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Procedia Engineering 100 (2015) 953 – 959

Procedia Engineering

www.elsevier.com/locate/procedia

25th DAAAM International Symposium on Intelligent Manufacturing and Automation, DAAAM 2014

Use of Monte Carlo Modified Markov Chains in Capacity Planning

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Abstract

The purpose of the paper is to demonstrate the use of Monte Carlo modified Markov chains in capacity planning. After a brief review of available literature on the subject of Monte Carlo simulations and Markov chains, the benefits of this method are illustrated on a practical example, where demand as well as the machine availability is dictated by Markov chains. Monte Carlo simulations are used to provide an additional view on the problem. Conclusions are presented in a well arranged manner, using graphs as the main output. These graphs provide the perfect wake up call for upper management since they clearly show lost profits due to ineffectiveness within the model. A sensitivity analysis is also performed in order to determine the variables that affect the model the most.

Peer-review under responsibility of DAAAM International Vienna

Keywords: Monte Carlo simulations; Markov chains; random walk; capacity planning; machine availability

1. Introduction

Determining the optimal capacity for production has been at the center of attention ever since large scale production was conceived and implemented into practice. With the modern economy putting pressure on efficiency and minimizing waste, companies cannot afford to operate more machines than absolutely necessary or to hire more worker than absolutely necessary just to have spare capacity in case the calculations were incorrect or imprecise.

Markov chains have been used in a variety of fields to better describe the state of various things while knowing the previous state of the observed problem. Monte Carlo simulation increases the value of using Markov chains in that it can modify otherwise discrete Markov Chains to include stochastic aspects. Monte Carlo simulation can also be used to construct random walks on the Markov chains to better illustrate the calculation to the decision-makers of

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Peer-review under responsibility of DAAAM International Vienna
doi:10.1016/j.proeng.2015.01.454

upper management by providing a more intuitive and iterative approach.

The purpose of this paper is to illustrate the value of modifying Markov chains with Monte Carlo simulation aspects in order to provide more information about the observed reality and the problem at hand. This illustration is performed using an example from the field of capacity planning, showing different results for various methods of calculation before comparing them and providing conclusions. This practical illustration is preceded by a brief overview of available literature in order to familiarize the readers with the field of Markov chains and the Monte Carlo simulations.

2. Brief literature review

In general, a Markov process can be defined as any process in which the current state of things depends solely on the state of things immediately preceding the current one. In other words, in a Markov process it is possible to completely neglect any results except the very last one when predicting the future results. The process moves from one state to another depending solely on the starting state. It can then move to any other state with a given probability. A Markov chain is a Markov process in which the probabilities of the process going from one state to another do not change with time. Therefore, a Markov chain will behave the same way no matter at what point in time it is observed – the only thing of importance is the very last state of things in the process [1].

A Markov chain moves through different states from a possible set of states, $S = \{s_1, s_2,\ldots,s_r\}$. The process starts in of these states s_i and moves to another one s_j at the next time step with a probability of p_{ij}. In a Markov chain as opposed to a general Markov process, the probabilities pij, also called transition probabilities, do not change with time elapsed. In addition to the movement from one state to another, the process may also stay in the same state s_i with a probability of p_{ii}. A matrix containing all of the values for p_{ij} for different values of i and j is called the transition matrix [2].

There are, of course, special types of Markov chains that can be described and analyzed. An absorbing Markov chain is a Markov chain in which at least one absorbing state (state that the Markov chain cannot leave, i.e. pii = 1) exists and if from every state it is possible to get to the absorbing state. It follows that in an absorbing Markov chain the process stabilizes once it reaches one of the absorbing states. On the other hand, in an ergodic Markov chain it is possible to get from every state to every state, which means the process never stabilizes [3].

The probability that the process is present in each of the possible sets at a given point in time can be calculated as a product of the vector denoting the state of the process at time 0 and the transition matrix to the power of n, where n is the time step for which the calculation takes place. As n approaches infinity, the vector denoting the present state of the process converges to a stationary vector, assuming the Markov process is ergodic [4].

Markov chains have so far been used in a wide variety of fields to study a huge amount of different subjects, for example in investment decision making [5], stock selling problems [6], or wind speed forecasting [7].

