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Dynamic Mathematical Model of Two-Way Bellow Actuator

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Abstract

In this paper is reviewing an elaboration of dynamic mathematical model of two-way bellow actuator involving gas dynamics theory. This actuator is based on bellow actuators produced by "Festo". The main idea of elaboration is based on consideration of gas processes using some equations of gas dynamics. The emphasis was made on simplification and linearization of these equations due to high complexity of them. As a result is received a reliable dynamic mathematical model of a two-way bellow actuator. As an example is shown a Matlab modelling of vibration stand actuator.

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Keywords: linearization; mathematical model; bellow actuator

1. Introduction

New materials and technologies allow receive elements and structures which can be used for creation of nontraditional actuating motors with high technical-economic indicators. To them relate bellow actuators produced by "Festo". To utilize them in closed-loop systems it is necessary to have a reliable dynamic mathematical model which also describes different processes inside bellow actuator. There are some materials about PAMs mathematical models [1, 2, 3, 4] but few information about such of bellow actuators.

Aim of this article is to elaborate accurate and reliable dynamic mathematical model involving gas dynamics. Development of such model is obvious for closed-loop systems simulations and calculations. This mathematical model allows developers calculate different actuators based on such elements. Two-way bellow actuator represents two antagonistic connected bellow actuators. At first it is necessary to work out an one-way mathematical model.

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Constructively bellow actuator is an axisymmetric elastic membrane armored with a system of inextensible cords. Butts are hermetically secured in conjunctive elements (Fig. 1). When compressed gas enter into a bellow actuator appear high pushing force in consequence of membrane deformation and growth an axial dimension.

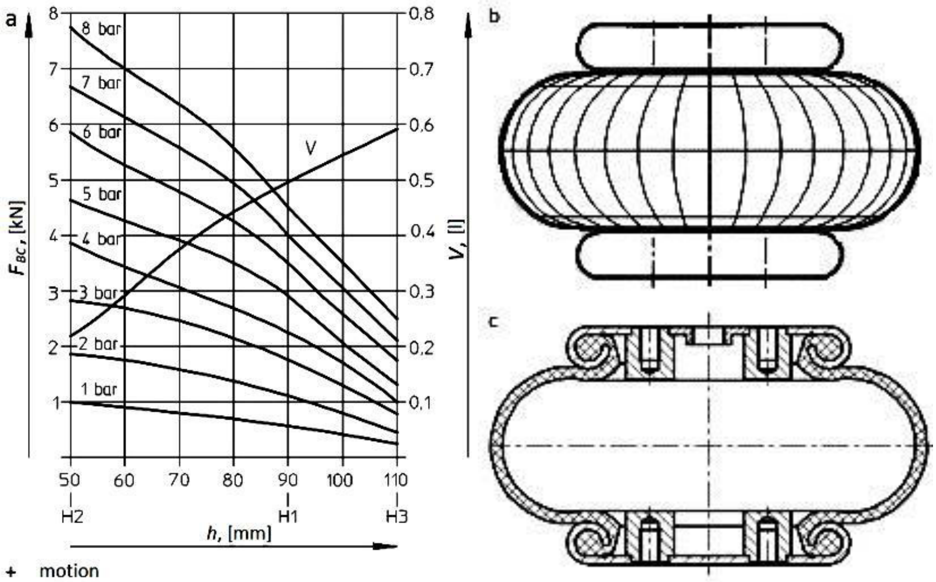

Fig. 1. (a) load bearing characteristics, bulk property; (b) design; (c) construction of bellow actuator.

2. Nonlinear dynamic mathematical model

Nonlinear dynamic mathematical model includes following equations:

- force F_{BA} and its correlation between change of height h and pressure p describes with an equation [5]:

$$F_{BA} = k_F\left(k_{hp}\Delta p - h_0\right) \tag{1}$$

where F_{BA} – effort of bellow actuator; k_F – coefficient of rigidity of load bearing characteristic; k_{hp} – proportionality constant between stroke and increase of pressure Δp; Δp – increase of pressure; h_0 – initial height of bellow actuator.

