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APPLICATION OF LINEAR PROGRAMMING TO INCREASE THE EFFICIENCY OF CONSTRUCTIONAL VENEER PRODUCTION

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Abstract

The paper presents the application of linear programming to increase production efficiency in a furniture factory based on constructional veneer technology. Veneer production classifies as novel wood processing technology and it is mainly an automated process. Insufficient quantities of delivered thermal energy for veneer drying result in discontinuity in the production process. Inadequate veneer humidity causes production problems and exploitation of veneer-based products. The problem of using human resources in the newly created situation was also considered. Algorithms for solving problems are given in the paper.

Keywords: linear programming; integer programming; wood; algorithm; veneer drying.

1. Introduction

Wood and wood-based raw materials are becoming increasingly popular materials in product design. Therefore, high performance with high dimensional stability and durability are expected from these materials. Veneers are of particular importance because they have high elasticity even if they are obtained from a hard type of wood [12].

Veneer production historically belongs to newer technologies for wood processing. According to the method of production and purpose, there are three types of thin sheets of wood or veneers. These are peeled, sliced and constructional. Sliced veneer is mainly used for surface finishing of wood-based panels. Structural or peeled veneer is used for the production of structural elements such as various panels based on veneer, seats or chair backs, armrests and the like.

In this paper, the focus is on the analysis of the technological process of production of structural veneer. The advantage of using products based on this veneer compared to solid wood is in improving strength and especially elastic properties. The problems with elements made of solid wood are: in smaller or larger radii of curvature in wooden elements and relatively small percentages of wood utilization. Also, the problem with massive wood bending is seen in smaller diameters and the loss of elastic properties with dynamic loading.

In the production of structural veneers, as well as other types of veneers, the drying phase plays a very important role. The consequences of insufficiently dried veneer appear in the later stages of processing and exploitation of the product. That is why many authors have dealt with the problem of veneer drying. In the paper [10], a prediction of the optimal drying temperatures of veneers for the production of pressed veneers is given. Some authors analyzed the influence of veneer drying temperature on certain physical properties and formaldehyde emissions in birch plywood [3]. In [1], Baldwin investigated the process of veneer drying and preparation for the veneer pressing phase. An interesting experimental investigation and calculation of the influence of ambient air parameters during drying on temperature, moisture content and resulting deformations and stresses in wood with the aim of determining the optimal drying speed is given in [9]. In the paper [5], the method of finite volumes was used to obtain non-stationary fields of temperature, displacement and stress in wood, which are the result of changes in the parameters of the surrounding medium, i.e. the regime of heat treatment of wood.

In the production of veneers, it is important to provide a sufficient amount of thermal energy in order to ensure the application of appropriate drying regimes. The discontinuity of production is caused by insufficient heat energy, which prevents the application of appropriate modes. Therefore, there is an imperative to make a certain investment in technological equipment, i.e. means of production. In order for the investment to be financially justified, an expert analysis of technological processes and appropriate mathematical decision-making tools are needed.

The research is focused on the application of certain scientific knowledge in the field of production organization and management. The challenge is to transfer knowledge when managing certain processes [11].

The paper will use integer programming as a subtype of linear programming. Integer programming, even though it is a mathematical method, is used today as a tool for business decision-making where it is necessary to make the right decision based on the given criteria, taking into account the availability and limited resources in the factory. Thus, optimization can be understood both as a way of setting an engineering problem and as a specialized mathematical decision support tool. By introducing linear programming as a mathematical tool for solving certain production problems, it has a professional contribution and scientific contribution to the profession. The results of the research will help in making an objective assessment of the investment.

The paper will also focus on human resources. The issue of human resources has particularly come to the fore in the COVID19 pandemic. A mathematical model for the redistribution of executors to other operations in order to maintain the existing production capacity in changed conditions as well as to overcome production discontinuity based on the redistribution of human resources was discussed in [7].

The paper is organized as follows. In section 2, the physical model of the problem considered in the paper is given. The basics of linear programming are presented in section 3. The mathematical model for the specific factory is presented in section 4. The analysis of the results is presented in section 5. In section 6, the results of the implementation research are presented and recommendations for further research are given.

