DOI: 10.2507/34th.daaam.proceedings.072

# NUMERICAL MODEL FOR IDENTIFICATION OF FIBER-REINFORCED CONCRETE PARAMETERS FROM THE CRACK-MOUTH OPENING DISPLACEMENT

Ivica Kožar & Tea Sulovsky

**This Publication has to be referred as:** Kozar, I[vica] & Sulovsky, T[ea] (2023). Numerical Model for Identification of Fiber-Reinforced Concrete Parameters from the Crack-Mouth Opening Displacement, Proceedings of the 34th DAAAM International Symposium, pp.0544-0550, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-902734-41-9, ISSN 1726-9679, Vienna, Austria
DOI: 10.2507/34th.daaam.proceedings.072

**Abstract**

Determining material parameters is crucial for understanding their physical properties, improving the design process, and optimizing costs. Due to the challenges of measuring parameters in fibre-reinforced concrete, numerical models and inverse analysis are employed for parameter identification. Laboratory tests, including bending and compressive strength tests, were conducted on concrete samples, recording three parameters: vertical displacement, crack mouth opening displacement, and applied force. The laboratory test results served as a reference for developing a numerical model of micro-reinforced concrete, which generated simulated data used as input for an inverse model based on the Levenberg-Marquardt method. Model verification involved comparing the results with the data obtained from the laboratory tests.

**Keywords:** fiber-reinforced concrete; mathematical model; inverse analysis; parameter estimation

## 1. Introduction

The identification and quantification of parameters for engineering materials play a significant role in improving the understanding of their physical properties, enhancing design methods, and optimizing costs. The process of identifying parameters often requires numerous laboratory tests that can be time-consuming, financially draining, and often inadequate. In the case of fibre-reinforced concrete, characterized by the replacement of traditional steel reinforcement with short fibres, the parameters describing the concrete mixture, steel fibres, and their mutual interaction are often physically immeasurable values.

One of the methods for indirectly determining unknown parameters is the use of computer models that can replace costly experimental tests. However, the heterogeneity of concrete presents a significant limitation when describing this material using numerical methods [1], which is further complicated by the addition of fibres to the mixture. In previous research, two approaches have been used to address this problem: the application of an appropriate finite element model where fibres are discretized and placed along the edges of finite elements [2], and the use of the Fiber Bundle Model (FBM) for composite materials [3].

This paper presents a new deterministic numerical model that, through inverse analysis based on the Levenberg-Marquardt algorithm, identifies the material parameters of fibre-reinforced concrete that affect the overall response of the finished concrete element.

## 2. Laboratory testing

To gain insight into the behaviour of beams made of fibre-reinforced concrete, laboratory tests were conducted. Two tests were performed: three-point bending of beams and testing of the compressive strength of cubes. All test specimens were made using self-compacting concrete. The compressive strength test was carried out on standard concrete samples with dimensions of 150x150x150 mm by applying a load until the cube failure, following the standard for determining the compressive strength of cured concrete test specimens [4]. The measurement results are presented in Fig. 1. and their mean value was taken as the reference value for the compressive strength of concrete.

