DOI: 10.2507/33rd.daaam.proceedings.xxx

DETERMINING THE INFLUENCE OF MODEL PARAMETERS ON THE CHOOSING OF AN OPTIMAL SIZE RANGE OF PNEUMATICALLY ACTUATED LINEAR MODULES FOR SPRAYER ROBOTS

Ivo Malakov, Velizar Zaharinov, Stilian Nikolov, Reneta Dimitrova & Grigor Stambolov

This Publication has to be referred as: Malakov, I[vo]; Zaharinov, V[elizar]; Nikolov, S[tilian]; Dimitrova, R[eneta] & Stambolov, G[rigor] (2022). Determining the influence of model parameters on the choosing of an optimal size range of pneumatically actuated linear modules for sprayer robots, Proceedings of the 33rd DAAAM International Symposium, pp.xxxx-xxxx, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-902734-xx-x, ISSN 1726-9679, Vienna, Austria

DOI: 10.2507/33rd.daaam.proceedings.xxx

Abstract

The paper presents results from a sensitivity analysis on the solution of the problem for size range optimization of pneumatically actuated linear modules. To that end, the optimization problem is solved for different values of chosen parameters included in the mathematical model of the problem, while keeping the values of the other parameters constant. An assessment is made regarding the influence of the studied parameters on the optimal solution. Sensitivity analysis (also called post-optimal analysis) is of a substantial practical significance, and is one of the important stages of an approach for designing optimal size ranges.

Keywords: sensitivity analysis; post-optimal analysis; size ranges; optimization; sprayer robots.

1. Introduction

The choice of an optimal size range, predefines in great deal, the good economic results for the manufacturer, as well as for the users of mass-produced technical products from different industry branches [1], [2], [3], [4], [5], [6]. It is related to defining the elements of such a size range, that with minimum costs and/or maximum efficiency (profit) in the area of manufacturing and operation, completely satisfies given product demand regarding quantity [1], [2], [7], [8], [9]. Solving this problem is related to numerous difficulties [1], [7].

One of the main difficulties comes from the necessity for forecasting demand for products with different values of their main parameters, and building a functional dependency between a chosen optimality criterion/criteria and the influencing factors. Solving these problems is related to collecting and processing of a vast quantity of heterogeneous technical and economic information, referring to the connections of the designed size range with its environment throughout the different stages of the size range's lifecycle, and requires substantial expenditure of time, labour, and highly qualified specialists from different fields – marketing, design, manufacturing, operation, etc. These activities are carried out in the earliest design stages, which stages are characterized by uncertainty and incomplete information regarding some of the influencing factors, and uncertainty increases with increase in product complexity [2], [10], [11], [12], [13]. Moreover, the level of uncertainty can rise additionally, because defining costs for materials, energy, salaries, etc. is made in conditions of dynamically changing domestic and international economic situation. As a consequence of this, the created model may not reflect in full the real conditions, and the optimal solution to the problem may have an approximate value [2].

On the other hand, for companies it is particularly important to study different possible situations related to the change in demand of products, production volume, costs, manufacturing conditions, operation, etc.

All of this determines the necessity for studying the dependency between the optimal solution and the change in the problem's parameters (coefficients and elements of the objective function, demand, production volume, feasible application areas, etc.) [14], [15]. To that end, the optimization problem is solved while one or several parameters are changed in certain limits, and the rest of the parameters remain constant. The analysis of the obtained results will allow for [16], [17], [18]:

- studying the influence of the individual parameters of the mathematical model on the solution of the optimization problem, and determining the most important ones, for which the most accurate information must be obtained. This will allow for less resources to be spent when determining the values of the rest of the model's parameters;
- obtaining information regarding the behaviour of the problem's solution in different situations, related to change in some model's elements – demand, production volume, etc.;
- analysing the workings of the developed mathematical model, and estimating if it is properly built;
- modifying (calibrating) the mathematical model for the purpose of adequately reflecting the real conditions, etc.

Subject of examination is the optimal size range of pneumatically actuated linear modules for sprayer robots (Fig. 1), determined after solving the problem by means of a proposed approach [3], [7].

The aim of the paper is to show the results from a sensitivity study of the solution to the problem for choosing an optimal size range of pneumatically actuated linear modules for sprayer robots (Fig. 1).

Fig. 1. Sprayer robot using a pneumatically actuated linear module for vertical motion [19]

2. Mathematical model

The mathematical model of the problem for choosing an optimal size range has the form:

