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Optimizing the Reference Signal in the Cross Wigner-Ville Distribution Based Instantaneous Frequency Estimation Method

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Abstract

An algorithm for optimizing the spread parameter of the reference signal (RS) in the recently proposed Cross Wigner-Ville distribution (XWVD) based method for the signal components instantaneous frequency (IF) estimation is presented. XWVD between the analyzed signal (AS) and the reference signal yields a scaled image closely resembling the IF laws of the signal components, provided a proper selection of the RS parameters has been made. The resulting image consists of the cross-terms between the AS and the RS, formed under strict geometrical rules which are subsequently exploited for signal components IF estimation. Since the method's performance strongly depends on the spread of the RS, an automatic optimization procedure based on differential evolution is proposed.

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Keywords: Wigner-Ville; reference; optimization; instantaneous frequency

1. Introduction

Signals are all around us. Speech, music, video, bird song, radio diffusion, radar, these are just a few examples. Useful information is often simultaneously present both in time and frequency, and to complicate the matter even further, frequency content is also often time varying - i.e. non-stationary, in contrast with stationary signals like, for instance, musical tones of constant frequency or voltage in power lines.

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In order to decode such non-stationary signals joint time-frequency (TF) analysis and processing methods are required and many such methods have been developed over time [1]. Generally they fall into one of the two categories of time-frequency distributions (TFD); the linear time-frequency distributions and the quadratic (bilinear or energy) TFDs [1].

Extracting useful information from TFDs often means revealing how signal's frequency content changes over time, i.e. finding the instantaneous frequency (IF) laws of the signal components. Signal component could, in general, be considered energy continuity in time without abrupt changes in frequency. Therefore, signals can be mono-, or multi-component in nature. Linear TFDs, such as short-time Fourier transform (STFT), are often used as a first choice of tools in TF analysis due to their simplicity in usage and interpretation. Yet linear TFDs lack in TF resolution and that is primarily why non-linear methods like the Quadratic class of TFDs have been introduced [1]. But non-linearity carries the associated problems of interference terms [1].

The core representative of the Quadratic class of TFDs is the Wigner-Ville distribution (WVD). Possessing many mathematically useful properties it is nevertheless corrupted by interferences or cross-terms caused by its bilinear nature whenever the signal under analysis is multi-component [2]. The cross-terms make the interpretation of the TF image more challenging by obscuring key signal features.

Cross-terms are in general considered as undesirable and majority of research has been directed towards their mitigation. Plethora of distributions has been proposed based on specific filtering of the WVD [1]. However, cross-terms do carry useful information [2], and possess some interesting exploitable geometrical properties, as it was shown in [3] and [4].

Building on the aforementioned properties, a simple, yet highly precise method for signal components IF laws estimation was proposed [5, 6]. Its key details are described in this paper along with a method for optimizing its most important parameter.

2. Instantaneous frequency laws from cross Wigner-Ville distribution

WVD of a real signal $s(t)$ can be calculated as [1]:

$$W_z(t,f)=\int_{-\infty}^{\infty} z\left(t+\frac{\tau}{2}\right)z^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi f\tau}\,d\tau \tag{1}$$

where z(t) is the analytic associate of $s(t)$. With $s(t)$ being a mono-component signal of linear frequency modulation (LFM), WVD defined in Eq. (1) will produce near perfect localization of signal energy in the time-frequency plane. However, for a multi-component signal, for instance a signal with two components $x(t)$ and $y(t)$, the quadratic superposition principle will take place [1]:

$$W_{x+y}(t,f)=W_x(t,f)+W_y(t,f)+2\cdot\mathrm{Re}\{W_{x,y}(t,f)\} \tag{2}$$

with

$$W_{x+y}(t,f)=\int_{-\infty}^{\infty} x\left(t+\frac{\tau}{2}\right)y^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi f\tau}\,d\tau \tag{3}$$

being the cross WVD (XWVD) of $x(t)$ and $y(t)$, and also the source of the notorious cross-terms.