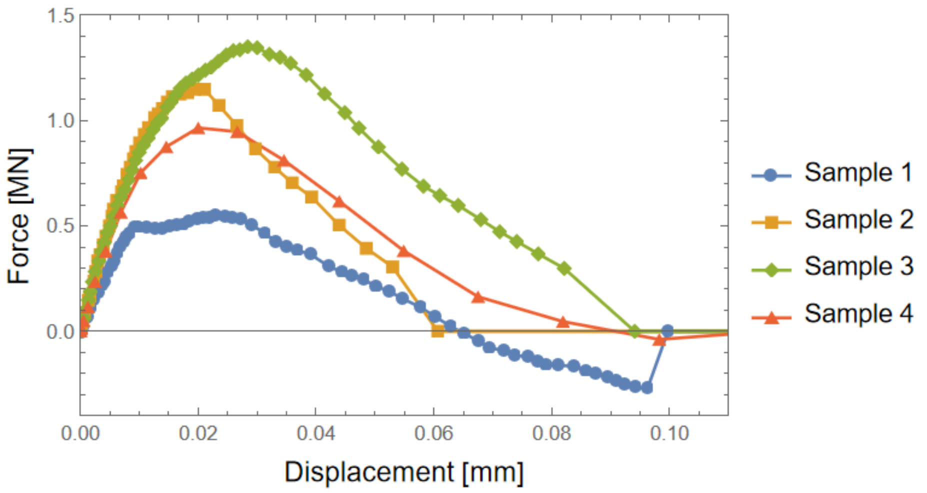


Fig. 1. Concrete’s force-displacement diagram

Samples for three-point bending tests were categorized into three groups: beams without fibres, beams with regular fibres, and beams with rough-surfaced fibres. When preparing fibre-reinforced concrete, fibres are usually added during the mixing of wet ingredients, resulting in their homogeneous but unpredictable distribution within the finished mixture. To eliminate the uncertainty of fibre positions within test specimens, fibres were placed at predefined positions using auxiliary elements before pouring the concrete mixture. After 28 days, the auxiliary elements were removed, leaving a "notch" on the beam that predefined the crack position and facilitated the observation of fibre behaviour during bending. All three-point bending tests were conducted with displacement control until the beam failure, following the standard for determining the flexural strength of cured concrete test specimens [5]. Measured quantities included vertical displacement, crack mouth opening displacement (horizontal displacement), and applied force. The test results, which were later used for the validation of the numerical model, are shown in Fig. 2. to 4.