For a given demand set $\bar{N} = \{\bar{N}^1, \bar{N}^2, \dots, \bar{N}^l, \dots, \bar{N}^{\bar{L}}\}$, $l = 1 \div \bar{L}$, of products $\bar{Z} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_l, \dots, \bar{x}_{\bar{L}}\}$, $l = 1 \div \bar{L}$, and their allowable possibilities for satisfying the different demand types, i.e. the elements of the applicability matrix $\Phi_{\bar{L}\times\bar{L}} = \|\varphi_m^p\|_{\bar{L}\times\bar{L}}$, find L^*, $Z^* = \{x^*_{l_1}, x^*_{l_2}, \dots, x^*_{l_j}, \dots, x^*_{l_{L^*}}\}$, $Z^* \subseteq \bar{Z}$, $j \in \{1,2,\dots,\bar{L}\}$, $N^* = \{N^{*l_1}, N^{*l_2}, \dots, N^{*l_j}, \dots, N^{*l_{L^*}}\}$, $L^* \leq \bar{L}$, for which a chosen effectiveness (optimality) criterion has a minimum value:

$$minR(L, \bar{x}_1, \dots, \bar{x}_l, \dots, \bar{x}_{\bar{L}}, \bar{N}^1, \dots, \bar{N}^l, \dots, \bar{N}^L, \Phi_{\bar{L}\times\bar{L}}) = \sum_{j=1}^{L} G\left\{x_{l_j}, N^{l_j}\left(\varphi_m^p, N^p\right)\right\} \tag{1}$$

for the following conditions:

$$\sum_{j=1}^{L} N^{l_j} = \sum_{j=1}^{L^*} N^{*l_j} = \sum_{l=1}^{\bar{L}} \bar{N}^l = N_0, \tag{2}$$

$$x_{l_L} = x^*_{l_{L^*}} = \bar{x}_{\bar{L}}, \tag{3}$$

$$x_{l_j} \in \bar{Z} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_l, \dots, \bar{x}_m, \dots, \bar{x}_{\bar{L}}\}, \forall j = 1 \div \bar{L}, \quad (4)$$

where R is total costs for all elements in the size range; $\bar{L}$ - the number of elements of the initial size range $\bar{Z} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_l, \dots, \bar{x}_m, \dots, \bar{x}_{\bar{L}}\}$, determined after demand research, $dim\bar{Z} = \bar{L}$; L - the number of elements in the currently analysed size range $Z = \left\{x_{l_1}, x_{l_2}, \dots, x_{l_j}, \dots, x_{l_L}\right\}$, $j \in \{1,2,\dots,\bar{L}\}$. There exists a simple representation between the elements of the current and initial size range, whereby to each element x_{l_j} corresponds an element $\bar{x}_m$. The elements of the current size ranges are a combination of L, $L = 1 \div \bar{L}$, elements of $\bar{L}$ number of possible elements of the initial size range, taking into account their allowable application ranges; L^* - the number of elements in the optimal size range $Z^* = \{z^{*l_1}, z^{*l_2}, \dots, z^{*l_j}, \dots, z^{*l_{L^*}}\}$, $j = 1 \div L^*$, $L^* \le \bar{L}$; $\Phi_{\bar{L}\times\bar{L}} = \left\|\varphi_m^p\right\|_{\bar{L}\times\bar{L}}$ - applicability matrix, which elements give information about the possible allowable application ranges for the products in the initial size range $\bar{Z} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_{\bar{L}}\}$ for satisfying the separate demand types $\bar{N} = \{\bar{N}^1, \bar{N}^2, \dots, \bar{N}^{\bar{L}}\}$, where $\varphi_m^p = 1$, if the product of m-th type, $m = 1 \div \bar{L}$, can satisfy the demand $\bar{N}^p$ of product $\bar{x}_p$, $p = 1 \div \bar{L}$, and $\varphi_m^p = 0$ otherwise; $N^{l_j}(\varphi_m^p, N^p)$ - the demand of product x_{l_j} element of the currently analysed size range $Z = \left\{x_{l_1}, x_{l_2}, \dots, x_{l_j}, \dots, x_{l_L}\right\}$, whose index l_j corresponds to the index m of product $\bar{x}_m \in \bar{Z}$, element of the initial size range, where $x_{l_j} = \bar{x}_m$, and $N^{l_j}(\varphi_m^p, N^p) = \sum_{p=l_{j-1}}^{l_j} \varphi_{l_j}^p N^p$; N_0 - total production quantity of all elements in the size range; N^{l_j} - the quantity of l_j-th product size, $N^{l_j} \in N = \{N^{l_1}, N^{l_2}, \dots, N^{l_L}\}$, where N is the demand set for the products of the current size range.

Condition (2) means, that all analysed size ranges, including the optimal, must satisfy all demand in terms of quantity, condition (3) means, that every size range, including the optimal, must include the element from the initial size range with the maximum value of its main parameter $\bar{x}_{\bar{L}}$, and condition (4), that the elements of all size ranges are chosen from the set of elements of the initial size range, determined after demand research.

The demand for each clamp size included in the initial size range $\bar{Z} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_l, \dots, \bar{x}_{\bar{L}}\}$, is determined after market research, i.e., the elements of the set $\bar{N} = \{\bar{N}^1, \bar{N}^2, \dots, \bar{N}^l, \dots, \bar{N}^{\bar{L}}\}$, $l = 1 \div \bar{L}$, where $\bar{N}^l$ is the quantity of the required products of size $\bar{x}_l$. The obtained results are shown in Fig. 2. The product's main parameter values are input along the abscissa – the stroke length, and along the ordinate - the corresponding quantity.