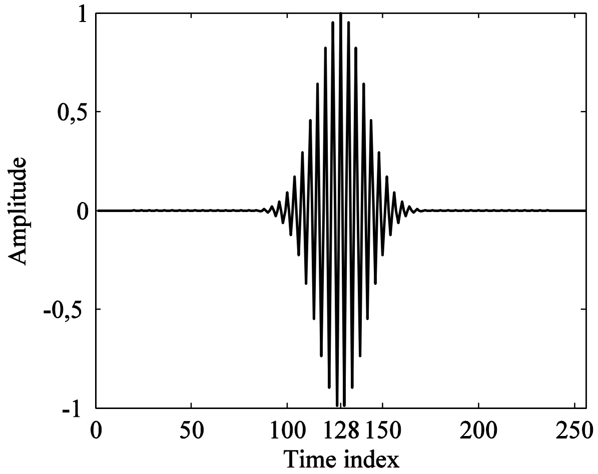

Fig. 1. Example reference signal in time domain. The time-frequency center has been set to 128 and 0,25, respectively. The spread of the Gaussian is 32 samples.

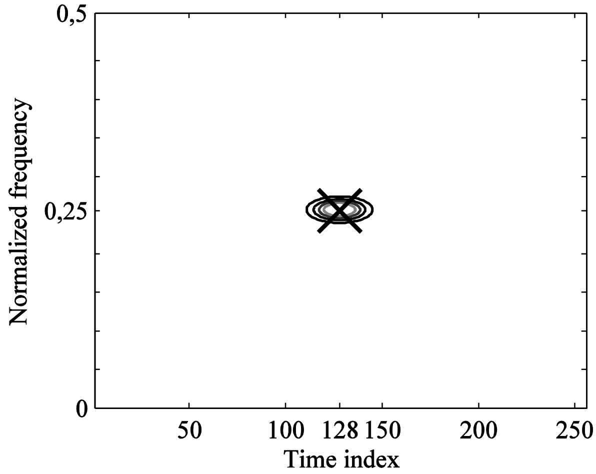

Fig. 2. The WVD of the reference signal in Fig. 1. × marks the time-frequency center.

In the Quadratic class of TFDs any signal component yields an auto-term (the WVDs of the individual components), and any pair of components produces a cross-term [2], which is evident from Eq. (2). If only the WVD of a signal is considered, a clear distinction between the signal terms and the interference (cross-terms) is not immediately evident. With the ever-present noise, TF image becomes even more cluttered since noise also obeys Eq. (2).

Yet cross-terms do not appear at random positions. Strict geometrical rules govern their whereabouts so that they appear exactly half-way on a straight line between any two points in the TF image. From this, it follows that the cross-terms TF coordinates can be calculated according to [2]:

$$t_i = \frac{t_1 + t_2}{2}, \tag{4}$$

$$f_i = \frac{f_1 + f_2}{2} \tag{5}$$

with t_i and f_i representing the time and frequency coordinates of the cross-term and (t_1, f_1) and (t_2, f_2) are the coordinates of the points that interfere. However, neither are known in general. But knowing the coordinates of one of the interfering points and the coordinates of the cross-terms, it would be possible to localize the other signal points. Based on the aforementioned, one such method was proposed [5]. Its steps are briefly outlined below:

1. Form the RS of length N as a Gaussian atom of unit amplitude. This provides the reference (t_1, f_1) coordinates. Example of a reference signal can be seen in Fig. 1, and its WVD in Fig. 2,
2. Calculate the XWVD of the RS and the AS. A scaled image of the true signal components formed by the absolute value of the cross-terms is revealed if the RS is chosen appropriately (see Fig. 3),
3. Extract the envelope of each of the time slices of the XWVD. For each time slice of the XWVD perform peak detection. The detected peaks represent t_i and f_i coordinates in Eq. (4) and (5), respectively,
4. Link the detected peaks into individual cross-components on the basis of energy continuity in time,
5. Perform up-scaling of the cross-terms coordinates:

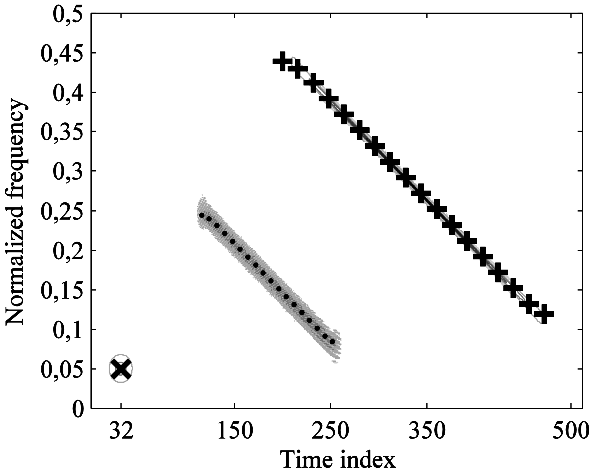

Fig. 3. The WVD of the sum of the RS and the AS with the cross-terms located in between: × marks (t_1, f_1) center of the RS, • marks the detected peaks of the cross-terms, and + marks the estimated coordinates of the AS [5].

