CALIBRATION OF TAPE MEASURES WITH SMALL MEASUREMENT UNCERTAINTY

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Abstract: *Tape measures are probably the most wildly used length measuring instruments in the world. The measures, used by surveyors, contractors and other professionals in building and construction industry, measurements of land areas, legal investigations etc. have to be calibrated and for legal purposes verified, to ensure traceability and avoid various controversies, misalignments and other problems. In the paper the precise calibration procedure is presented. In spite of extremely small measurement bench with the length of only three metres, by the use of precise measurement equipment (laser interferometer and video system) and thorough uncertainty evaluation based on experimental and theoretical work, very small calibration uncertainty is achieved. The procedure was successfully accredited and accepted as Calibration and measurement capability (CMC) into key comparison database at BIPM.*

Key words: *tape measure, calibration, measurement, uncertainty*

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1. Introduction

Among other, more precise length measuring instruments, most length calibration laboratories also perform tape measures calibrations. Usually long measurement benches, build specially for this purpose, are used for the task. Its length ranging in from several metres up to 30 m, modern measurement bench is equipped with linear position encoder and cart with a positioning system. Positioning systems differ largely, starting from simple magnifying glass over optical microscope to enhanced video systems with line recognition.

For a small calibration laboratory purchase of a special tape calibration bench was not economically viable, so we adapted our length measuring device Zeiss ULM 3000. The ULM was equipped with existent laser interferometer, video probing system (VPS) and fixtures for tapes and weights.

2. Calibration procedure

2.1 Application and limitations of the procedure

Tape measures, in legal documents called "material measures of length" (in the following text "measures"), are simple instruments comprising scale-marks whose distances are given in legal units of length. They can be used in legal and non-legal metrology. The procedure concerns different kinds of measures, as defined by the Measuring instruments directive (MID, 2004). Steps taken when calibrating measures up to 200 m will be described. Different designs of measures influence the way of fixing the measure for the calibration. Tractive force for loading the measures during calibration is for some measures stated on the measure, for all others it is prescribed by MID. This procedure and its uncertainty is limited for the use of the LI, but no major changes are needed when using calibration bench with an encoder.

2.2 Pre-calibration tasks

Prior to calibration, measure has to be visually checked for any obvious defects like scratches or other defects (e.g. corrosion), which would impede the calibration. Measure is cleaned with alcohol and checked, if graduation and numbering is complete, readable and undeletable. Temperature stabilisation at 20 °C ± 0,5 °C is performed for minimum of six hours.

2.3 Fixation of the measure

The measure is fixed in accordance with the kind of the measure in a special fixture. The measures differ substantially at the end and can be divided in three groups:

- Measures with end hook, ring or handle;
- Measures with floating tang (for both inside and outside dimensions measurement);
- Dipping tapes with sinkers.

While at the measure with a floating tang or a sinker a measurement is bounded by a tang's surface or sinker's tip, scale at the measure with end hook starts several centimetres after the hook. Fixing of this end of the measure must be carried out accordingly to the measure end; special fixtures are needed.

Because of the bench length of only 3 m, measures longer than 3 m are measured in more steps. In the second and in the following steps the measure is fixed with clamps. On the free hanging end the measure is loaded with a weight that corresponds to the specified tractive force. If tractive force is not specified by manufacturer and marked on the tape, for measures of 5 m and longer MID prescribes the tractive force of 50 N. The tractive force is established by the fixing of the weight on the loose end of the measure.

Dipping tapes with sinkers are loaded with a weight, that equals the sinker's mass (normally 0,5 kg or 1 kg).

2.4 Adjusting the measure

Before measurement the measure shall be adjusted parallel to the axis of measurement. The adjustment is performed using the horizontal (x) axis of the coordinate cross of VPS. The upper line of the double cross is positioned on the top of the reference mark of the measure. After that the VPS is positioned to the end mark of the measure. It is checked whether the line is touching the top of the mark on the measure. If this is not the case, the measure is moved slightly until the cross line touches the top of the mark. The procedure is repeated so until the cross touches both marks (reference and the end mark) on the top.