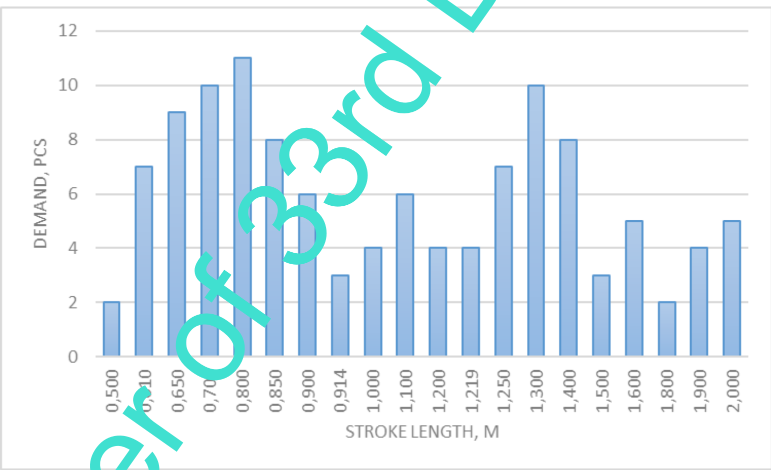

Fig. 2. Product demand from market research

3. Solving the problem – choosing an optimal size range

For choosing an optimal size range it is necessary to determine the functional relationship between the components of the optimality criterion R, and the influencing factors. The following relationship is obtained:

$$\mathrm{R} = \sum_{j=1}^{L} \left[\left(10271,7 + 925,7x_{l_j} + 8,651\right)\left(\frac{4}{N^j}\right)^{0,32} + 9304,32x_{l_j}^2 + 77,47\sqrt{30x_{l_j}} + 154,945x_{l_j} + +66,67\right] N^j + \left(4323,9 + 308,57x_{l_j} + 2,88\right)\left(\frac{4}{N^j}\right)^{0,32} \quad (5)$$

R is determined on the basis of production analysis, and production and operational costs data for pneumatically actuated linear modules. The relationship for determining total costs is obtained through the application of methods for statistical data analysis and regression analysis.

With the aid of developed tools (mathematical model of the optimization problem, cost models for production and operational costs, algorithm and application software) the problem (1) – (4) is solved.

The results are shown in Tab. 1 [20], where R^* are the minimum (optimal) total costs, EUR; L^* - - the number of elements in the optimal size range; $R_{\bar{L}}^{\bar{L}}$ - - the total costs of the size range including all possible sizes, i.e., of the initial size range, EUR.

Indicators	R^*	L^*	$R_{\bar{L}}^{\bar{L}}$	$\frac{R_{\bar{L}}^{\bar{L}}-R^*}{R_{\bar{L}}^{\bar{L}}}$
Optimal size range	2 531 731,07	9	2 796 363,88	9,46%

Table 1. Results from solving the problem

The data for the chosen optimal size range are shown in Tab. 2. The size range includes $L^* = 9$ sizes.

N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Size $x_{l_j}^* \in X^*$, $j = 1 \div 9$	$x_{l_1}^*$	$x_{l_2}^*$	$x_{l_3}^*$	$x_{l_4}^*$	$x_{l_5}^*$	$x_{l_6}^*$	$x_{l_7}^*$	$x_{l_8}^*$	$x_{l_9}^*$
Size $\bar{x}_m \in \bar{X}$, $l \in \{1 \div \bar{L}\}$	$\bar{x}_1 = 0{,}65$	$\bar{x}_6 = 0{,}80$	$\bar{x}_9 = 1{,}00$	$\bar{x}_{12} = 1{,}25$	$\bar{x}_{14} = 1{,}20$	$\bar{x}_{15} = 1{,}40$	$\bar{x}_{17} = 1{,}60$	$\bar{x}_{19} = 1{,}90$	$\bar{x}_{20} = 2{,}00$
Application range	50 - 80	180 - 360	360 - 560	560 - 650	750 - 840	1000	1050 - 1400	1600 - 2200	2200 - 2500

Table 2. Optimal size range

The production quantities of each element of the optimal size range (1), and the elements of the initial size range (2) are shown in Fig. 3. The total costs curve, determined in the process of solving the problem, is shown in Fig. 4.

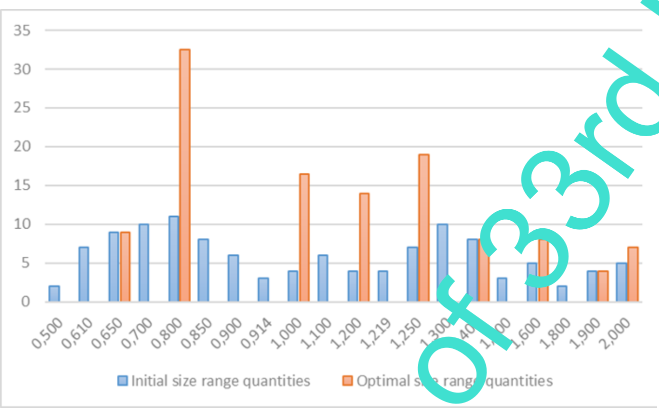

Fig. 3. Production quantity of the elements of the optimal size range (red), and the initial size range (blue)