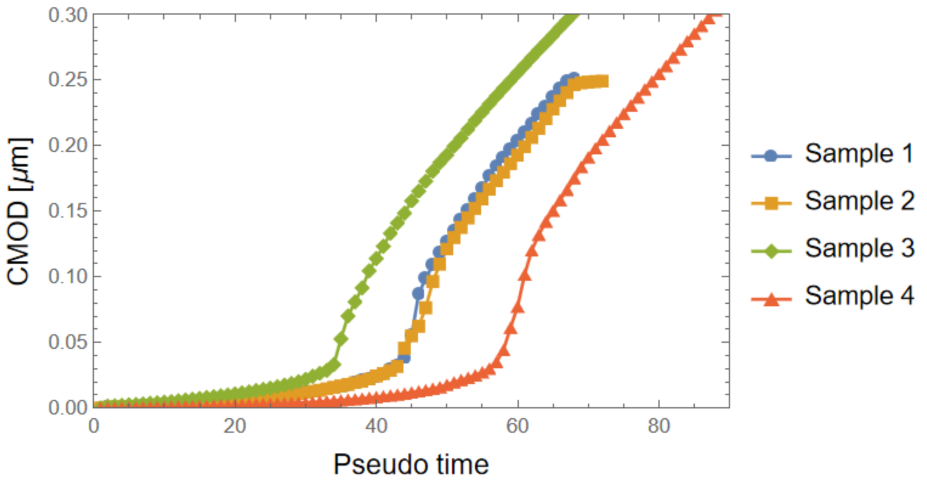


Fig. 2. Crack-mouth opening displacement diagram for beams without fibres

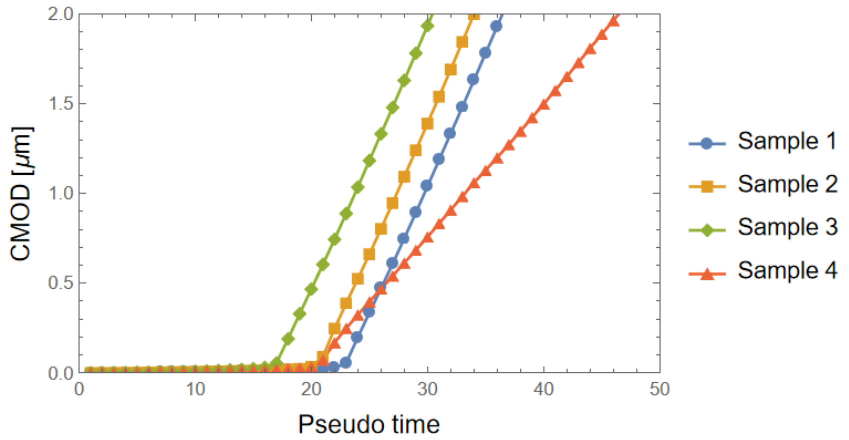


Fig. 3. Crack-mouth opening displacement diagram for beams with regular fibres

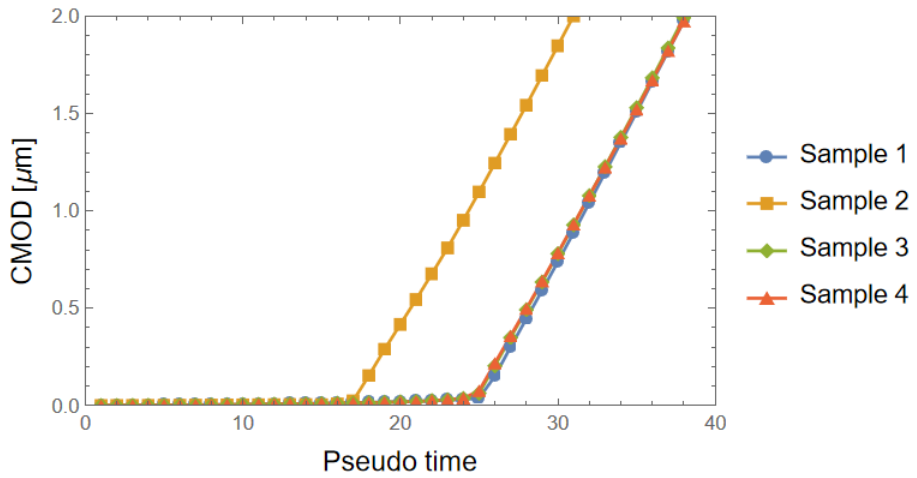


Fig. 4. Crack-mouth opening displacement diagram for beams with rough-surfaced fibres

## 3. Crack-mouth opening displacement

Crack Mouth Opening Displacement (CMOD) is the horizontal deformation of the beam that occurs during bending, measured at the bottom of the crack, as shown in Fig. 5.

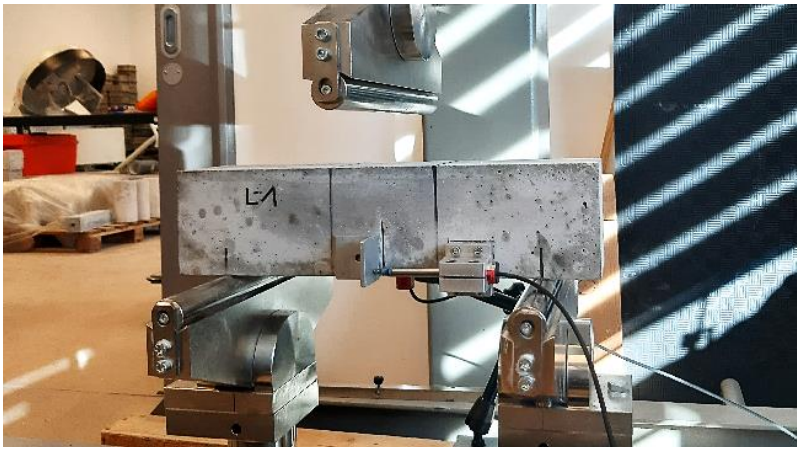

Fig. 5. Three-point bending test configuration with a crack mouth opening displacement gauge installed

Among all the results obtained from laboratory testing, CMOD is the only parameter that directly relates to the cross-sectional and longitudinal aspects of the beam. By analyzing this parameter, it is possible to gain insight into the influence of material parameters, such as those describing fibres in the concrete, on the global response of the beam under loading.

## 4. Mathematical model of fibre-reinforced concrete

The mathematical model of fibre-reinforced in this research is a forward model based on principles laid out by Menke (2018) [6] and it aims to reproduce certain beam behaviour parameters that relate to material parameters and experimental results and is suitable for the formulation of an inverse model [7]. As the heterogeneity of the fibre-reinforced concrete represents a great challenge for numerical modelling, the material is simplified by describing the concrete and the fibres individually, assuming homogeneity of each. Concrete is divided into layers, following the approach introduced by Kožar et al. in their work from 2021 [8], and is mathematically described using the force-displacement diagram:

$$f_b(x, a_t, b_t, a_c, b_c) = \begin{cases} a_c x E_b exp(-b_c x) & if x < 0 \\ a_t x E_b exp(-b_t x) & if x \geq 0 \end{cases} \quad (1)$$

