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Dynamics of a Frozen Sea Buckhorn Berry Motion in a Separator

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Abstract

After removing frozen sea buckthorn berries from the branches of sea buckthorn tree in a separator (patent EE05717B1), the berries fall onto separator's moving conveyer belt under a certain angle of inclination, thus guiding the berries in the collector of separator. This paper presents the mathematical model of the berry motion along the conveyer and after falling from it. The nonlinear differential equations describing the berry motion on the conveyer and after falling from it take into account air resistance depending on the square value of berry's absolute velocity. All these equations were solved numerically by using Runge-Kutta method of 4th order on Mathcad worksheet. This paper focuses on berry motion depending on belt velocity, friction coefficient between belt and berry, and inclination angle of the belt. Composed video clip visualizes berry motion and demonstrates how the berry moves from the initial position in the same direction as conveyer belt velocity, down to lower edge of the belt and falls from conveyer. Results of this paper can be used by engineers designing conveyors and by educators teaching agricultural machinery.

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Keywords: berry separator; mathematical modelling; berry motion; visualization; Mahtcad; belt conveyor

1. Introduction

Moving different materials on belt conveyors is a very common solution used in various industries because of their high efficiency in comparison to other transport methods [1]. Therefore, belt conveyors are widely researched and developed.

Ibishi et al. showed the method for calculating the tension force of the belt. The purpose of the method is to recognize the tension force upper limit, exceeding of which may damage belt transportation equipment [2]. Zanoelo

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and et al. studied mate leaves dryer and proposed to control the velocity of the belt by the mathematical drying model to ensure the accepted range of moisture in mate leaves [3]. There are energy models that can be used for efficiency optimization of the belt conveyor´s operation [4]. In recent studies by Göttlich et al., mathematical model was used to simulate a conveyor belt where the cargo is separated by means of a rigid singularizer. They also validated their model with real data tests [5].

This paper aims at providing a model and simulation of continuous belt conveyor on sea buckhorn berry separator (Fig. 1). The function of this belt conveyor is to separate the frozen berries, leaves and small branches which cannot be removed with sieve. The belt carries leaves and branches away while berries roll over the belt due to its inclination (Fig. 1). This paper presents the possibility to model mathematically berry motion on the conveyor belt. Calculations are based on the numerical method Runge-Kutta that was also used by above-mentioned authors in the field of conveyor modelling. The motion of the berry is visualized by graphs and by a video clip. Results of this paper can be used for designing berry conveyors and for analysing their motion. It also introduces some methods to be used with math program Mathcad, which is used also in other studies for engineering calculations [6] [7].