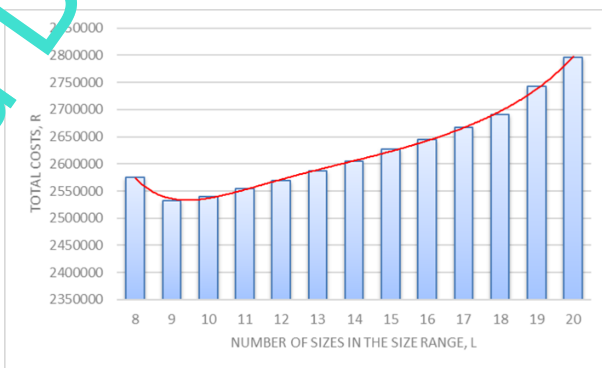

Fig. 4. Change in the total costs

The found optimal size range of the product "Pipe clamp" is characterized by:

- reduction in the number of sizes from 20 to 9, i.e., with 55%;
- reduction in the total costs in comparison to the size range including all possible sizes with 281 815,82 EUR, i.e. with 9%;

4. Sensitivity analysis of the optimal solution

A sensitivity analysis of the optimal solution is carried out in relation to the change in the values of the mathematical model's parameters. The main goal is to determine the influence of the separate components and coefficients on the optimization problem's solution, and to determine the most important ones, for which the most accurate information must be obtained.

To that end, numerical experiments are carried out, which consist of solving the problem (1) – (4) for different values of the main coefficients and components, included in the mathematical model.

The sensitivity analysis includes the following experiments:

A. Changing the demand function set $\bar{N} = \{\bar{N}^1, \bar{N}^2, \dots, \bar{N}^l, \dots, \bar{N}^{\bar{L}}\}$ while keeping the total production quantity N_0;

B. Changing the quantity of production N_0;

C. Changing the application ranges of the separate elements of the initial size range, given by means of the applicability matrix $\Phi_{\bar{L}\times\bar{L}} = \|\varphi_m^p\|_{\bar{L}\times\bar{L}}$;

D. Changing the coefficient, which takes into account the production scale factor;

E. Changing the coefficient, which takes into account the production learning rate.

4.1. Changing the demand function

Problem (1) – (4) is solved for different demand functions as follows: experiment A0 – demand shown in Fig. 2 (initial problem); A1 – demand shown in Fig. 5, A2 – demand shown in Fig. 6, and A3 - demand shown on Fig. 7. For these experiments the overall production quantity is not changing, i.e., the sum of the demands for the separate sizes for each experiment has the same value. The solutions found are shown in Tab. 3.

The aim is to study different market states, where there is a higher demand for larger sizes (A1), average in magnitude sizes (A2), and small sizes (A3). From Fig. 2 can be seen, that the demand function for the initial problem has two peaks – towards the small and average sizes.

In Fig. 8 the total costs are graphically represented in relation to the number of sizes for different demand functions. It is obvious, that the curves depicting the total costs function for different demand distribution, significantly change in value. The shape of the curves changes near the optimum, and afterwards it is similar. It can be seen, that when changing demand, the number of sizes included in the optimal solution can change, and from Tab. 3 it can be determined, that the composition of the size range also changes. Therefore, deviations of the demand distribution from the actual one, leads to significantly different problem solutions. Thus, for obtaining reliable results from the optimization, it is necessary to carefully and in detail collect data regarding customer orders, i.e., market research, and to correctly define the demand function.

Experiment	**A0**	**A1**	**A2**	**A3**
L^*	9	10	9	9
R^*	2 531 731,07	3 343 715,18	2 719 746,87	2 309 722,77
$R_{\bar{L}}^{\bar{L}}$	2 796 363,88	3 549 897,56	2 959 954,02	2 546 364,56
$x_{l_j}^* \in X^*$	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,50; 1,60; 1,80; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,50; 1,60; 1,90; 2,00	0,65; 0,80; 0,914; 1,25; 1,20; 1,50; 1,60; 1,90; 2,00

