles, that is from 0.9163 to 1.0913.

This means that the amplitude of the Ω vibrations is approximately proportional to the winding angle which will change with time because this angle is different for layers that are unwinding from front towards rear edge and those that are unwinding as the unwinding point moves from the rear towards front edge.

The effect of the apex angle of the cone on the dimensionless velocity is shown in Fig. 6. It is clear that the effect of the typical apex angle on the dimensionless angular velocity is negligible for all practical purposes.

5. Conclusion

We show that the winding angle influence on the vibrations in yarn unwinding from packages. We have derived the expressions for the time dependence of the dimensionless angular velocity of the balloon rotation around the package axis, during the yarn unwinding from cylindrical and conical packages. In both cases the unwinding in the backward direction is faster than the unwinding in the forward direction, because for the unwinding in the backward direction the dimensionless angular velocity is higher. We show dependence dimensionless angular velocity of the winding angle. During unwinding the values of vibration between unwinding backward and forward. By increasing the winding angle the dimensionless angular velocity is getting higher. We showed that large vibrations of dimensionless angular velocity occur during yarn unwinding. We also showed that the amplitude of dimensionless angular velocity increases with large winding angle.

To reach lower vibrations in the yarn , we need to respect winding angle, because it has minor act at unwinding ($5° \leq \phi \leq 10°$). Optimization could reduce costs in many textile processes.

In no case can we avoid the oscillations of dimensionless angular velocity near the edges of a package, when the direction of motion of unwinding and lift-off points changes. Our future research will be based to the ration between the difference of the dimensionless angular velocity for unwinding in either direction and their average. In cross-wound packages the difference between the two dimensionless angular velocities may be up to a few ten percent, therefore the influence of the winding angle is sizable .

6. References

Barr, A. E. D., Catling, H. (1976). Manual of Cotton Spimmimg, Volume Five. Butterworth

Fraser, W. B., Ghosh, T. K., Batra, S. K. (1992). On unwinding yarn from cylindrical package. Proc. R. Soc. Lond. A, 436 479-498.

Fraser, W. B., (1992). The effect of yarn elasticity on an unwinding ballon. J. Tex. Inst, 83 603-613.

J.D.Clark, W.B.Fraser, R. Sharma and C.D.Rahn. (1998). The dynamic response of a ballooning yarn:theory and experiment. Proc. R. Soc. Lond. A, 454 2767-2789.

Kothari,V.K.,Leaf G.A.V. (1979). The unwinding of yarns from packages, PartII: The theory of yarn-unwinding.J.Text.Inst 70 (3)95-105.

Mack, C. (1953). Theoretical study of ring and cap spinning balloon curves(with and without air drag).J.Text.Inst 44 483-498.

Padfield, D. G. (1956). The Motion and Tension of an Unwinding Thread. Proc. R. Soc., vol. A245, 382-407.

Praček, S. Sluga, F. Možina, K. Franken, G. (2011). Unwinding from cylindrical packages, Annals of DAAAM for 2011 & Proceedings of the 22 st International DAAAM Symposium, 23-26th November 2011, Vienna, Austria, ISSN 1726-9679, ISBN 978-3-901509-83-4, Katalinic, B. (Ed), pp. 0917-0918, Published by DAAAM International Vienna, Vienna

Praček, S. Sluga, F. Možina, K. Franken, G. (2011). Cylindrical packages simulations, Annals of DAAAM for 2011 & Proceedings of the 22 st International DAAAM Symposium, 23-26th November 2011, Vienna, Austria, ISSN 1726-9679, ISBN 978-3-901509-83-4, Katalinic, B. (Ed), pp. 0919-0920, Published by DAAAM International Vienna, Vienna