Monte Carlo simulation stochastically estimates a predefined output based on at least partially stochastically defined inputs. "The name Monte Carlo was applied to a class of mathematical methods first used by scientists working on the development of nuclear weapons in Los Alamos in the 1940s. The essence of the method is the invention of games of chance whose behavior and outcome can be used to study some interesting phenomena. While there is no essential link to computers, the effectiveness of numerical or simulated gambling as a serious scientific pursuit is enormously enhanced by the availability of modern digital computers" [8].

[9] used the card game called patience to describe the main idea of the method. Although it is theoretically feasible to estimate the odds of winning the game of patience by using simple probability methods, it would be an extremely arduous and unnecessarily lengthy process. By choosing a strategy, a Monte Carlo simulation can be used to simulate and playing the game over and over with that strategy several million times. Following that, the resulting distribution should well enough describe the odds of winning.

Since its conception in mid-20th century, Monte Carlo simulations have grown in popularity among the scientific community, becoming a mainstay of stochastic decision making tools, used in many fields including real options [10], air pollution [11], workspace-visualization [12], or hydro power plants availability [13], among a myriad of others.

Capacity planning has been described comprehensively and in detail in [14], with [15] offering a system dynamics view of the problem and [16] discussing the problem in business networks. A model of total costs for determining optimal capacity can be found in [17].

3. Model background and description

An unspecified company is facing cash-flow difficulties for the upcoming financial year. The demand for its products is unstable as is the price that the company can charge its customers. There are 2 other competing companies on the market and the competition is fierce, though the customers are somewhat loyal to the chosen brand. The cash-flow planning process has so far yielded an expense value of \$19 million per month, assuming the company runs 4 fully operated production machines with a capacity of 50 000 units per month per machine. Each machine has an observed failure rate which can either result in the production of a defect or it can stop production at the given machine altogether until repaired.

Marketing research based on historical observations has determined that the number of units purchased by each customer per month follows a Normal distribution with a mean of 3 units and a standard deviation of 0,1 unit. The price charged per unit similarly follows a Normal distribution with a mean of \$200 and a standard deviation of \$10. The market itself is not of a fixed size but rather fluctuates. At the start of the financial year the market is estimated to include 100 000 potential customers with a fluctuation following a Normal distribution with a mean of 0% and a standard deviation of 0,05%. The market is known for rather low marketing campaign efficiency and it is segmented among the 3 competitors based on a loyalty factor following a Markov chain. At the beginning of the model's observation, the market share of all 3 competitors is the same, with the market being equally distributed between all of the rivals. The transitional matrix of the Markov chain describing the buying behavior of any given customer given the information about their behavior in the previous time step is shown in Fig. 1 (S1 through 3 denote the demand for the goods of the competitor 1 through 3 respectively, with S1 denoting the company observed in this example and S2-3 denoting the competitors.

	S1	**S2**	**S3**
S1	0,5	0,2	0,3
S2	0,1	0,3	0,6
S3	0,4	0,4	0,2

Fig. 1. Transitional matrix for the demand Markov chain.

Each of the company's machine has a capacity of 50 000 units per month with a failure rate following a Poisson distribution with a mean of 100 defects per month. At the same time, each machine can either be operational or not operational during a given month. At the beginning of the financial year, all machines are fully functional. Their behavior from that point on is dictated by a Markov chain with a transitional matrix shown in Fig. 2. Note that each machine is differently reliable which results in different transitional matrices for each machine.

Machine 1 operational	Yes	No
Yes	0,95	0,05
No	0,9	0,1

Machine 2 operational	Yes	No
Yes	0,9	0,1
No	0,95	0,05

Machine 3 operational	Yes	No
Yes	0,999	0,001
No	0,5	0,5

Machine 4 operational	Yes	No
Yes	0,85	0,15
No	0,99	0,01

Fig. 2. Transitional matrices for machine reliability Markov chains.

4. Monte Carlo modified Markov chains

The market share of all 3 companies at the beginning of the model's observation is the same – each competitor controls a third of the market. Fig. 3 shows the probabilities for each state at the given point in time (in this case there are 12 months in the observed time span, hence 12 time steps in Fig. 3).

n	1	2	3	4	5	6
S1	0,333333	0,343333	0,343333	0,344033	0,344033	0,344082
S2	0,3	0,303333	0,301	0,301233	0,30107	0,301086
S3	0,366667	0,453333	0,355667	0,354733	0,354897	0,354831

n	7	8	9	10	11	12
S1	0,344082	0,344086	0,344086	0,344086	0,344086	0,344086
S2	0,301075	0,301076	0,301075	0,301075	0,301075	0,301075
S3	0,354843	0,354838	0,354839	0,354839	0,354839	0,354839

Fig. 3. State probabilities at different points in time.

