MODELING AND COMPUTATIONAL ANALYSIS OF MACHINE TOOL SPINDLE UNITS

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Abstract: The paper discusses the problem of modeling and analysis of machine tool spindle units. A computational method is presented, which enables the separation and optimization of the constructional variables of any variant of the spindle unit. The computational analysis employed the statistical method of constructional variables separation and the optimization of significant variables by the Monte Carlo method. The separation of the constructional variables and the optimization of the significant variables of the spindle unit of a medium-size turning lathe was made. For the real model of the turning lathe spindle unit, a modified solution is proposed, which is characterized by better properties. In order to verify the obtained results, an attempt was made to analyze the spindle unit using the finite element method (FEM) using an off-the-shelf computer program.

Key words: spindle units, constructional variables, significance of variable, optimization, FEM

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1. Introduction

In computer aided design of machine tool spindle units one of the important stages is the analytical determination of the evaluation indices of their properties and capabilities with an aim to make sure that the proposed design solution is accurate in static, dynamic, and thermal points of view. The dependence of the spindle unit evaluation indices on the design variables that describe the physical (computational) model is very complex. It is difficult to determine a priori those variables which are to be changed in order to obtain an improvement in the value of the evaluation indices. This entails the necessity of defining those variables in the design process, which have the greatest effect on the values of the evaluation indices. A separated subset of significant design variables should represent decision variables in the successive stages of modification or optimization.

A computational method has been developed, which enables the decision variables to be separated out of the design variables of any arbitrary variant of the spindle unit (SU) of a machine tool. The separation of the design variables is based on the statistical methods of multiple linear regression and the procedure of the elimination of model variables by the a posteriori method. The method of the separation of spindle unit design variables is presented on the example of the existing constructional solution of a medium-size machine tool.

In the computational analysis, a statistical method of design variables separation (Wolny, 1992) and a procedure for the optimization of significant variables by the Monte Carlo method were used. The verification of the obtained computation results was performed by modeling the spindle unit by the FEM method using an off-the-shelf computer program.

2. Method of the separation of spindle unit design variables

For the separation of the design variables of the spindle unit, statistical methods were used, which, as combined with the computer aiding methods, form the applied theory of experiment. The experimental quantities were replaced by a subset of the spindle unit design variables (as generated using a generator of pseudo-random numbers with an equal probability distribution) and that of the results of calculations performed for the cases of matching the above variables (Wolny, 1992).

A block diagram of the method of separating the design variables of the spindle unit is shown in Fig. 1. The prepared set of input data is entered to the computational system either from a computer keyboard or by reading out the disk file. After entering the data, a mathematical model should be defined, which best fits the set of input values. The mathematical model may be any linear model or a nonlinear model that is reducable to a linear one. It is also required to enter the limiting value of the significance coefficient, \( \alpha_{\text{lim}} \), as the so called elimination level.

The separation of the SU design variables was based on the multiple linear regression method and on the procedure of model variable elimination by the a posteriori method. The main computational block of the statistical methods used performs:
• Calculations of the determination coefficient $R^2$, and its significance level $\alpha_F$;
• Calculations of regression coefficient $A_i$, for particular independent variables;
• Calculations of the confidence levels of regression coefficients using Gauss’ multipliers;
• Calculations of the confidence level $\alpha_i$, of regression coefficients $A_i$, for the design variables.

Fig. 1. Block diagram of the method of spindle unit design variables separation
The evaluation of the significance $\alpha_F$ of determination coefficient $R^2$ and comparison with the limiting significance level $\alpha_{lim}$ are a measure for matching the mathematical model to the set of input data. Of the two mathematical models whose significance levels $\alpha_F$ are of the same order of magnitudes, the one with a greater value of the determination coefficient fits better to the set of input values. In the case where $\alpha_F > \alpha_{lim}$, the computation procedure will send out the “wrong model” message, after which a change in the initially adopted model should be made. The calculation of the regression coefficients $A_i$ and design variables together with their significance $\alpha_i$, as against the limiting value of the significance level $\alpha_{lim}$, form a basis for the elimination of insignificant variables by the a posteriori method.

If the significance of a particular regression coefficient of the design variable $A_i$ is greater than the elimination level $\alpha_{lim}$ ($\alpha_i > \alpha_{lim}$), then that variable is assumed to be insignificant. Upon passing on to the procedure of eliminating the model variables by the a posteriori method, the calculations proceed in the following stages:

I. Calculations of the regression equation containing all independent variables;

II. Testing of the significance of all equation variables using the $t$ - Student test;

IIa. If there are variables for which $\alpha_i > \alpha_{lim}$, then the removal of a variable with the greatest value of $\alpha_i$ occurs followed by a return to point I;

IIb. If for all variables $\alpha_i \leq \alpha_{lim}$, then the procedure terminates the calculations and adopts a regression equation according to the final calculation.