2.5 Measurement

Distances between the zero- mark and 10 to 30 random scale-marks including the end- mark are measured. The number of measuring points depends on the length of the scale. The positioning system is always set to the axis of a mark as shown in Figure 5 using double co-ordinate cross. The light slots on both sides of the line shall be equal.

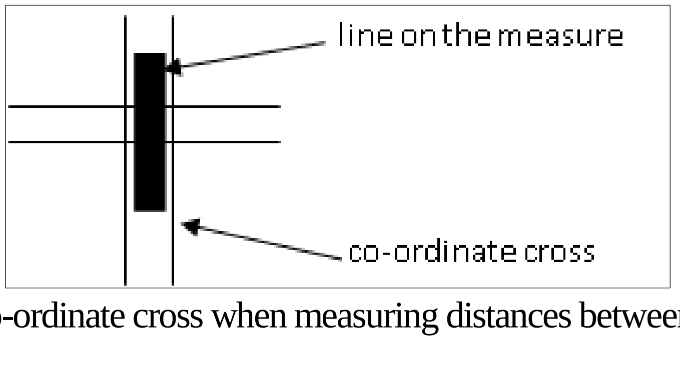

Fig. 1. Position of the co-ordinate cross when measuring distances between scale marks

If the zero point is defined by an end surface, the reference scale-mark for the measurement (scale-mark from which distances are measured) is the closest mark indicating a round measure (e.g. 10 mm, 50 mm, etc., depending on the type of the measure).

Measured values are recorded by LI software and copied into Excel file for final calculations, after the measurement is completed.

3. Correction of the measurement result

3.1 Correction of the measure's thermal extension

Temperature of the measure is measured on the base plate in two points using the material temperature contact sensors of LI. LI's software carries out on-line correction of thermal extension.

3.2 Correction of Abbe error

The angle between the guide on the bench and the cart carrying VPS and LI optics was measured by electronic level. Two levels were positioned as shown in Fig. 4, first fixed on the guide, while second was on the cart that was moved along the whole length of the bench. Maximum Abbe error, caused by the pitch of the cart, was calculated and the value taken into the uncertainty evaluation.

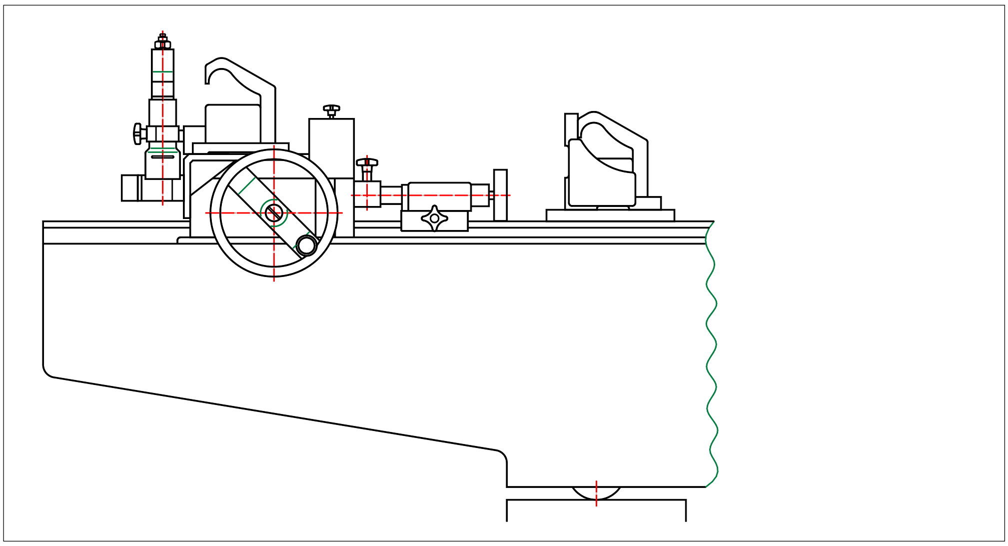

Fig. 2. Measurement of the pitch angle of the cart carrying VPS and LI optics

4. Measurement uncertainty

4.1 Mathematical model of measurement

Deviation *e* (measurement result) is given by the expression (Acko, 2012):

$$e = L_m \cdot (1+\alpha_m \cdot \theta_m) - L_{LI} + e_{cos} \cdot + e_{mp} + e_a - e_F \qquad (1)$$

where:

- e - deviation (measurement result) at 20 °C
- L_m - path length between the reference position of the video probing system and the measurement position of the video probing system
- α_m - linear temperature expansion coefficient of material measure of length
- θ_m - temperature deviation of the measure of length from 20 °C

L_{LI} - corrected length shown by LI
e_{cos} - cosine error of measurement (supposed to be 0)
e_{mp} - dead path error (supposed to be 0)
e_a - Abbe error caused by angular deviation of the video probing system (supposed to be 0)
e_F - error due to the tractive force

4.2 Standard uncertainties of the estimations of the input values

Equation (4.1) in (EA-4/02, 1999) in our case gets the following form:

$$u_c^2(e) = c_{Lm}^2 u^2(L_m) + c_{\alpha m}^2 u^2(\alpha_m) + c_{\theta m}^2 u^2(\theta_m) + c_{LLI}^2 u^2(L_{LI}) + \\ + c_{ecos}^2 u^2(e_{cos}) + c_{emp}^2 u^2(e_{mp}) + c_{ea}^2 u^2(e_a) + c_{eF}^2 u^2(e_F) \quad (2)$$

c_i are partial derivatives of the function (1):

$$c_{Lm} = \partial f/\partial L_m = 1+\alpha_m \cdot \theta_m \approx 1;\ \text{if}\ \theta_{max}=\pm 1\ °\text{C} \quad (3)$$
$$c_{\alpha m} = \partial f/\partial \alpha_m = \theta_m \cdot L_m \quad (4)$$
$$c_{\theta m} = \partial f/\partial \theta_e = \alpha_m \cdot L_m \quad (5)$$
$$c_{LLI} = \partial f/\partial L_{LI} = -1 \quad (6)$$
$$c_{ecos} = \partial f/\partial \delta e_{cos} = 1 \quad (7)$$
$$c_{emp} = \partial f/\partial e_{mp} = 1 \quad (8)$$
$$c_{ea} = \partial f/\partial e_a = 1 \quad (9)$$
$$c_{eF} = \partial f/\partial e_F = 1 \quad (10)$$

Standard uncertainties of influence (input) values are calculated (estimated) for applied equipment and method as well as for supposed measurement conditions.

4.2.1 Uncertainty of the path length between the reference position of the video probing system and the measurement position of the video probing system $u^2(l_m)$

The uncertainty is composed of the positioning uncertainty in the reference point $u(pos_{ref})$ and of the positioning uncertainty in the measurement point $u(pos_{mea})$. It is supposed that both uncertainties are equal: $u(pos_{ref}) = u(pos_{mea}) = u(pos)$ (Druzovec et al., 2009). Total uncertainty is then:

$$u(l_m) = u(pos) \cdot \sqrt{2} \quad (11)$$

Standard positioning uncertainty was determined by statistical evaluation (Acko, 2003). Three persons have made more than 60 positioning into reference point. Standard deviation of these measurements that is accepted as standard uncertainty was:

$$s = u(pos) = 1{,}8\ \mu\text{m} \quad (12)$$

Since the measurements for determining positioning uncertainty were made on the standard line scale with better line quality and since measurements on end surfaces shall be made by using single co-ordinate cross, the determined standard uncertainty was be increased by factor 2. Total standard uncertainty is therefore:

$$u(L_m) = 5{,}1\ \mu\text{m} \tag{13}$$

Furthermore, if the measure is longer as 3 m, it should be measured in more steps. When repositioning the measure, additional positioning uncertainty appears. In such cases additional component of $u(L_{m3})$ = 3,6 μm is added for every 3 m segment.