2. Physical model

The physical model includes the following stages of veneer production:

- heat treatment of wood;
- peeling logs;
- cutting the veneer to a certain width and removing errors in construction and wood processing;
- veneer drying;
- disposal of dry veneer.

Figure 1. shows the principle scheme of the technological process of structural veneer manufacturing. Focus of this paper is on the veneer drying.

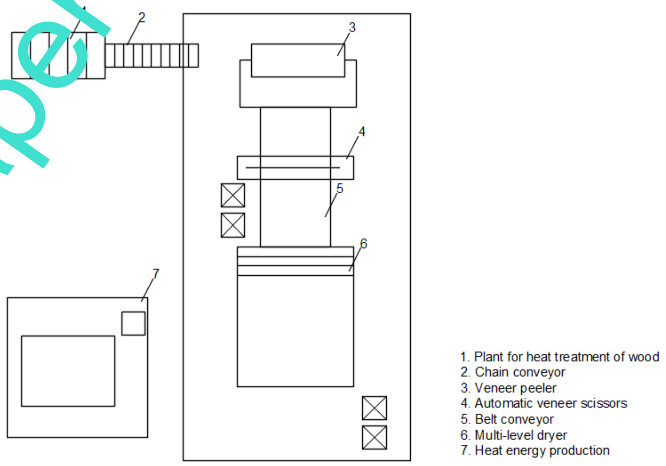

Fig. 1. The principle scheme of the technological process

3. Linear programming

Optimization plays a very important role in the production process. It is actually a new approach to engineering synthesis, i.e. process or product development based on optimality. Optimization procedures are applied in a whole range of linear and non-linear problems, of which we will mention only the most important ones: the problem of optimal investment decisions, the transport problem, the problem of resource allocation, the problem of optimizing mechanical structures, the problem of optimizing heat flows, the problem of optimal management of systems and others. Process optimization in the engineering sense is not a simple procedure and the optimization scheme is shown in Figure 2.