Where $a_t$ and $b_t$ are the parameters describing the behaviour of concrete in tension, $a_c$, and $b_c$ are the parameters describing the behaviour of concrete in compression, and $E_b$ is the modulus of elasticity of concrete. The equation relating displacement and force for steel fibres is as follows:

$$f_a(x, F_t, E_u, E_d) = \begin{cases} x E_b exp(-b_c x) & if x < 0 \\ x E_u & if x > 0 \wedge x < x_{elast} \\ (x - x_{elast}) E_d & if x \geq x_{elast} \wedge x < x_{limit} \\ 0 & if x > x_{limit} \end{cases} \quad (2)$$

The parameters describing the behaviour of fibres are the ultimate force $F_t$ and the modulus of elasticity of steel fibres under loading $E_u$ and unloading $E_d$. Equations (1) and (2) are independent of each other but are related within the equilibrium equations:

$$\begin{aligned} F(\epsilon, \kappa) &= \Delta h \sum_{i=1}^{sloj} f_b[(h_i - \epsilon h) tg(\kappa)] + \Delta a\, f_a(h_a - \varepsilon\, h) = 0 \\ M(\epsilon, \kappa) &= \Delta h \sum_{i=1}^{sloj} (h_i - \epsilon h) f_b[(h_i - \epsilon h) tg(\kappa)] + \Delta a\, (h_a - \varepsilon\, h)\, f_a(h_a - \varepsilon\, h) = 0 \end{aligned} \quad (3)$$

Where the upper equation represents the force balance, and the lower equation represents the moment balance. From the equilibrium equations, the position of the neutral axis of the beam $\varepsilon$ and the crack mouth opening displacement angle $\kappa$ can be determined. These two values are then used in the expression for the crack mouth opening displacement:

$$d_{cmod} = (1 - \varepsilon) \cdot \mathrm{h} \cdot tg(\kappa) \quad (4)$$

Where h represents the height of an individual layer of the beam, the results obtained through numerical modelling of expression (4) are presented in Fig. 6.

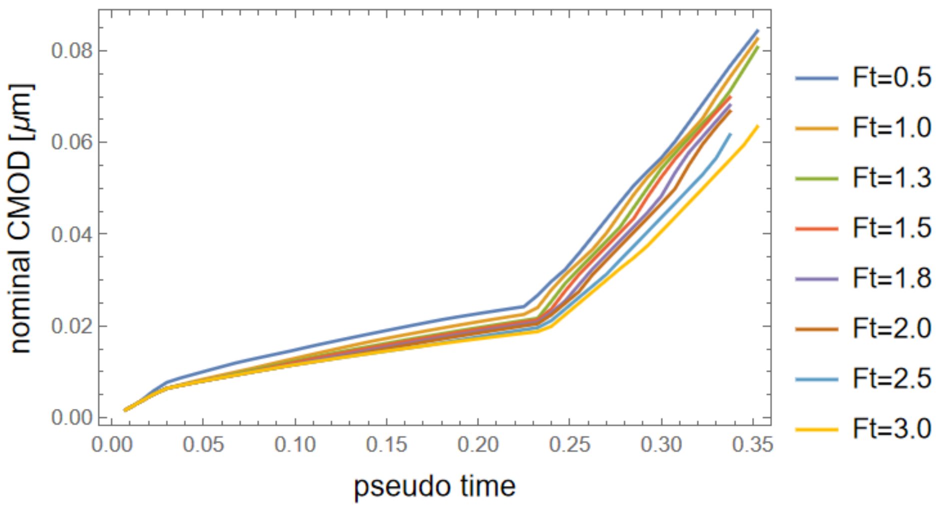


Fig. 6. CMOD Results for $E_b$ = 40 GPa and $F_t$ values ranging from 0.5 kN to 3.0 kN

Obtained results from the forward model were successfully validated using the laboratory testing data, as well as the crack-mouth opening displacement results generated using the data-driven stochastic model formulated by Kožar et al. in their work from 2021 [9].