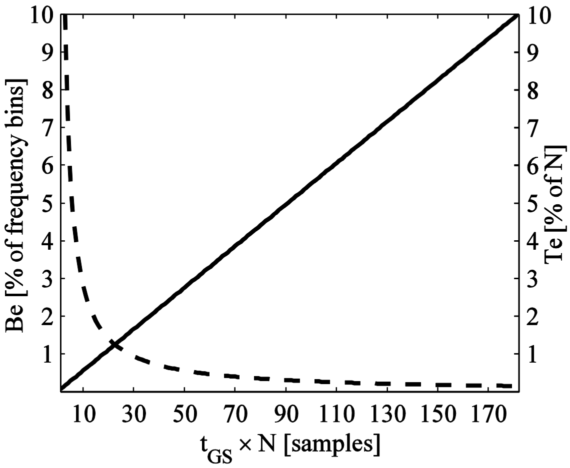

Fig. 4. Functional relationship between the t_{GS} and the resulting *Be* (dashed) as the percentage of the total frequency bins, and *Te* (solid) as the percentage of the signal length [7].

$$t_2 = 2t_i - t_1 \tag{6}$$

$$f_2 = 2f_i - f_1 \tag{7}$$

6. interpolate and smooth the scaled coordinates from the previous step to obtain the IF estimate of the AS.

Values of t_2 and f_2, resulting from Eq. (6) and (7), represent the estimated time and frequency coordinates of the AS component.

3. The reference signal optimization

For the described method the choice of the reference signal is of crucial importance. Gaussian shape eases peak detection due to a known shape, its time spreading regulates the concentration of the cross-terms in the TF plane and its good localization properties, both in time and frequency provide a firm reference for estimation of the IF laws. The influence of the RS parameters on the method's performance was more thoroughly investigated in [7]. It was found that the location (t_1, f_1) of the RS in the TF plane could be chosen arbitrarily and that the Gaussian shape is preferable also due to the lowest bandwidth-time duration product [1], i.e. the lowest spreading in time and frequency directions, therefore having the best theoretical concentration in TF plane [1]. The most influential RS parameter, denoted t_{GS} in the analytical expression for the RS [7]:

$$RS(t) = e^{-\left(\frac{\sqrt{\pi}}{t_{GS}}\right)^2 t^2} \cdot e^{j2\pi f t} \tag{8}$$

was found to be its time spreading, defined as the full width at half-maximum of the peak in the envelope of the RS. This spreading governs the concentration of the cross-terms, much like the window function in the STFT [1].

Based on definitions of the effective bandwidth (*Be*) and the effective time duration (*Te*) [1], a functional relationship between the selected t_{GS} and the resulting *Be* and *Te* as a percentage of the total signal length (*N*) was determined (see Fig. 4) [7]. From the extensive tests on the synthetic multi-component signals it was concluded that for a good resolution in frequency *Be* should be below 1 percent of the total number of frequency bins. In order to achieve this less that 1 percent cross-term spreading in frequency, *Be* is required to be at least 1 percent of the total signal length which then translate to the required minimum t_{GS}. Increasing t_{GS} benefits the frequency resolution but a strong degradation can be observed in time localization beyond certain value. A practical limit for the *Te* value in the range of 1 to 5 percent of the total signal length was found to be acceptable in terms of time resolution degradation.

Ultimately the choice of t_{GS} value depends on the nature of the analyzed signal, which is unknown in general case. Therefore the analysis from [7] can serve only as a guideline. In order to achieve the best possible cross-terms concentration, i.e. optimize the value of t_{GS} for the analyzed signal, herein we propose an automatic method based on cost function minimization and differential evolution algorithm.

3.1. The cost function

By recognizing that cross-terms concentration governed by t_{GS} plays a dominant role in the IF estimation of the proposed method, a suitable measure that would reflect conditions in the TF image was sought after. We reasoned that the sharper the peaks in the envelope of the XWVD, the better the concentration of the cross-terms is. Therefore we propose a ratio of total XWVD energy and energy in its maxima as a suitable concentration measure:

$$\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|W_{RS,AS}(t,f)\right| dtdf}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\max\left\{\left|W_{RS,AS}(t,f)\right|\right\} dtdf} \tag{9}$$

with smaller values indicating better concentrations.