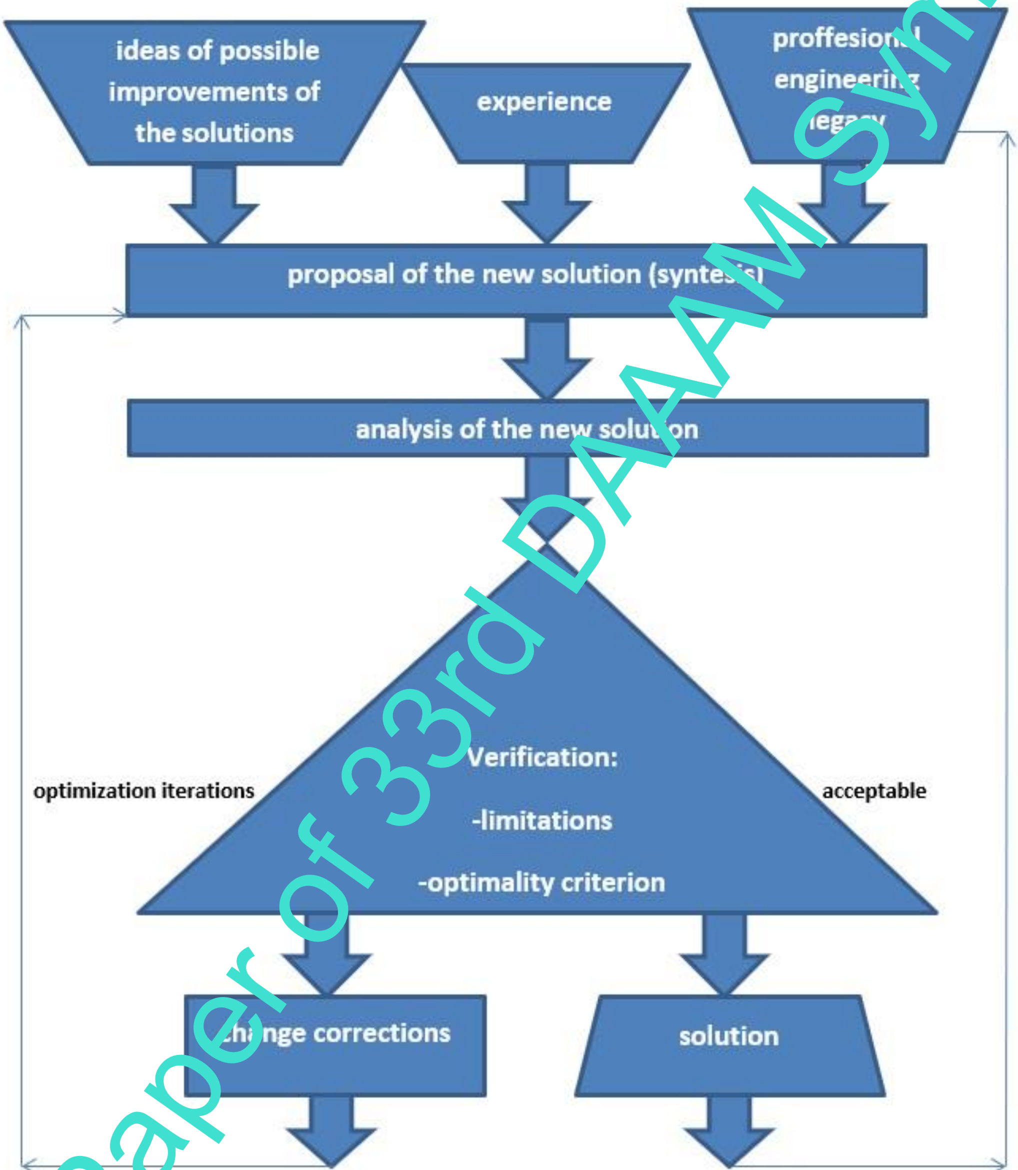

Fig. 2. The Optimization Scheme

In order to choose the most suitable solution and scientifically verify that solution, it is necessary to pose the problem mathematically. Optimization or mathematical programming is a branch of mathematics that studies the maximization and minimization of real functions of a real variable. Linear programming is a very important field in optimization. We can find its beginnings with Fourier, after whom one of the methods for solving linear programming problems got its name. Namely, it is about Fourier-Motzkin elimination. In 1939, Leonid Kantorovich developed the linear programming technique with the aim of reducing military costs and increasing enemy losses in World War II. Until 1947. linear programming methods were not available to the general public. In 1947. George B. Dantzig published the simplex method, which is one of the most important methods in linear programming. After the Second World War, linear optimization methods are used in everyday practice, because many practical problems in operations research can be expressed as linear programming problems.

Linear programming is a mathematical method for minimizing or maximizing a linear objective function with constraints in the form of linear inequalities, and with non-negativity constraints for the variable.

We can write the general form of the minimum problem as follows:
Minimize the objective function

$$Min\ z = \sum_{j=1}^{n} c_j x_j \quad (1)$$

with constraints

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_{i,} \quad i = 1,2, \ldots, m \quad (2)$$

$$\sum_{j=1}^{n} d_{ij} x_j \geq e_{i,} \quad i = 1,2, \ldots, l \quad (3)$$

$$\sum_{j=1}^{n} f_{ij} x_j = g_{i,} \quad i = 1,2, \ldots, p \quad (4)$$

and the negativity condition

$$x_j \geq 0, \ j = 1,2, \ldots, n,$$

where
c_j- the coefficient of the objective function j -th variable;
x_j- structural variables;
b_i, d_i, g_i- the quantity of the i -th constraint, i.e. the coefficients on the right side of the equations, i.e. the inequalities;
$a_{ij}, d_{ij,} f_{ij}$ – the amount of the i -th restriction required for the unit of the j -th variable.

In order to write down the problem in matrix form, we introduce the following notations:

$$\boldsymbol{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{l1} & \cdots & d_{ln} \end{bmatrix}, \quad \boldsymbol{F} = \begin{bmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{p1} & \cdots & f_{pn} \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}, \quad \boldsymbol{e} = \begin{bmatrix} e_1 \\ \vdots \\ e_l \end{bmatrix}, \quad \boldsymbol{g} = \begin{bmatrix} g_1 \\ \vdots \\ g_p \end{bmatrix}.$$

where $\boldsymbol{c}$ objective function coefficient vector, $\boldsymbol{x}$ vector of unknowns and structural variables, $\boldsymbol{A}, \boldsymbol{D}, \boldsymbol{F}$ matrices of structural coefficients, $\boldsymbol{b}, \boldsymbol{e}, \boldsymbol{g}$ are vectors of free members.