4.2.2 Uncertainty of linear temperature expansion coefficient $u(\alpha_m)$

Because scales can be made of different materials, a deviation interval of $\pm 4\cdot 10^{-6}$ °C^{-1} is estimated (Druzovec et al., 2008). Standard uncertainty at supposed rectangular distribution is:

$$u(\alpha_m) = (4\cdot 10^{-6}\,°\text{C}^{-1})/\sqrt{3} = 2{,}3\cdot 10^{-6}\,°\text{C}^{-1} \tag{14}$$

4.2.3 Uncertainty of the temperature deviation $u(\theta_m)$

We shall consider the standard uncertainty of temperature measurement, which in our case is 0,005°C (Godina et al., 2010) and uncertainty of the difference between the temperature of fixing base (where the temperature is measured) and the temperature of the tape measure. Standard uncertainty is therefore:

$$u(\theta_m) = 0{,}05\ °\text{C} \tag{15}$$

4.2.4 Uncertainty of LI indication $u(l_{LI})$

Uncertainty of our LI indication was calculated (Acko, 2012) to be:

$$u(l_{LI}) = 0{,}01\mu\text{m} + 0{,}2\cdot 10^{-6}\cdot L \tag{16}$$

4.2.5 Uncertainty caused by cosine error (e_{cos})

Maximum expected value after precise positioning is 0,5 μm/m (Medic et al., 2012). The standard uncertainty is then:

$$u(e_{cos}) = 0{,}25\cdot 10^{-6}\cdot L \tag{17}$$

4.2.6 Uncertainty caused by dead path $u(e_{dp})$

For measurements with LI, this component is negligible.

4.2.7 Uncertainty caused by Abbe error $u(e_a)$

This component is caused by inclinations of the cart, carrying VPS and LI reflector, during the measurement path. Considering equipment dimensions (distance between measure and CCD sensor), cart inclination of 1 μm/m causes the Abbe error of 0,08 μm. The angles were measured with an electronic level; maximum angle difference along the measurement path was 27 μm/m. Therefore, the greatest Abbe error is e_a = 2,2 μm.

If Abbe error is corrected by angle measurements and if it is supposed that the uncertainty is composed of the uncertainty caused by the level *u*(*lev*) and of the uncertainty of determination of the error dependence from the angle *u*(*d*), total uncertainty is calculated by the equation:

$$u(e_{a1}) = \sqrt{u(lev)^2 + u(d)^2} \tag{18}$$

If standard uncertainty of the level is *u*(*lev*) = 1 μm/m (calibration certificate), than *u*(*lev*) = 0,08 μm (from test measurements). Uncertainty of determination of the error dependence from the angle was calculated as standard deviation of repeated test measurements and is *u*(*d*) = 0,05 μm.

Total uncertainty is than: $u(e_{a1})$ = 0,09 μm.

If it is considered that the Abbe error is corrected twice (in zero point and in measurement point), than the final result is:

$$u(e_a) = u(e_{a1}) \cdot \sqrt{2} = 0{,}13\ \mu\text{m} \tag{19}$$

4.2.8 Uncertainty due to the tractive force u(eF)

A short test on a plastic tape has shown that an increase of the tractive force of 10 N (when added to the nominal force of 20 N) causes an extension in the 2 m tape segment of approx. 800 μm. Additional tests with 200 g and 1200 g weights have shown linear relation between the force and the extension. If relative deviation of the tractive force due to the uncertainty of the weight and friction is assumed to be within the limits of ±0,3 % (0,06 N at the nominal force of 20 N), the change in tape length would be within an interval of ±2,4 μm/m. Since the deformation interval of plastic tapes is greater than of metal tapes, it can be used for all materials. Standard uncertainty at supposed rectangular distribution is therefore:

$$u(e_F) = (2{,}4\ \mu\text{m/m}) / \sqrt{3} = 1{,}4\ \mu\text{m/m} \tag{20}$$

or

$$u(e_F) = 1{,}4 \cdot 10^{-6} \cdot L \tag{21}$$

Since exact circumstances by repositioning of the tape and the influence of the generated force on the tape extension and bending over a small hanging wheel, especially with harder tapes, are not well known, the standard uncertainty is increased. The following value is put in the uncertainty budget:

$$u(e_F) = 2 \cdot 10^{-6} \cdot L \tag{22}$$

4.3 Combined standard measurement uncertainty

Combined standard uncertainty of the estimations of the input values in the best possible measurement conditions can be expressed by the equation (calculated from Tables 1 and 2):