Formulating the problem of optimal t_{GS} selection as minimization of Eq. (9) with Eq. (9) considered as a function of t_{GS}:

$$\min_{t_{GS}} f(t_{GS}), \quad 0 < t_{GS} \cdot N \le N \tag{10}$$

an automatic procedure for finding the global minimizer of Eq. (9) could be devised according to Eq. (10). All that is needed is a suitable optimization algorithm.

3.2. The optimization algorithm

Gradient based optimization methods although recognized as effective are often inconvenient. The complexity of determining function derivatives and the problem of the associated numerical noise due to finite precision frequently makes them unsuitable in practice [8]. Opting for a robust, simple to implement global optimization algorithm that would not require derivatives, amongst the existing ones, we decided on the differential evolution.

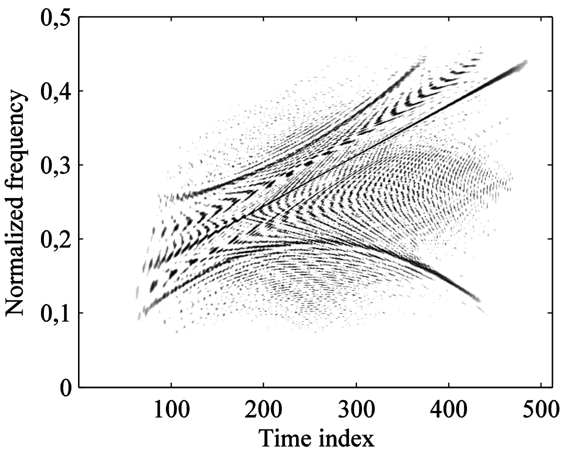

Fig. 5. The WVD of the test signal in [6]. The signal consists of two parabolic FM components and a LFM chirp located in between embedded in 5 dB AWGN. Signal length is 512 samples.

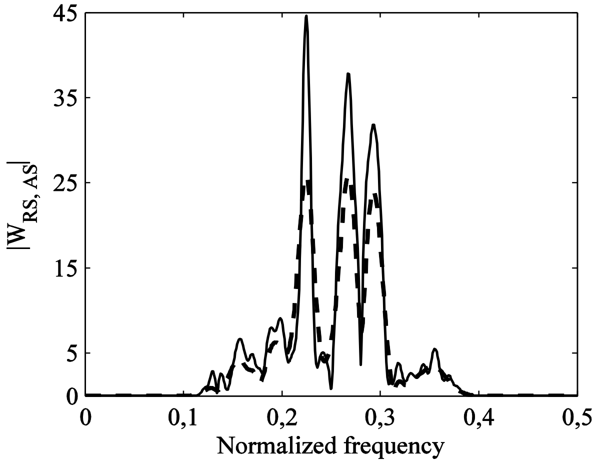

Fig. 6. The XWVD time slices of the test signal in Fig. 5 at the time index 256. The result form [6] with t_{GS} of 26 (dashed) is overlaid with the optimized t_{GS} of 43,48 (solid). Better concentration is evident with the optimized t_{GS}.

Differential evolution (DE) was first described in [9] and belongs to the branch of evolutionary algorithms like the evolution strategies (ES) and genetic algorithms (GA). Ever since it has been gaining popularity by demonstrating its effectiveness on a wide variety of benchmark and real life problems. It works by maintaining a population of potential solutions called individuals which are continuously improved - evolved through intelligent use of differences between individuals [8]. At the core of this algorithm there are three basic operations: differential mutation, recombination and selection which are very simple to implement and apply in an iterative procedure. Also there are only three control parameters that guide/affect the evolution: population size (*NPOP*), difference amplification (*F*) and the probability of recombination i.e. cross-over (*Cr*). General overview of the algorithm as implemented for the purpose of finding the global minimizer of Eq. (10) follows suggestions from [8] and is given next (for a more detailed description please cf. [8] or [10]):

1. set *NPOP*, *F*, *Cr*- the control parameters
2. set boundary constraints on the individuals
3. initialize population to random values inside boundaries and evaluate fitness (cost function)
4. while not stopping conditions, for every individual, iterate:
    a. chose at random three individuals, different from the current one
    b. create a trial individual from the three random individuals by means of differential mutation and recombination
    c. verify boundary constraints, reinitialize if not feasible
    d. evaluate trial individual
    e. select better solution, trial or current, replace current with trial if needed
5. repeat from 4.