We can write the linear programming problem in matrix form as follows:

$$Min\ \boldsymbol{z} = \boldsymbol{c}^T \boldsymbol{x} \quad (5)$$

$$\boldsymbol{Ax} \leq \boldsymbol{b} \quad (6)$$

$$\boldsymbol{Dx} \geq \boldsymbol{e} \quad (7)$$

$$\boldsymbol{Fx} = \boldsymbol{g} \quad (8)$$

$$\boldsymbol{x} \geq \boldsymbol{0} \quad (9)$$

Vector $\boldsymbol{x}$, which satisfies the constraints (6), (7) and (8) represent a solution to the problem. A problem solution that additionally satisfies condition (9) is called a possible problem solution. A possible solution to the problem that satisfies condition (5) is called an optimal solution to the problem.

Two methods are used to solve linear programming problems:

- the graphical method, which is the simplest, but which is limited to two-dimensional problems;
- simplex method, which is used when we have more than two variables.

The graphic method consists in the graphic representation of the lines that determine the conditions in the plane $x_1 0 x_2$. The conditions determine some polygon whose vertices are possible solutions. By checking the value of the objective function at the vertices of the polygon, we obtain optimal solutions from a set of possible solutions.

The simplex method is a finite, iterative and general method for solving linear programming problems. In the case of the simplex method, one starts from some basically possible solution and based on the criteria of optimality, one examines whether that solution is optimal. If the solution is optimal, the procedure is over. In case we did not get the optimal solution, the simplex method finds a new basic possible solution whose solution corresponds to the neighboring extreme point and at that point the objective function receives a better value. If the minimum problem is being solved, then the objective function takes on a smaller value. We repeat the process until we get the optimal solution. When solving a problem, non-basic variables are introduced. Non-basic variables are a set of variables whose value is set to zero. Basic variables are a set of variables whose value is calculated, and as a rule, they are different from zero. When solving problems using the simplex method, special tables are introduced, which make it easier to find the optimal solution. More information on linear programming can be found in [2], [4], [6] and [8].

A special subtype of linear programming is integer linear programming which offers a solution in integer form. Three types of problems that require the application of integer linear programming are: the fixed cost problem, the optimal investment decision problem, and the knapsack problem. Every problem that needs to be solved in such a way is solved using the standard simplex method. After obtaining an optimal solution that is not integer, it is necessary to use methods of integer forms or methods of branching and fencing to obtain a final, integer solution that is treated as optimal. However, there are simpler examples from practice where, due to the integer number of variables, it is easy to write appropriate algorithms that quickly provide an optimal solution to the problem. We will present two such algorithms in the next section.

4. Mathematical model for a specific factory

From the physical model shown in the technological scheme in Figure 1, it can be seen that in the production of structural veneer, a multi-level dryer and a plant for the production of thermal energy play a significant role. In the specific case, the multi-level dryer consists of 6 sections, and the plant for the production of thermal energy consists of a 4 MW boiler. In the production process, production discontinuity occurs, because for the production of raw veneer from one shift, it is necessary for the dryer to work in three shifts. There are two ways to solve this problem, namely the installation of additional dryer sections and the purchase of an additional boiler for the production of thermal energy. The number of possibly supplied dryer sections is limited by the money for investment and the size of the place where the acquired sections would be located without disrupting the production process. A maximum of 3 sections can be purchased for a specific case. On the other hand, there are occasional stoppages in production related to boiler malfunctions, the losses are large and in the order of 50,000 euros. It means that the possible purchase of a 1MW or 2MW backup boiler should be considered. Reduction of production costs is an important data for making an investment decision. New investments will affect both possible reductions in the number of shifts and costs per shift. Since the investments are planned for the calendar year, it is more likely that a failure will occur during the year than during the month, the costs of labor production were observed on an annual basis.