Table 3. Results from solving the problem with different demand functions

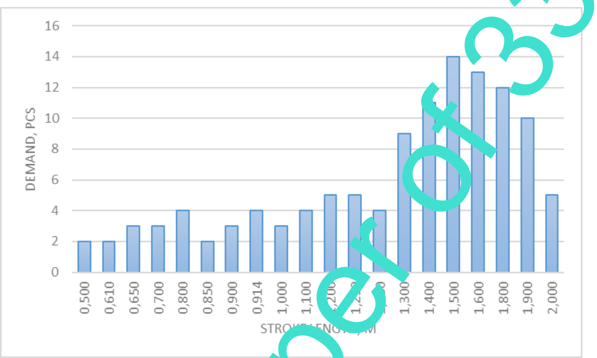

Fig. 5. Demand function for A1

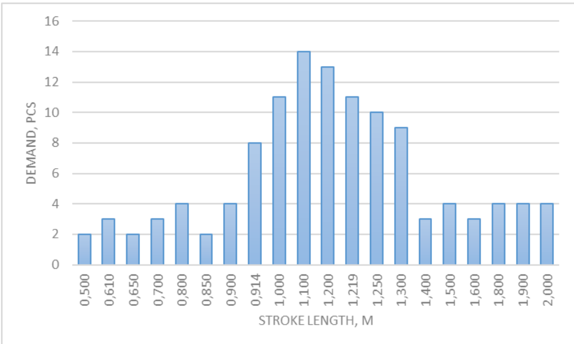

Fig. 6. Demand function for A2

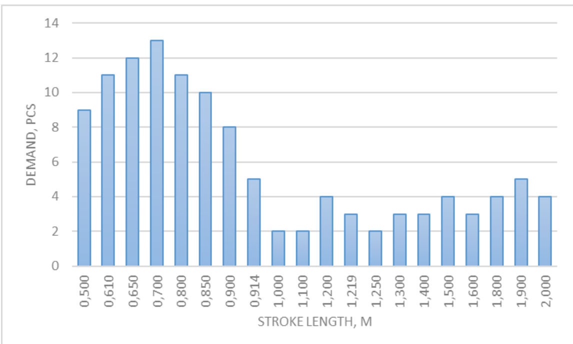

Fig. 7. Demand function for A3

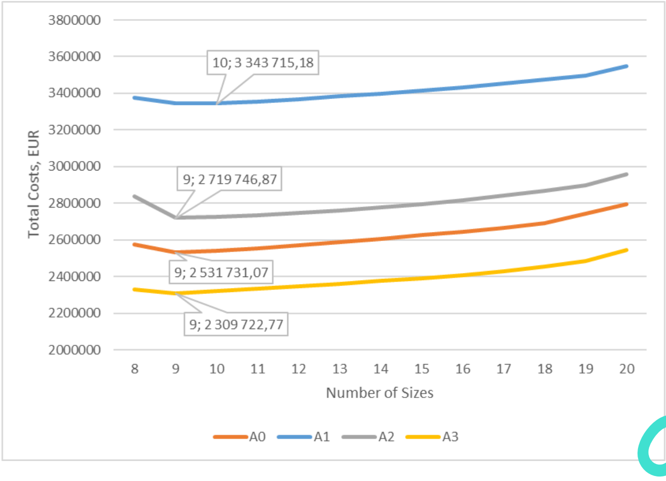

Fig. 8. Change in total costs for different demand functions

4.2. Changing the production quantity

Problem (1) – (4) is solved for different production quantities as follows: B0 – production quantity of the initial problem; B1 – doubling the production quantity; B2 – tripling the production quantity; B3 - quadrupling the production quantity; B4 – halving the production quantity. For these experiments the demand distribution does not change. The found solutions are shown in Tab. 4.

Experiment	**B0**	**B1**	**B2**	**B3**	**B4**
L^*	9	10	10	10	9
R^*	2 531 731,07	4 676 012,12	6 730 565,95	8 924 015,64	1 489 262,16
$R_{\bar{L}}^{\bar{L}}$	2 796 363,88	5 030 333,79	7 165 510,28	9 434 768,02	1 699 298,41
$x_{l_j}^* \in X^*$	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 0,914; 1,10; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 0,914; 1,10; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 0,914; 1,10; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00

Table 4. Results from solving the problem with different production quantities

The following results are obtained for the total costs R^* (Fig. 9):

- Doubling, tripling, and quadrupling the production quantity leads to increase in the total costs accordingly with 85%, 166%, and 253% with respect to the total costs for the optimal size range of the initial problem;
- Halving the production quantity leads to lowering of the total costs with 41% with respect to the total costs for the optimal size range of the initial problem.

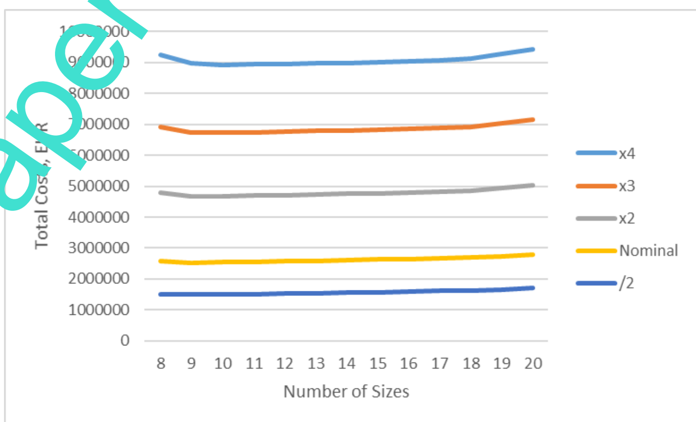

Fig. 9. Change in total costs for different production quantities

Additionally, when increasing the production quantity, the number of sizes in the optimal size range increases with one, and its composition changes. Lowering the production quantity does not change the composition of the size range in comparison to the optimal one.

4.3. Changing the application ranges of the separate elements of the initial size range

Problem (1) – (4) is solved for different application ranges of the separate elements of the initial size range. The change is made in two directions: adding and removing application fields. The changes made are shown in Fig. 10 for each of the experiments C0-C6, and are colour coded in the figure. The experiment C0 is the initial problem. For experiments C1-C3 fields are added starting with the initial problem and gradually adding one field for every following experiment until the smallest range covered with one size becomes four fields. For experiments C4-C6 fields are removed starting with the initial problem and gradually removing one field for each following experiment, until the largest range covered with one size becomes three fields.

The experiments' results are shown in Tab. 5.