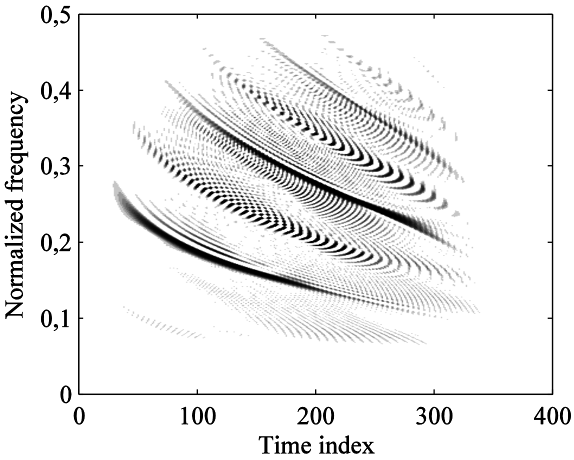

Fig. 7. The WVD of the echolocation signal as emitted by the large brown bat.

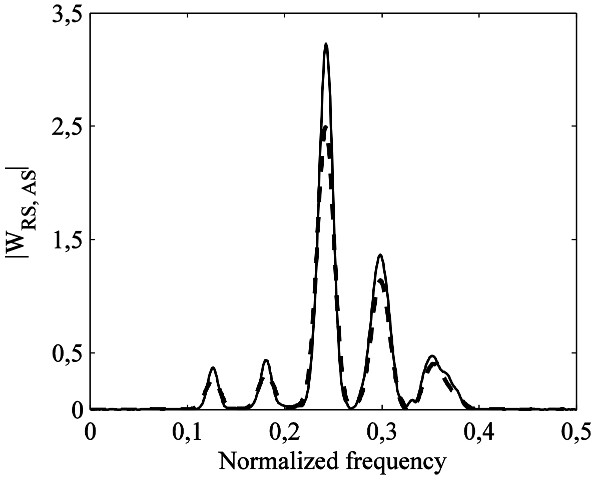

Fig. 8. The XWVDs time slices of the bat echolocation signal at the time index 236. The result with t_{GS} of 26 (dashed) is overlaid with the result using the optimized t_{GS} of 37,19 (solid).

4. Results

Setting DE control parameters according to suggestions in [8] and [10] (*NPOP=40*, *F=0,85*, *Cr=0,5*) and deciding to run the evolution for 300 iterations, the optimization algorithm was tested on the example from [6] whose WVD is shown in Fig. 5. The signal from [6] is a multi-component mixed class signal consisting of two parabolic frequency modulated components and a LFM chirp embedded in 5 dB zero mean additive white Gaussian noise (AWGN). The value of 26 for t_{GS} in [6] was set on the basis of expert knowledge and the value produced by the proposed automatic procedure is 46,48. Overlaying the results of the time instant ($t_i=256$, see Fig. 6) where the signal components are the closest indicates clear advantage in achieving better cross-component concentration and resolution in the case of the automatic procedure.

Furthermore we tested the proposed method on the echolocation signal as emitted by the large brown bat. Fig. 7 shows the WVD of the echolocation signal. In Fig. 8, time slices of the XWVDs ($t_i=236$, corresponding to the time instant 272 in WVD, cf. Eq. (4)) as obtained with t_{GS} set to 26 and with the optimized t_{GS} of 37,19 are overlaid. Again, better concentration has been achieved with the spread of the reference signal tailored to the analyzed signal.

5. Conclusion

Along with an overview of the recently proposed cross Wigner-Ville distribution based instantaneous frequency estimation method this paper has presented a procedure for determining the optimal spread of the Gaussian atom in the reference signal. Differential evolution paired with the proposed cross-terms concentration measure in an automated procedure yields results that are in accordance with the previous analysis and has the additional benefit of a reference signal automatically tailored to the analyzed signal without the need for an expert knowledge.

As a future research, an in-depth analysis of the differential evolution control parameters and their influence on the proposed method's performance will be conducted along with the extensive tests on the synthetic and real-life signals.

Acknowledgements

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