Therefore, the objective function to be minimized has the following form

$$z = z_I + z_L + z_{SH} \tag{10}$$

where

$$z_I = 3000x_1 + 15000x_2 \tag{11}$$

$$z_L = 12500(2 - x_2)+25000 \tag{12}$$

$$z_{SH} = 36000(3 - 0.\dot{3}\,x_1 - 0.5x_2) \tag{13}$$

z_I, z_L i z_{SH} are direct investment function, function of losses due to boiler failure and dryer costs due to workers shift change, respectively. x_1 is the number of acquired dryer sections and x_2 power of the new boiler. The coefficients in formula (11) along with x_1 and x_2 correspond to the price of a section or a 1MW boiler on the market.

Formula (10) can be written in another form

$$z = 3000x_1 + 15000x_2 + 12500(2 - x_2) + 36000(3 - 0.\dot{3}\,x_1 - 0.5x_2)+25000 \tag{14}$$

After sorting we obtain the objective function z in form of

$$z = 158000 - 9000x_1 - 15500x_2 \quad (15)$$

The linear programming problem for the specified veneer factory has the form

$$\min z = 158000 - 9000x_1 - 15500x_2 \quad (16)$$

$$x_1 + x_2 \leq 5 \quad (17)$$

$$0 \leq x_1 \leq 3 \quad (18)$$

$$0 \leq x_2 \leq 2 \quad (19)$$

Conditions (17), (18) and (19) are determined by the size and place for installing the purchased new sections of the dryer and the funds that the factory has decided to invest in solving production optimization.
In matrix form, the linear programming problem looks like

$$\min z = 158000 - 9000x_1 - 15500x_2 \quad (20)$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (22)$$

Since this is a linear programming problem with two variables, it is suitable for solving by the graphical method. The graphical method is shown in Figure 3.

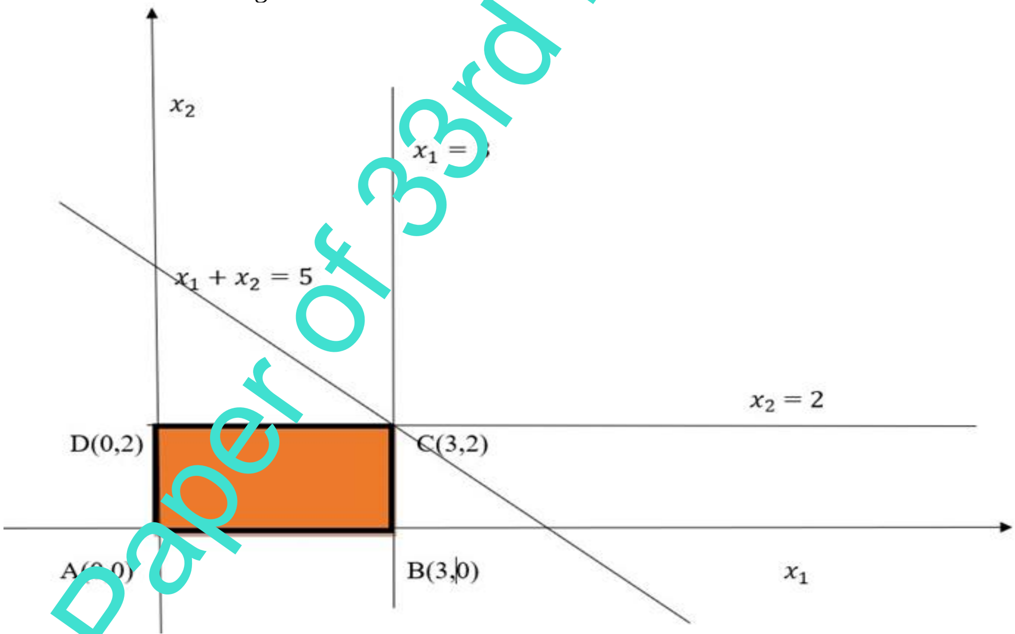

Fig.3. The graphical method

So, it is necessary to check the value of the function z at the points *A(0,0)*, *B(3,0)*, *C(3,2)* and *D(0,2)*, respectively, and choose the smallest value among them. Since *z(0,0)=158000*, *z(3,0)=131000*, *z(3,2)=100000* and *z(0,2)=127000*. Comparing the values of the function at the vertices of the quadrilateral, we conclude that the optimal solution is obtained at the point *C(3,2)* for which the objective function has the value *z=100000*. This means that $x_1 = 3$ and $x_2 = 2$, that is, it is necessary to acquire three dryer sections and a 2MW boiler. We should note that in this case the dryer will work only in one shift.