$$u = 5{,}1\ \mu m + 1{,}8 \cdot 10^{-6} \cdot L \tag{23}$$

This uncertainty is however valid only for measures up to 3 m. If the standard uncertainty is calculated for the longest measure of length, which is 200 m, the value would be:

$$u(200\ m) = 631{,}6\ \mu m \tag{24}$$

Value X_i	Estimated value	Standard uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
L_m	1 mm	5,1 μm	normal	1	5,1 μm
α_m	$10 \cdot 10^{-6}$°C^{-1}	$2{,}3 \cdot 10^{-6}$°C^{-1}	rectangular	$0{,}3 \cdot 10^{3}$ μm°C	0,0007 μm
θ_m	0°C	0,22°C	normal	0,01 °C^{-1}μm	0,0022 μm
L_{LI}	1 mm	0,01 μm	normal	1	0,01 μm
e_{cos}	0	$0{,}25 \cdot 10^{-3}$ μm	normal	1	0,0025 μm
e_a	0	0,13 μm	rectangular	1	0,13 μm
e_F	0	0,002 μm	rectangular	1	0,002 μm
				Total	**5,1 μm**

Tab. 1. Standard uncertainties on the lower limit of measurement range (1 mm)

Value X_i	Estimated value	Standard uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
L_m	3000 mm	5,1 μm	normal	1	5,1 μm
α_m	$10 \cdot 10^{-6}$°C^{-1}	$2{,}3 \cdot 10^{-6}$°C^{-1}	rectangular	$0{,}9 \cdot 10^{6}$ μm°C	2,07 μm
θ_m	0°C	0,22°C	normal	30 °C^{-1}μm	6,6 μm
L_{LI}	3000 mm	0,61 μm	normal	1	0,61μm
e_{cos}	0	0,75 μm	normal	1	0,75 μm
e_a	0	0,13 μm	rectangular	1	0,13 μn
e_F	0	6 μm	rectangular	1	6 μm
				Total	**10,5 μm**

Tab. 2. Standard uncertainties on the upper limit of measurement range (3000 mm)

In this case additional positioning uncertainties, which appear every three metres, were considered. If the combined uncertainty is calculated by linearization of the equation (calculations for points 1mm and 200 m), combined standard measurement uncertainty is:

$$u = 5{,}1\ \mu\text{m} + 3{,}1\cdot 10^{-6}\cdot L \quad (25)$$

When considering measures up to three metres, the difference between equations (23) and (25) is negligible, so equation (25) will be used for all measures.

4.4 Expanded measurement uncertainty

Factor k=2 is used for the calculation of the expanded uncertainty. Considering the experiences and the results of several international comparisons in which our laboratory participated, the expanded uncertainty of measure calibration it is rounded up to:

$$U = 10\ \mu\text{m} + 7\cdot 10^{-6}\cdot L \quad (26)$$

5. Traceability

Measurement equipment used for calibration:

- Universal length measuring device Zeiss ULM 3000 - no calibration necessary (only used as a base)
- Video probing system (VPS) Renishaw VP 2- no calibration necessary (used only for positioning)
- Laser interferometer (LI) HP 5528 A - calibrated in an accredited laboratory UM/FS/LTM
- weights 0,5 kg, 1 kg, 2 kg in 5 kg - checked by weighing at UM/FS/LTM

6. Conclusion

Procedure, presented above, was already successfully accredited and accepted as Calibration and measurement capability (CMC) into key comparison database at BIPM (***a). CMC's in key comparison database at BIPM are quantities, for which calibration and measurements certificates are recognized by institutes participating in the CIPM Mutual Recognition Arrangement (MRA), from 85 world countries (***b).

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***a http://kcdb.bipm.org/AppendixC/country_list.asp?Iservice=L/DimMet.2.3.7 - BIPM Key Comparison Database, Appendix C: Calibration and Measurement Capabilities of National Metrology Institutes, Length. *Accesed on: 2012-09-03*

***b http://www.bipm.org/en/cipm-mra/ - CIPM Mutual Recognition Arrangement. *Accesed on: 2012-09-03*

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