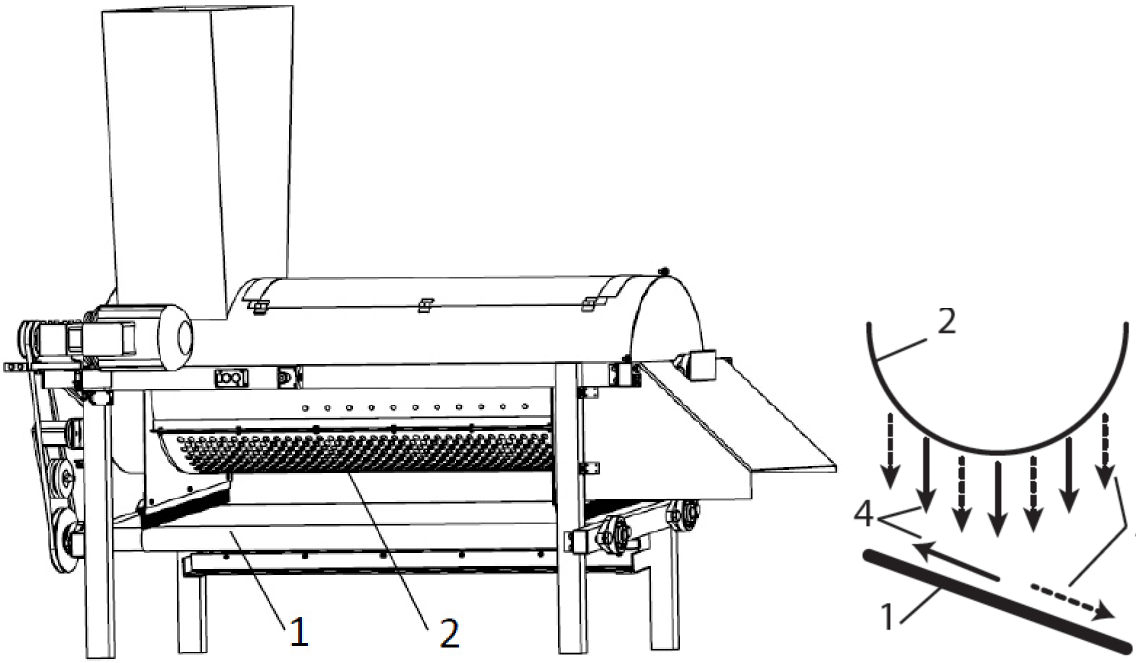

Fig. 1. Left: device for separation of deep-frozen sea buckthorn berries from twigs [8]. Right: principal scheme of the sea buckthorn berries separator sieve and conveyor belt. Where: 1 - inclined conveyor belt, 2 - sieve, 3 – sea buckhorn berries, 4 - leaves and small branches.

Nomenclature

v_k	velocity of the conveyor belt, m/s
l	width of the belt, m
s	axis, with origin at the point B along the conveyor belt, m
s_0	initial value on s-axis, m
s'	berry's velocity, m/s
β	inclination angle of the conveyor belt, degrees
m	berry particle mass, kg
g	acceleration of gravity, m/s^2
f	friction coefficient between berry particle and conveyor belt

ρ	air density, kg/m^3
c_d	air resistance constant
A	surface of the frontal area, m^2
s''	berry particle acceleration, m/s^2
t	time, s
t_0	the time when berry motion begins, s
t_N	the time when berry motion ends, s
N	the number of subintervals within the interval (t_0, t_N)

2. Materials and Methods

The kinematic scheme of the cleaning belt conveyor of buckhorn berry separator is shown on Fig 2.

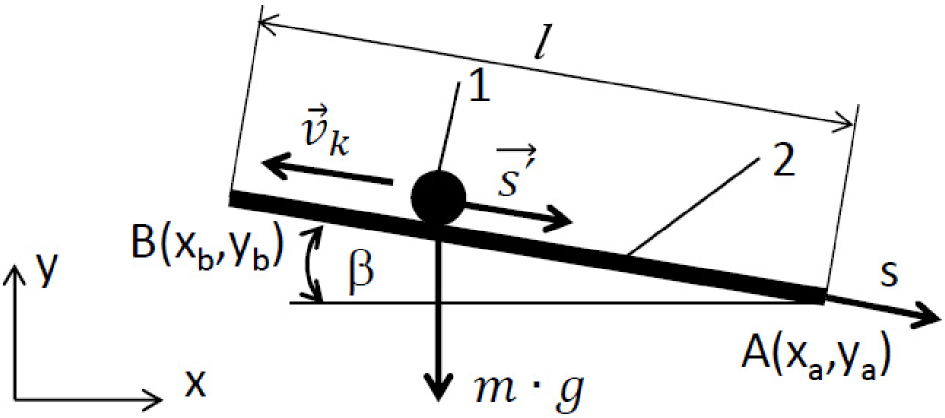

Fig. 2. The computational scheme of belt conveyor: 1 – berry particle, 2 – conveyor belt between points AB, l – width of the belt (distance between points AB), v_k – velocity of the conveyor belt, s – axis, with origin at the point B along the conveyor belt, s' - berry's velocity, β – inclination angle of the conveyor belt, m – berry particle mass, g – acceleration of gravity.