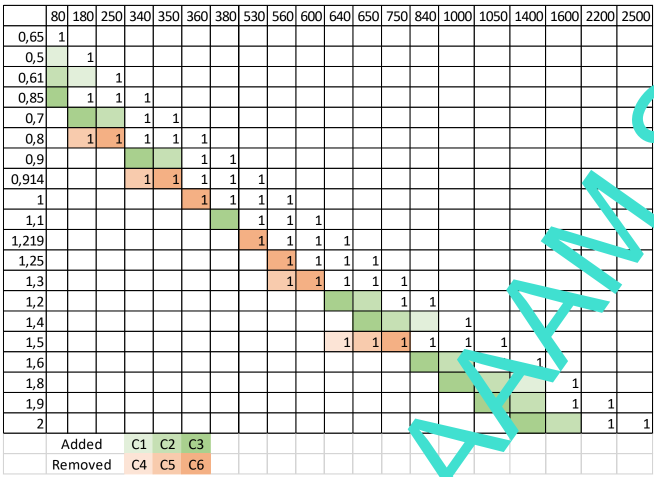

Fig. 10. Changing application ranges

Exp.	C0	C1	C2	C3	C4	C5	C6
L^*	9	9	8	[illegible]	9	10	11
R^*	2 531 731,07	2 511 562,97	2 485 160,05	2 [illegible]522,33	2 531 731,07	2 549 321,07	2 571 879,25
$x^*_{i_j} \in X^*$	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,50; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,50; 0,70; 0,914; 1,219; 1,20; 1,40; 1,60; 2,00	0,50; 0,70; 0,914; [illegible]; 1,20; 1,40; 1,60; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,50; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,50; 0,61; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00

Table 5. Results from solving the problem with different application ranges

Exp.	C0	C1	C2	C3	C4	C5	C6
C0	0	1	5	5	0	1	2
C1	1	0	3	4	1	1	2
C2	5	3	0	1	5	5	6
C3	5	4	1	0	5	5	6
C4	0	1	5	5	0	1	2
C5	1	1	5	5	1	0	1
C6	2	2	6	6	2	1	0

Table 6. Differences between the elements of the optimal size ranges, found by changing the application ranges, number of different elements

The results show that when increasing the application ranges, the value of the total costs for the separate experiments decreases compared to the initial problem – for C1 with 1%, for C2 with 2%, and for C3 with 3%. When decreasing the application ranges the total costs increase compared to the initial problem's solution – for C4 the solution coincides with C0 and there is no change, for C5 the increase is 1%, and for C6 it is 2%. The total costs for the size range including all elements do not change. These increases and decreases are relatively small (less than 5%) and their significance can be argued.

On the other hand, the type and number of elements included in the optimal solution, significantly changes with changing the application ranges. This can be seen from Tab. 6, in which are presented the differences between the elements of the found optimal size ranges for the separate experiments.

4.4. Changing the coefficient, which takes into account the production scale factor

The coefficient taking into account the production scale factor in the initial problem (5) is equal to 4 (the value in the numerator of the expression $\frac{4}{Nj}$). The optimal solution's sensitivity is studied related to the change in this coefficient, expressing the production scale factor. The experiment is carried out for the following cases: D0 – coefficient value 4 (initial problem); D1 – coefficient value 20; D2 – coefficient value 40. The solutions found are shown in Tab. 7.

Experiment	D0	D1	D2
L^*	9	9	8
R^*	2 531 731,07	3 143 959,47	3 514 311,91
$R_{\bar{L}}^{\bar{L}}$	2 796 363,88	3 630 495,55	4 145 115,41
$x_{l_j}^* \in X^*$	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,80; 2,00	0,65; 0,80; 0,914; 1,30; 1,50; 1,60; 1,80; 2,00

Table 7. Results from solving the problem with different coefficients taking into account the production scale factor

Solving the optimization problem when changing the coefficient taking into account the production scale factor, shows the following results (Fig. 11): when changing the value of the coefficient from 4 to 20, the total costs for the optimal size range increase with 24%, and when changing the coefficient to 40, they increase with 39%. The optimal number of sizes changes from 9 when the value of the coefficient is 4 to 8, for the experiment with coefficient of 40.

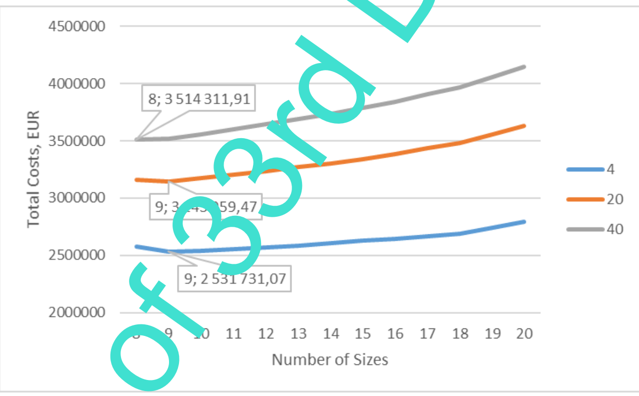

Fig. 11. Change in the total costs for change in the coefficient taking into account the production scale factor

The type of the elements constituting the optimal solution changes the most between experiments D1 and D2. Between experiments D0 and D1 the change is minimal (in one element).