The result of using the method of the separation of $SU$ design variables are regression equations containing all independent variables, the most important design variables of the spindle unit. The separated significant design variables of the spindle unit, while considering the adopted separation criterion, should represent decision variables in the successive stages of modification or optimization.

### 3. Example of using the method of the separation of spindle unit design variables

The separation of the design variables of the spindle unit was made on the example of a medium - size machine tool. For the existing constructional solution (Fig. 2.a), a substitute $SU$ model was developed (Fig. 2.b).

The computational analysis was carried out in five different points of position of the cutting forces in three lathe working spaces: A, B, and C. In the performed analysis, the region of the working spaces was separated from the spindle tip by the chuck length, $l_u = 200$ mm (Fig. 3). The component and resultant values of spindle unit displacement were determined in the points of position of the cutting forces in three regions of the working spaces. The values of the active loads in the substitute lathe $SU$ model were determined for a representative cutting rate of $V_{rep} = 140$ m/min in straight turning (Weck, 1985).

Based on the substitute lathe $SU$ model (Fig. 2.b), a simplified model was built (Fig. 4), which was then assumed as a basis for further considerations.

As a result of the analysis of the existing $SU$ constructional solution, and to facilitate any changes to be made, an initial set of ten design variables has been established. For the fixed set of $SU$ design variables, a plan of numerical
experimentation was set up. The region of the permissible values of particular variables was limited by the minimum and maximum values of the variables. Having in mind the constructional limitations of the existing lathe SU solution, variable variation ranges were defined for the case of the simplified model under analysis (Fig. 4):

1. $125 \leq DZW2 \leq 145$ with a step 1 mm;
2. $150 \leq DZW3 \leq 180$ with a step 1 mm;
3. $476 \leq ZP2 \leq 526$ with a step 1 mm;
4. $295 \leq ZS1 \leq 345$ with a step 1 mm;
5. $872 \leq ZKO \leq 922$ with a step 1 mm;
6. $2.87 \leq WKS \leq 9.15$ with a step 0.175 rad;
7. $500 \leq CP2 \leq 2500$ with a step 150 N/\(\mu\)m;
8. $200 \leq CP3 \leq 1000$ with a step 50 N/\(\mu\)m;
9. $200 \leq CP4 \leq 1000$ with a step 50 N/\(\mu\)m;
10. $200 \leq CP5 \leq 1000$ with a step 50 N/\(\mu\)m.

Fig. 2. Machine tool spindle unit: a) construction of a real system, b) substitutive model
With the fixed variation ranges, thirty values for each variable were generated using the generator of pseudo-random numbers with equal probability distribution, while assuming a specified step of discretization. For the generated variable values, static displacements of the axis $SU$ were calculated in points where this axis intersects a plane normal to it, which passes through the five points of application of the cutting forces in three regions of working spaces (Fig. 3) – according to the computational methodology.

The separation of the ten design variables of the lathe $SU$ was made based on the statistical method described in section 2. The input data for the computation system were sets of design variable values generated according to the adopted numerical experimentation plan, as well as the values of $SU$ axis static displacements as calculated in five points of the three working space regions: A, B, and C (Fig. 3).

For the entered data, a mathematical model was defined, which fitted best to the sets of input quantities. In the case of the numerical experiment under consideration, a decision was made to select an algebraic polynomial with linear summands. Using the computational capabilities of the separation method, a statistical verification of the significance of the particular ten design variables of the spindle unit was performed.

The procedure of eliminating insignificant $SU$ variables was carried out according to the a posteriori method for the limiting value of significance, $\alpha_{\text{lim}} \leq 0.05$. This is a representative significance level value that is used in machine construction engineering.

Fig. 3. Coordinates of cutting force application points in the areas A, B, and C of the field of machining
Those $SU$ design variables for which the significance of approximating function coefficients was greater than the limiting significance level were omitted in the elimination procedure. The result of the statistical procedure were regression equations containing the most significant $SU$ design variables for five points of the working spaces regions: A, B, and C.

As a criterion of $SU$ design variables separation the value of the radial component of relative displacement between the work piece and the tool was adopted in the actual position of the application of the cutting forces. The displacement value determines the dimensional and shape accuracy of the turned surface.