Let us now state the corresponding algorithm

Algorithm 1

```
m=158000
for i =0:3
    for j=0:2
        if i+j>0
            z=158000-9000*i-15500*j;
            if z<m
                m=z;
                x_1 = i;
                x_2 = j;
            end
        end
    end
end
x_1, x_2, m
```

In this example, we have a dryer that is not fully automated. If we want to check that the acquisition of equipment for automatic filling veneer, the price of which is 3000 euros, is economically worthwhile, we introduce a new variable x_3 which can take the integer values 0 and 1. On this occasion, we do not want to invest more funds for the acquisition of means of production than in the previous case. Now the new integer linear problem will take form of

$$minz = 158000 - 3480x_1 - 8300x_2 - 43200x_3 \quad (23)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 3 \\ 3 \\ 2 \\ 1 \end{bmatrix} \quad (24)$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (25)$$

Appropriate algorithm is

Algorithm 2

```
m=158000
for i =0:3
    for j=0:2
        for k=0:1
            if i+j>0
                if i+k<4
                    z=158000-3480*i-8300*j-43200*k;
                    if z<m
                        m=z;
                        x_1 = i;
                        x_2 = j;
                        x_3 = k;
                    end
                end
            end
        end
    end
end
x_1, x_2, x_3, m
```

Here we have obtained an optimal solution $x_1 = 2, x_2 = 2, x_3 = 1$ a $minz = 91240$.

5. Result analysis

In this section, the optimization results from the previous section are analyzed. Due to the discontinuity in the production of structural veneer, a decision was made to purchase dryer and boiler sections of 1MW or 2MW. At the beginning of the operation, for the operation of the dryer, which works in three shifts and receives heat energy through a 4MW boiler, at least 158000 euros were needed for the costs of possible loss and the costs of working in the dryer's shifts. The problem was solved by using integer linear programming. By applying Algorithm 1, it was found that the optimal solution is the purchase of three dryer sections and a 2MW boiler. Then the costs of possible losses and labor costs in the dryers are reduced to 100000 euros. So for an investment in production assets of 24000 euros, we get an improvement of 26000 euros in the first year, and the improvement starting from the second year is as much as 50000 euros. It should be pointed out that the work of the dryer was reduced to one shift, while it used to work in three. So the entire production can work in one shift. The labor of workers in one shift is also very important for their psychophysical health, and this directly reduces the costs of sick leave. On the other hand, workers who worked in multiple shifts can now be moved to other work tasks with proper scheduling.

In the second part, we dealt with the justification of purchasing equipment for automatic filling veneer, the price of which is 3000 euros. By applying integer programming with the same investment for production assets of 24000 euros, the optimal solution is obtained in the form of the purchase of 2 dryer sections, a 2MW boiler and equipment for automatic filling veneer, the price of which is 3000 euros. Then the costs of possible losses and work in shifts amount to 91240 euros. It means that by applying mathematical programming, we have found high-quality optimal solutions.

6. Conclusion

Veneer drying is the most important but also the most demanding phase in the production of structural veneers. Production discontinuity usually occurs due to an inadequate amount of heat energy in the dryer. This is solved in two ways: by acquiring additional sections of the dryer and by using a boiler of a certain power. In order to estimate the required number of dryer sections and the power of the boiler to be purchased, we used integer linear programming. Through linear programming, we proved that the number of shifts in the dryer can also be reduced.

If the dryer is not fully automated, which includes the supply of veneers to the drying sections; it has been proven here that investing in automation significantly reduces costs in the long term. The use of linear programming to improve production efficiency was considered in the specific example of a factory for the production of the structural veneers. Linear programming gave us two suitable algorithms for providing help in the making of investment decisions. Further research goes in the direction of applying multi-criteria linear programming and methods of fuzzy multi-criteria linear programming for the optimization of technological procedures and the production of structural veneer.

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