Increasing the scaling factor leads to increase in total costs and corresponding decrease in the number of sizes included in the optimal size range, shifting the optimal solution to the left (Fig 11).

4.5. Changing the coefficient, which takes into account the production learning rate

The coefficient, which takes into account the production learning rate has a value of 0,32 for the initial problem (the power in the expression $\left(\frac{4}{Nj}\right)^{0,32}$). The sensitivity of the optimal solution is studied relative to changes in the coefficient for the following cases: E0 – coefficient value of 0,22; E1 - coefficient value of 0,27; E2 - coefficient value of 0,32 (initial problem); E3 - coefficient value of 0,37; E4 - coefficient value of 0,42. The solutions found are shown in Tab. 8.

Solving the optimization problem with different values of the production learning rate coefficient, shows the following results (Fig. 12): when changing the coefficient's value from 0,22 to 0,27 the total costs decrease with 2,38%. When changing the coefficient to 0,32 total costs decrease with 2,25%. When changing the coefficient to 0,37 the costs decrease with 2,13%, and for a value of 0,42 the decrease is 2,01%. The optimal number of sizes remains the same for these changes. The elements included in the optimal solutions for the experiments do not change. Thus, when changing the production learning rate, the production costs decrease.

Experiment	E0	E1	E2	E3	E4
L^*	9	9	9	9	9
R^*	2 652 968,41	2 589 931,75	2 531 731,07	2 477 943,50	2 428 186,13
$R_{\bar{L}}^{\bar{L}}$	2 849 280,97	2 822 208,09	2 796 363,88	2 771 706,05	2 748 194,29
$x_{l_j}^* \in X^*$	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00	0,65; 0,80; 1,00; 1,25; 1,20; 1,40; 1,60; 1,90; 2,00

Table 8. Results from solving the problem with different production learning rate coefficients

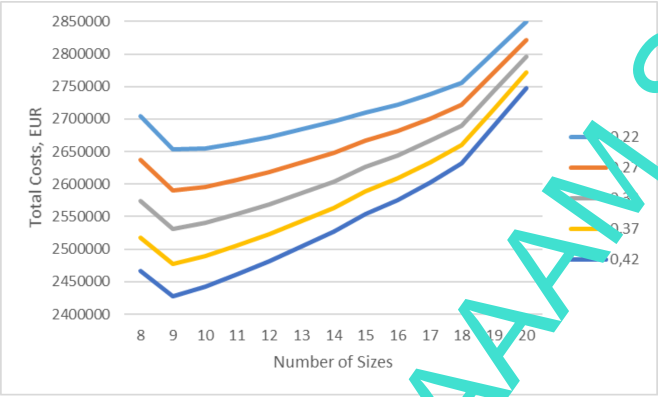

Fig. 12. Change in the total costs for change in the production learning rate coefficient

5. Conclusion

The following important results are presented in the current paper:

- A mathematical model is proposed of the problem for choosing an optimal size range for a technical product with constraints on the applicability of the elements in the size range.
- The optimal size range of pneumatically actuated linear modules is found for predefined application ranges for the elements of the size range.
- A study is made of the influence of the mathematical model's parameters on the optimal solution, and the following is established:
- the mathematical model adequately reflects the conflicting interests of manufacturer and users. Through it a size range is found, that satisfies all demand with minimum costs. The solution is a compromise between the users' requirement for bigger size range density that satisfies in full their demand, and the manufacturer's requirements for lower density size ranges aiming for bigger batches and lowering the costs for the manufactured products;
- the demand function must be determined with the greatest precision, in order to obtain reliable results;
- increasing the production quantity changes the number and type of sizes in the optimal size range, and decreasing the production quantity leads to no changes. The total costs change as such: when production quantity increases, they increase, and when production quantity decreases – costs also decrease;
- the application range significantly influences the optimal solution;
- increasing the production scale factor leads to increase in total costs, and a decrease in the number of sizes constituting the optimal size range;
- increasing the production learning rate decreases total costs, but do not change the composition and number of elements of the optimal solution.
- Through the conducted experiments the sensitivity of the elements included in the optimal solution can be studied and analysed regarding changes in the different components of the mathematical model. Even in cases with the same number of elements, different combinations are possible, having close values of their main parameter and with completely or partially overlapping application ranges. This confirms the necessity for post-optimal analysis for studying the optimum and possible close to the optimum solutions. The decision maker can have a greater preference for these solutions, due to the presence of secondary considerations and factors, which are not included, or are difficult to be taken into account, in the mathematical model.

A further development is conducting sensitivity analysis on size range optimization problems with multiple parameters. The inclusion of multiple main parameters raises significantly the mathematical model's complexity, and consequently the complexity of the experiments, that have to be conducted, both algorithmically and for solving.