- equatoin of forces which influence on control object:

$$F_{BA} = m\frac{d^2h}{dt^2} + F_L + k_{fr}\frac{dh}{dt} + F_E; \tag{2}$$

where F_L – effort to overcome the load; F_E – effort of material elasticity; m – control object mass; k_{fr} – viscous friction coefficient.

On Fig. 2 is shown experimental reaction of transient process of bellow actuator, based on mechanical muscle [4]. It has similar thickness of a membrane to bellow actuator. We suppose that the same reaction of bellow actuator would be similar. The reaction is not oscillatory and contains three areas: acceleration, constant velocity and retard. We would research area with constant velocity because there bellow actuator has the best efficiency, what is the most important for using bellow actuators in technical systems.

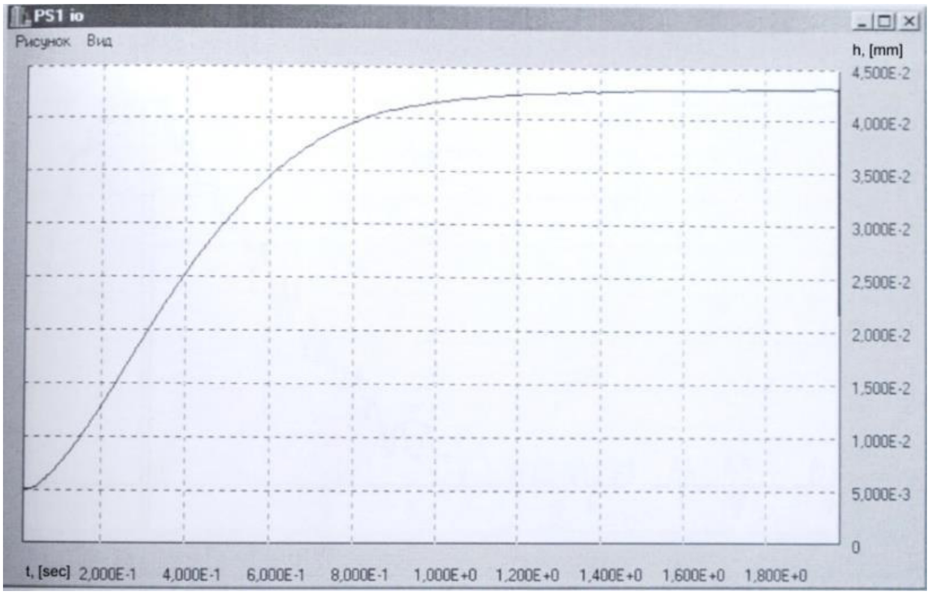

Fig. 2. Step reaction of control object.

- state equation [6]:

$$\rho = f(p, T) \tag{3}$$

where p – pressure; ρ – density; T – temperature of a compressed gas.

- continuity equation [6]:

$$\frac{d\rho}{dt} + \rho div u = \rho q_m \tag{4}$$

where q_m – supply velocity of gas mass per unit mass.

- momentum conservation law [7]:

$$\rho \frac{du}{dt} = \rho P_m - grad\ p + \left(\mu_V + \frac{\mu}{3}\right) grad\ div\ u + \mu \nabla^2 u \tag{5}$$

where P_m – bulk forces vector per unit mass; ∇^2 – Laplacian; x, y, z – rectangular coordinate system; $dx/dt = u_x$, $dy/dt = u_y$, $dz/dt = u_z$.

- dynamic viscosity equation [7]:

$$\mu = \rho \nu \tag{6}$$

where ν – kinematic viscosity.