## 5. Inverse model

Levenberg-Marquardt Method is an iterative technique for solving nonlinear least squares problems to estimate parameters. This method iteratively adjusts a function's parameter until it finds a value that minimizes the error between predicted and actual data. The general notation for the sum of squares in the equation describing the model function fitting, $\hat{y}(x, p)$, to the independent variable $t$ and a vector of $n$ parameters $p$, given a set of m data points $(t_i, y_i)$, is:

$$SS = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (5)$$

Where $y_i$ is the known (measured) data, and $\hat{y}_i$ is the target function. If $\hat{y}(x, p)$ is nonlinear in the model parameters $p$, the minimization process becomes iterative.

$$\frac{\partial SS}{\partial p} = 0 \quad (6)$$

Substituting expression (5) into (6) yields:

$$\frac{\partial SS}{\partial p} = -2\sum_{i=1}^{n}(y_i - \hat{y}_i)\frac{\partial \hat{y}_i}{\partial p} \quad (7)$$

Where $\frac{\partial \hat{y}_i}{\partial p}$ is the Jacobian matrix of dimensions $m$x$n$, representing the local sensitivity of the function to variations in parameter $p$. This is also known as the sensitivity coefficient at each measurement point $\chi_p$ and can be expressed as:

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)\chi_p = 0 \quad (8)$$

The perturbed value of the parameter with each iteration is denoted as $\Delta p$. The target function with $p + \Delta p$ is then written as:

$$\hat{y}_i(p + \Delta p) = \hat{y}_i(p) + \frac{\partial \hat{y}_i}{\partial p}\Delta p \quad (9)$$

When (9) is substituted back into (8), the perturbation value of parameter $p$ is obtained as:

$$\sum_{i=1}^{n}\left(y_i - \hat{y}_i(p) - \frac{\partial \hat{y}_i}{\partial p}\Delta p\right)\chi_p = 0 \quad (10)$$

From expression (10), the value of the perturbation of the parameter $p$ is obtained as:

$$\Delta p = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i(p))\chi_p}{\sum_{i=1}^{n}\chi_p{}^2} \quad (11)$$

## 6. Determination of the material parameters Eb and Ft

The previously described method is applied to determine the values of the modulus of elasticity of concrete $E_b$ and the threshold force of the steel fibre $Ft$. Expression (11) is rewritten as:

$$\Delta E_b = \frac{\sum_{i=1}^{n}(y_i - d_{cmod}(E_b, F_t))\chi_{E_b}}{\sum_{i=1}^{n}(\chi_{E_U})^2} \quad (12)$$

for the minimization with respect to $E_b$, and

$$\Delta F_t = \frac{\sum_{i=1}^{n}(y_i - d_{cmod}(E_b, F_t))\chi_{F_t}}{\sum_{i=1}^{n}(\chi_{F_t})^2} \quad (13)$$

for the minimization with respect to $F_t$. The calculation begins with initial values $E_{b,0}$ and $F_{t,0}$. Expression (4) is used as the target function, and sensitivity coefficients obtained from expression (8) are:

$$\chi_{E_{b,1}} = \frac{y_1 - d_{cmod}(E_{b,0}, F_{t,0})}{\Delta E_b} \quad (14)$$

for the parameter $E_b$ and

$$\chi_{F_{t,1}} = \frac{y_1 - d_{cmod}(E_{b,0}, F_{t,0})}{\Delta F_t} \quad (15)$$

for the parameter $F_t$. Consequently, the expressions for the perturbation of both parameters are as follows:

$$\Delta E_b = \frac{\sum_{i=1}^{n}\left(y_i - d_{cmod}(E_{b,0}, F_{t,0})\right)\chi_{E_{b,1}}}{\sum_{i=1}^{n}\chi_{E_{b,1}}{}^2} \quad (16)$$

for parameter $E_b$, and

$$\Delta F_t = \frac{\sum_{i=1}^{n}\left(y_i - d_{cmod}(E_{b,0}, F_{t,0})\right)\chi_{F_{t,1}}}{\sum_{i=1}^{n}\chi_{F_{t,1}}{}^2} \quad (17)$$

for parameter $F_t$. Adding the initial assumption to expressions (16) and (17) for the respective parameters yields the final iteration values:

$$F_{t,1} = F_{t,0} + \Delta F_t \quad (18)$$

$$E_{b,1} = E_{b,0} + \Delta E_b \quad (19)$$

The process is then repeated with a new iteration and continues until local minimum values are found where both parameters converge. This procedure was applied to the data from Fig. 1., with $E_b$ = 45.0 GPa and $F_t$ = 1.8 kN. Through the inverse analysis, satisfactory results were obtained, with the resulting values of these parameters being $E_b$ = 43.12 GPa and $F_t$ = 1.71 kN, converging within 20 iterations. The resulting values are deemed satisfactory and they validate the inverse procedure.