6. Acknowledgments

This work was supported by the European Regional Development Fund within the Operational Programme "Science and Education for Smart Growth 2014 - 2020" under the Project CoE "National center of mechatronics and clean technologies" BG05M2OP001-1.001-0008

7. References

[1] Chivarov N., Stoev P., Yovkov S., Stoimenov N., Chivarov S., (2021) Specialized Gripper-Dispenser for Dosing Laboratory Mill With Grinding Bodies, 20th IFAC Conference on Technology, Culture and International Stability, TECIS 2021, Volume 52, Issue 25, September 14-17, IFAC-PapersOnLine, Publisher: IFAC Secretariat, ISSN:2405-8963, pp. 539-542

[2] Malakov, I. (2015). Optimization of parametric ranges of technical products, In: Complex automation of the discrete production, Malakov, I. & Nikolov, S., (Eds.), pp. 25-48, TU-Sofia, ISBN 978-619-167-155-3, Sofia

[3] Dichev, D., Zhelezarov, I., Karadzhov, T., Madzharov, N., Diakov, D. (2019) Method for measuring motion parameters of moving objects. Vide. Tehnologija. Resursi - Environment, Technology, Resources, 3, pp. 27-31

[4] Lotz, J. (2018). Beherrschung von Unsicherheit in der Baureihenentwicklung, Ph.D. Dissertation, Mechanical Engineering, TU-Darmstadt, Darmstadt, Germany

[5] Pahl, G., Beitz, W. & Feldhuzen, J. (2007). Konstruktionslehre. Methoden und anwendung, Springer-Verlag, ISBN 354-0-340-60-2, Berlin/Heidelberg

[6] Simpson, T. (2004). Product platform design and customization: status and promise, Artificial Intelligence for Engineering Design, Analysis and Manufacturing, Vol. 18, No. 1, (February 2004), pp. 3-20, DOI: 10.1017/S0890060404040028

[7] Malakov, I., Zaharinov, V. & Tzenov, V. (2015). Size ranges optimization, Procedia Engineering, Vol. 100, (January 2015), pp. 791-800, DOI: 10.1016/j.proeng.2015.01.433

[8] Malakov, I. & Boyadjiev, I. (2001). Choice of optimal range of parameters of technical products, In: Proceedings of the 12th International DAAAM Symposium, Katalinic, B., (Ed.), pp. 285-286, Published by DAAAM International, ISBN 3-901509-19-4, Vienna

[9] Mueller, D. (2011). A cost calculation model for the optimal design of size ranges, Journal of Engineering Design, Vol. 22, No. 7, (July 2011), pp. 467-485, DOI: 10.1080/09544820903449218

[10] Lotz, J., & Kloberdanz, H. (2014). Scaling under dynamic uncertainty using laws of growth, In: Proceedings of the International Symposium on Robust Design - ISoRD14, Howard, T. & Eifler, T., (Eds.), pp. 17-27, DOI: 10.4122/dtu:2083, Copenhagen

[11] Lotz, J.; Freund, T., Würtenberger, J. & Kloberdanz, H. (2016). Uncertainty in Size Range Development - an Analysis of Potential for a new Scaling Approach, International Design Conference - DESIGN 2016, , pp. 341-350, Dubrovnik

[12] Todorov, G.; Kamberov, K.; Ivanov, T. (2021) Parametric optimisation of resistance temperature detector design using validated virtual prototyping approach, Case Studies in Thermal Engineering, DOI: 10.1016/j.csite.2021.101302

[13] Todorov, G., Sofronov, Y., Dimova, K. (2022). Comparison Analysis Between Different Technologies for Manufacturing Patient-Specific Implants. In: Rackov, M., Mitrović, R., Čavić, M. (eds) Machine and Industrial Design in Mechanical Engineering. KOD 2021. Mechanisms and Machine Science, vol 109. Springer, Cham. https://doi.org/10.1007/978-3-030-88465-9_27

[14] Borgonovo, E. (2017). Sensitivity analysis, Springer, ISBN 978-3-319-52257-9

[15] Saltelli, A., Chan, K. & Scott, F. (2009). Sensitivity analysis, John Wiley & Sons, ISBN 978-0-470-74382-9, New York

[16] Rai, R. & Allada, V. (2003). Modular product family design: agent-based pareto-optimization and quality loss function-based post-optimal analysis, International journal of production research, Vol. 41, No. 17, (November 2003), pp. 4075-4098, DOI: 10.1080/0020754031000149248

[17] Razavi, S. & Gupta, H. (2016). A new framework for comprehensive, robust, and efficient global sensitivity analysis: Theory, Water Resource Research, Vol. 52, No. 1, (January 2016), pp. 423-439, DOI: 10.1002/2015WR017559

[18] Pianosi, F., Beven, K., Freer, J., Hall, J., Rougier, J., Stephenson, D. & Wagener, T. (2016). Sensitivity analysis of environmental models: a systematic review with practical workflow, Environmental Modelling & Software, Vol. 79, (May 2016), pp. 214-232, DOI: 10.1016/j.envsoft.2016.02.008

[19] BORUNTE Product Catalogue (2020), Vertical Sprayer, Guangdong BORUNTE Intelligent Equipment Co., Ltd.

[20] Zaharinov, V., Malakov, I., Nikolov, S., Dimitrova, R. & Stambolov, G. (2020). Choosing an Optimal Size Range of Pneumatically Actuated Linear Modules for Sprayer Robots, Proceedings of the 31st DAAAM International Symposium, pp.0159-0167, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-902734-29-7, ISSN 1726-9679, Vienna, Austria