As a result, combined equations for one-way system are following:

$$\begin{cases} \rho = f(p,T); & (a) \\ \mu = \rho v; & (b) \\ \frac{d\rho}{dt} + \rho divu = \rho q_m; & (c) \\ \rho \frac{du}{dt} = \rho P_m - grad\ p + \left(\mu_V + \frac{\mu}{3}\right) grad\ div\ u + \mu \nabla^2 u; & (d) \\ F_{BA} = m \frac{d^2 h}{dt^2} + F_L + k_{fr} \frac{dh}{dt} + F_E; & (e) \\ F_{BA} = k_F \left(k_{hp} \Delta p - h_0\right). & (f) \end{cases} \qquad (7)$$

Taking into account two antagonistic bellow actuators in combined equations (7) acquire such a form:

$$\begin{cases} \rho = f(p,T); & (a) \\ \mu = \rho v; & (b) \\ \frac{d\rho}{dt} + \rho divu = \rho q_m; & (c) \\ \rho \frac{du}{dt} = \rho P_m - grad\ p + \left(\mu_V + \frac{\mu}{3}\right) grad\ div\ u + \mu \nabla^2 u; & (d) \\ F_{BA} = m \frac{d^2 h}{dt^2} + F_L + k_{fr} \frac{dh}{dt} + F_E; & (e) \\ F_{BA} = k_{F_{lp}} \left(k_{hp_{lp}} \Delta p - h_{lp}\right). & (f) \end{cases} \qquad (8)$$

where h_{lp} – height of bellow actuator in linearization point; k_{Flp} and k_{hplp} – coefficients taken from linearization point. More detailed it would be covered further.

3. Accepted assumptions and limitations

I. Describing working medium gas flow is counted operating equally in all directions nevertheless consideration is limited only lengthwise the axis z and do not consider areas of acceleration and retard. Transient processes running in cross-section are neglected because its diameter change is less than linear dimension. Therefore influence of gas-dynamic processes in that directions appear less. In this case gas flow is counted steady;

II. Air temperature changes assume insignificant. Therefore air cinematic viscosity v can be counted constant. Volume viscosity μ_V and bulk forces vector P_m are neglected so far as in the working medium (air) they are insignificant. Pressure p changes insignificant;

III. Bellow actuator is hermetic and mass increase is even. Value gas mass flow G_m directed lengthwise the axis z at inlet end should be maximal and at opposite end E/Z;

IV. On purpose of simplification of values of pressure in each point inside bellow actuator is used concept of mean parameters value of pressure p, radius of curvature generatrix R and gas density ρ which are taken at bellow actuator middle section.

4. Linearized dynamic mathematical model of bellow actuator

Here is made linearization of combined equations (8) by reason of complexity of mathematical apparatus and because using it in overall view is labor-intensive. Linearization of mathematical model (8) forms is made including mentioned assumptions and limitations. Consideration of antagonistic bellow actuator force in linearization point can be made in different ways: adding antagonistic force in equation (7*e*) or changing coefficients in equation (7*f*). In this article is used second method because calculations are made for application in actuator of vibration stand where pressure changes insignificantly.

Results of linearization are following:

$$\begin{cases} \rho(p) = \rho_0 + k_{\rho p} p; & (a) \\ \frac{2}{\rho_{lp}} \frac{d\rho_{lp}}{dt} + k_r \frac{dR_{lp}}{dt} = \frac{k_u}{h_{lp}} \frac{Q_3}{k_V}; & (b) \\ F_{BA} = m\frac{d^2h}{dt^2} + F_L + k_{fr}\frac{dh}{dt} + k_E h; & (c) \\ F_{BA} = k_F\big(k_{hp}\Delta p - h_0\big). & (d) \end{cases} \quad (9)$$

Coefficient k_F can be found from the load bearing characteristics diagram by their reciprocal subtraction in neutral position. Here is Laplace transformation of combined equations (9):

$$\frac{k_F k_{hp} k_u \rho_{lp}}{2k_{\rho p} V_{lp}} \frac{Q_v(s)}{s} = \left[s^2 m + s k_{fr} + k_F \left(\frac{k_{hp} k_r}{k_{\rho p}} \frac{\rho_{lp}}{2\sin\frac{\alpha_{lp}}{2}} + 1 \right) + k_E \right] H(s). \quad (10)$$

This expression can be written simplier:

$$k_F k_{hp} \frac{C_1}{V_{lp}} \frac{Q_3(s)}{s} = \left[s^2 m + s k_{fr} + k_F\big(k_{hp} C_1 C_2 + 1\big) + k_E\right] H(s);$$

$$C_1 = \frac{k_u \rho_{lp}}{2k_{\rho p}}, C_2 = \frac{k_r}{k_u \sin\frac{\alpha_{lp}}{2}}.$$