The stationary vector is apparent from Fig. 3 and can be calculated with n approaching infinity as equal to (0,344086022; 0,301075269; 0,35483871). The data in Fig. 3 represent the companies' market shares at different points in time. For example, at the end of the financial year, the observed company is expected to hold 34,4086% of the entire market with one competitor slightly above 30% and another above 35%. These values will be important later on in the finished model.

The rest of the Markov chains described in the model's background determine the functionality of the machines used in production and thus the production capacity. The simulation used in this paper requires the knowledge of the machines' functionality at all 12 points in the observed time span. This is calculated in the following way. The state of the first machine at time 0 is known – the machine is functional. Therefore, during the first month the first machine will be functional with a 95% probability. The simulation takes this information into account and generates a random number following a uniform distribution at the (0; 1) interval. Depending on the results of the random number, the simulation either assigns the first machine a 1 representing functionality or a 0 representing non-functionality. In this way, the simulation assigns 1s and 0s to all machines at all points in time. The capacity available for production is a simple sum of products of the machine states and their respective capacities.

5. Simulation results

There were 1 000 000 runs completed using the simulation software Oracle Crystal Ball as an extension of the Microsoft Excel 2010 software. Fig. 4 shows the results for the amount of defective units produced in the last month of the observed time span.

These results follow the logical assumption that only the functional machines can produce defective units. In the very unlikely scenario that only one machine will be functional, only around 100 defective units will be produced. Do note that there is no overlap between the 1-machine-functional scenario and the 2-machines-functional scenario, whereas there is an overlap between all other pairs of scenarios – for example 350 defective units can be achieved both with 3 operational machines (in which case it could be considered unlucky) and 4 operational machines (in which case it would be considered lucky).

But, of course, the number of defective units is not the main variable that the company's planning team is interested in – that important variable is the income made via sales. That value is calculated using the price and the amount sold, which is shown in Fig. 5 for the last month of the observed time span. Note the interesting occurrence at the 100 000 mark.

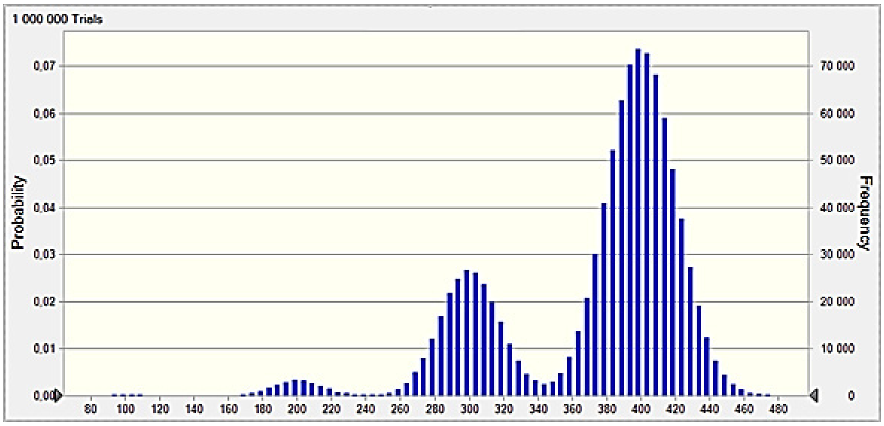

Fig. 4. Defective units in the 12th month.

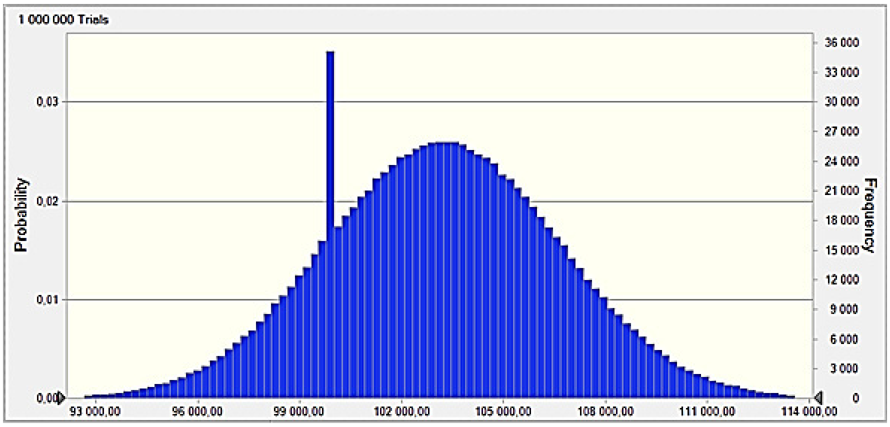

Fig. 5. Units sold in the 12th month.