Fig. 4. A simplified model of the machine tool spindle unit

The radial component of relative displacement between the work piece and the cutting tool was assumed to be equal to the component tangential displacement, $ZOY$, of the spindle unit.

It has been found on the grounds of the analysis of the regression equations that of the initial set of ten $SU$ variables the following five design variables are significant to the working area region, A, under consideration: $DZW2$, $ZKO$, $WKS$, $CP4$, $CP5$.

For the remaining two working area regions, B and C, it has been found that the following six significant variables occur in the set of ten $SU$ design variables: $DZW2$, $ZKO$, $WKS$, $CP2$, $CP4$, $CP5$. 

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The separation of significant lathe \(SU\) variables constitutes a basis for undertaking studies on the modification and optimization of the existing constructional solution. A modification of the \(SU\) constructional solution should proceed towards increasing the spindle diameter on the inter-support segment (\(DZW2\)), increasing the lateral rigidity of the centre and front supports bearings (\(CP2, CP4,\) and \(CP5\)), shortening the front spindle tip length (\(ZKO\)), and changing the angular coordinate of application of the resultant drive force (\(WKS\)). The values of the modified \(SU\) constructional solution should be contained within the previously established ranges of variation. The separation of significant variables out of the initial set of ten variables reduces considerably the size of the optimization task. The significant \(SU\) design variables should be assumed to be decision variables, whereas the regression equations should form the simple mathematical models of objective function, in accordance with the adopted variable separation criterion.

4. Optimization of the significant design variables

The application of the computational optimization methods required the decision variables, the limiting conditions and the optimization criterion to be defined, which were presented in a function form, i.e. as the objective function (Findeisen, 1980). For the case of optimization of design \(SU\) variable values in the area \(C\) of the machining field, the separation of six significant variables out of the initial set of ten variables considerably reduced the size of the computational task. The significant variables \(SU\) were taken as decision variables, while considering the established limitations (the actual headstock size), whereas the regression equations formed simple mathematical models of the objective function.

The tasks of optimization of significant machine-tool \(SU\) variables were carried out by the Monte Carlo method available in the CADEX: PROFES program package for the number of iterations \(N=10000\) (Polanski, 1992). After separating the significant variables and based on the optimization results an attempt was made to modify the actual machine-tool \(SU\) design solution.

Three solutions resulted, which were a trade-off between the computational analysis results and the design and constructional expertise and experience. Ultimately, a modified \(SU\) solution was proposed, which was characterized by the best static properties.

5. The analysis of the spindle unit by the finite element method

Based on the equivalent system of the spindle unit, a simplified model \(SU\), of the turning lathe was built (Fig. 5), which constituted a basis for the computational analysis by finite element method in the COMSOL computer system used. The geometry of the spindle model was created using CAD tools available in the Structural Mechanics module. The available 3D primitives in the form of cylinders were used, which were connected into a single object.

The active loads in the form of generalized forces resulting from carrying out the cutting process and a force originating from the drive were modeled.
The passive loads were reactions in the bearing supports modeled as surfaces with a uniformly distributed load corresponding to the rigidity of the bearings. Figure 6 shows deformations of the geometry of the turning lathe model SU analyzed. Locations which are the most susceptible to static deformations under the active loads are marked by change in color (Wolny, 2003).

![Fig. 5. Simplified model of lathe spindle unit for MES analysis](image1)

![Fig. 6. Geometry deformation of lathe spindle unit model](image2)

The performed SU computational analysis of the turning lathe has confirmed the rightness of use of the FEM as a means of verification of previously used computational methods.
6. Conclusion

The performed computational analysis of the design variables of the spindle unit (SU) of a medium-size turning lathe has shown that the separation of the design variables depends on:

- The adopted state of load,
- The established separation criterion, and
- The determination of the location where the value of displacement is to be calculated.

The state of load of the spindle unit should correspond to the real conditions existing during machining. This state is defined, in addition to all values describing the vectors of cutting forces, also by the coordinates of the origin of these forces. As a representative origin of the vectors of cutting forces in the working space region, a point should be assumed, which corresponds to the maximum in the frequency of occurrence of these forces in the lathe working space under consideration. As a criterion for the separation of SU design variables, the value of the radial component of relative displacement between the work piece and the tool should be assumed in the representative point of action of these forces. This displacement value determines the dimensional and shape accuracy of surfaces being turned.

The computational analysis of spindle units presented in the paper, consisting in the use of the design variables separation method, analytical optimization methods and computational verification by the finite element method (FEM), should constitute an integral part of computational systems in the computer-aided process of designing machine tools and other technological machines.

7. References


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