On basis of superposition principle can be found motion transfer function for two-way bellow actuator:

$$W_{BAtf}(s) = \frac{H(s)}{Q_v(s)} = \frac{k_{tf}}{s(T^2 s^2 + 2\xi T s + 1)};$$

$$k_{tf} = \frac{k_F k_{hp} C_1}{\big(k_F(k_{hp} C_1 C_2 + 1) + k_E\big) V_{lp}}; \; T = \sqrt{\frac{m}{k_F\big(k_{hp} C_1 C_2 + 1\big) + k_E}};$$

$$\xi = \frac{k_{fr}}{2} \frac{1}{\sqrt{m(k_F\big(k_{hp} C_1 C_2 + 1\big) + k_E)}},$$

where $Q_v(s)$ – Laplace representation of flow on bellow actuator inlet.
Value ξ defined by values of membrane elastic force and viscous friction coefficient.

5. Two-way bellow actuator modelling

Matlab modelling would be made on base of two-way bellow actuator which represents a vibration stand actuator. Initial data for modelling are following: load mass of 300 kg, frequency of 5 Hz and amplitude of 5 mm. Modelling was made on a sin wave. Was used PD controller. The result is shown on Fig. 3. Time delay appears because of compressibility of gas. From Fig. 3 we can see that calculations of dynamic mathematical model are accurate enough.

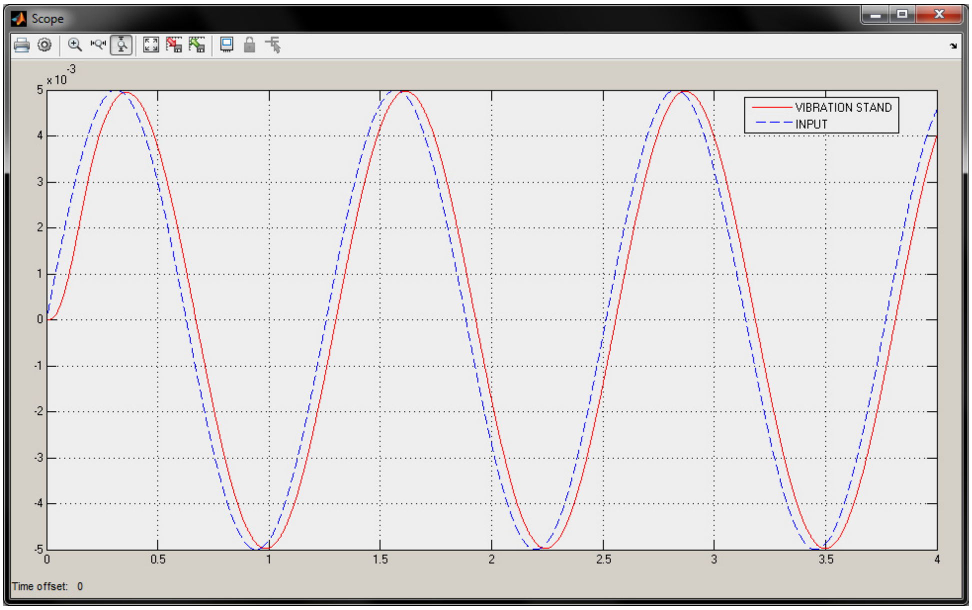

Fig. 3. Sin wave reaction of a vibration stand actuator.

Conclusion

In this paper is received a high reliable dynamic mathematical model of two-way bellow actuator. The main idea of development of mathematical model was a consideration of gas processes using some equations of a gas dynamics. Development of the model included:

- Analysis of bellow actuator characteristics
- Creation of nonlinear dynamic mathematical model, its simplification and linearization
- Creation of transfer function of two-way bellow actuator

The emphasis was made on simplification and linearization of these equations due to high complexity of them. Such model together with reliable models of electro-pneumatic throttle spool-type valves allow to receive more accurate pneumatic drive model.

Here is represented Matlab modelling on sin wave input for a vibration stand actuator.

An experimental part of this research for confirmation of received results is in progress now.

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