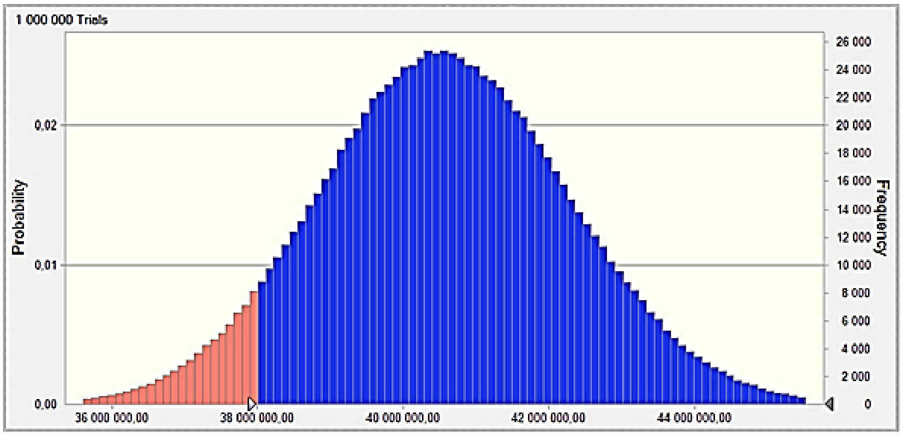

Fig. 6. Income in the 1st month.

With the demand fluctuating around 100 000 units, it is apparent that the company will be able to satisfy all of the demand only if at least 3 machines are fully functional or if 2 machines are fully functional and the company gets lucky with the amount of defective units produced in that given month. More importantly, the line extending above the standard bell curve in Fig. 5 clearly illustrates to upper management the sales lost from insufficient capacity. The line represents all of the customers that had to be turned away since the capacity was not big enough to satisfy the demand.

In order to determine the financial stability of the cash-flow plan, a per-month income is required. The total income at the end of the first month is shown in Fig. 6. There is only a 79,4346% chance that the income will be sufficient to cover the expenses in the first month of the planned financial year. However, this chance decreases each subsequent month, since the probability that the income will cover the expenses is greater than 50% each month and therefore with each subsequent month there a larger chance that a previous month generated profit that could cover the potential losses. The risk of financial instability at the end of the financial year is only 0,0111%. However, the same risk at the end of the first month is slightly greater than 20%, which should indicate that the company should prepare some kind of a loan in case the income doesn't cover the expenses.

A sensitivity analysis was also performed using the Monte Carlo simulation with each variable in the model sequentially lowered 10% and increased 10% to determine the impact of the change on the resulting cash-flow stability. The price per unit, market share, market size and individual demand changes all non-marginally influenced the final outcome. The pessimistic scenario for these variables increased the risk of negative cash-flow in the first month to 45,4512%, 43,3659%, 43,2145% and 37,9800% respectively.

Using this method results in a more comprehensive overview of the options available to the company, as well as a clearer interpretation of the results than other methods currently used in the field.

Conclusion

This paper presented an explanation on how Markov chains and Monte Carlo simulations can be utilized in capacity planning while illustrating the points using a simple case study example. This can be used as a starting point for further research in utilizing both discrete and stochastic methods in capacity planning. Even though both Markov chains and Monte Carlo simulations are widely used methods, their integration in the context of capacity planning presents a new way of looking at the problem of creating adequate and sufficient conditions for production in today's unstable, uncertain world.

By using Monte Carlo modified Markov chains, the paper uncovers an inherent risk in a company's cash-flow forecasted plan for the upcoming financial year by pointing out possible inadequacies in income when compared to the projected expanses. The presented graphical outputs clearly illustrates the lost sales form insufficient capacity in a way that can be easily understood by anyone, regardless of their knowledge of the subject or of Monte Carlo simulations and Markov chains.

Acknowledgements

This paper was created as a part of the SGS-2014-040 project – Modern approaches to innovation, entrepreneurship and management.

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