The Levenberg-Marquardt algorithm's success in finding optimal parameter estimates is sensitive to the choice of initial parameter values which influences the model's convergence. Future work will focus on enhancing the robustness of the initial parameter estimation process to mitigate this sensitivity and improve accuracy, as well as conducting a sensitivity analysis of the obtained parameters. Such analysis aims to determine the individual components and coefficients' influence on the optimization problem's solution and identify those with the most significant impact on the behaviour of the concrete element [10].

## 7. Conclusion

In this study, challenges of identifying material parameters in fibre-reinforced concrete were addressed, a task hampered by the limitations of traditional laboratory testing. A deterministic numerical model which employs the Levenberg-Marquardt algorithm for inverse analysis was introduced. This novel approach efficiently and accurately estimated material parameters, which was specifically presented on the modulus of elasticity of the concrete ($E_b$) and the threshold force of steel fibres ($F_t$). The results mathematically matched the values obtained through laboratory experiments, confirming the viability of the method.

Through this research, the problem of efficient parameter determination using a deterministic model was solved. This effectively eliminates the need for extensive, resource-intensive laboratory tests, reducing both time and cost and paves the way for further optimization and practical applications of fibre-reinforced concrete. Future work will focus on refining the inverse model and conducting a comprehensive sensitivity analysis of material parameters, with the goal of enhancing the performance and reliability of these materials in engineering applications.

## 8. Acknowledgments

This work was supported by project HRZZ 7926 "Separation of parameter influence in engineering modelling and parameter identification".

## 9. References

[1] Wang, S. R., Zhao, J. Q., Wu, X. G., Yang, J. H., & Liu, A. (2021). Meso-Scale Simulations of Lightweight Aggregate Concrete Under Impact Loading. International Journal of Simulation Modelling (IJSIMM), 20(2), pp. 291-302. doi:10.2507/ijsimm.20(2).487

[2] Smolčić, Ž., & Ožbolt, J. (2014). Meso scale model for fiber-reinforced-concrete: Effective bond-slip relationship of fibers. Proceedings of works (Faculty of Civil Engineering, University of Rijeka), 17, 197-212. https://doi.org/10.32762/zr

[3] Sampson, W. W. (2009). Modelling Stochastic Fibrous Materials with Mathematica. (Engineering Materials and Processes). Springer Nature., London, United Kingdom

[4] Testing hardened concrete – part 3: Compressive strength of test specimens (en 12390-3:2019), 2019, doi: 10.3403/30360097U

[5] Testing hardened concrete – part 5: Flexural strength of test specimens (en 12390-5:2019), 2019, doi: 10.3403/30360073U

[6] Menke, W. (2018). Geophysical Data Analysis (4th Edition). Academic Press Inc. https://doi.org/10.1016/B978-0-12-813555-6.00001-0

[7] Kožar, I., Malić Torić, N., & Rukavina, T. (2018). Inverse model for pullout determination of steel fibers. Coupled Systems Mechanics, 7(2), 197-209. https://doi.org/10.12989/csm.2018.7.2.197

[8] Kožar, I., Bede, N., Mrakovčić, S., & Božić, Ž. (2021). Layered model of crack growth in concrete beams in bending. Procedia Structural Integrity, 31, 134-139. https://doi.org/10.1016/j.prostr.2021.03.022

[9] Kožar, I., Bede, N., Bogdanić, A., & Mrakovčić, S. (2021). Data-Driven Inverse Stochastic Models for Fiber Reinforced Concrete. Coupled Systems Mechanics, 10(6), 509-520. https://doi.org/10.12989/csm.2021.10.6.509

[10] Ganin, P., Kobrin, A., Shilin, D., Moskvin, V., & Shestov, D. (2019). Synthesis of Real-Time Control Systems for Multilink Industrial Robots Based on Hybrid Neural Network Approach of Solution Inverse Kinematics Problem, Proceedings of the 30th DAAAM International Symposium, pp.0513-0517, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-902734-22-8, ISSN 1726-9679, Vienna, Austria