2.1. Berry motion on the belt

Berry particle motion can be modelled by using the following differential equation

$$m \cdot s'' = m \cdot g \cdot \sin\beta - f \cdot m \cdot g \cdot \cos\beta - \frac{1}{2} \cdot \rho \cdot (s' - v_k)^2 \cdot c_d \cdot A, \tag{1}$$

where f – friction coefficient between berry particle and conveyor belt,
ρ – air density,
c_d – air resistance constant,
A – surface of the frontal area,
s'' - berry particle acceleration.

Last part on the right in equation (1) models air resistance [9].

The initial conditions for equation (1) are

$$s(t_0) = s_0,\ s'(t_0) = s'_0 - v_k \tag{2}$$

where $t_0 = 0$ and $s'_0 = 0$.

The initial value s_0 must be chosen presuming that the solution meets the condition $s > 0$ (Fig. 2). To solve the equation (1) on Mathcad worksheet, it had to be presented as the following system of two equations of the first order

$$y'_0 = y_1,$$
$$y_0 = g\,sin(\beta) - f\,g\,cos(\beta) - \frac{\rho\,(y_1 - v_k)^2\,c_d\,A}{2\,M}, \tag{3}$$

where y_0=s, $y_1 = s'$.

Nonlinear system of equations (3) was solved numerically by using the algorithm of Runge-Kutta IV order, under conditions (2) by the following Mathcad function

$$Z(t_N, f, \beta, v_k) = \left| \begin{array}{l} D(t,y) \leftarrow \begin{bmatrix} y_1 \\ g\,sin(\beta) - f\,g\,cos(\beta) - \frac{\rho\,(y_1 - v_k)^2\,c_d\,A}{2\,M} \end{bmatrix} \\ Z \leftarrow rkfixed\left[\begin{pmatrix} s_0 \\ s'_0 - v_k \end{pmatrix}, t_0, t_N, N, D\right] \end{array} \right|$$

where the function

$$rkfixed\left[\begin{pmatrix} s_0 \\ s'_0 - v_k \end{pmatrix}, t_0, t_N, N, D\right]$$

is the differentiation equation solver. D - the notation of the function

$$D(t,y) = \begin{bmatrix} y_1 \\ g\,sin(\beta) - f\,g\,cos(\beta) - \frac{\rho\,(y_1 - v_k)^2\,c_d\,A}{2\,M} \end{bmatrix},$$

of the right side of the system (3). Here y is the vector $y = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$. t_0 – the time when berry motion begins, t_N – the time when berry motion ends, N - the number of subintervals within the interval (t_0, t_N), $\begin{pmatrix} s_0 \\ s'_0 - v_k \end{pmatrix}$ – the matrix of initial values. Function $Z(t_N, f, \beta, v_k)$ returns the matrix with columns $Z(t_N, f, \beta, v_k)^{<1>}$, $Z(t_N, f, \beta, v_k)^{<2>}$, $Z(t_N, f, \beta, v_k)^{<3>}$, where <1>, <2>, <3> denote the 1st, 2nd, 3rd column of this matrix. At that $t = Z(t_N, f, \beta, v_k)^{<1>}$ is the vector of the values of time, $s_t = Z(t_N, f, \beta, v_k)^{<2>}$ is the vector of the berry coordinates and $s'_t = Z(t_N, f, \beta, v_k)^{<3>}$ is the velocity vector of the berry. While t_N is also the value of time t, when the berry reaches point A (Fig. 2), its value can be calculated from equation

$$t_N = root[(Z(t_N, f, \beta, v_k)^{<2>})_N - l, t_N, 0.1, 10], \tag{4}$$

where the function $root(f(x), x, a, b)$ returns the solution of an equation $f(x) = 0$ in the closed interval $[a, b]$. In (4), $(Z(t_N, f, \beta, v_k)^{<2>})_N$ gives the Nth value of the coordinate s_t at point A (Fig. 2) and l is the width of the belt. The value is supposed to be in the closed interval [0.1, 10].

The Cartesian coordinates of the berry on belt during motion are

$$x_t(t_N, f, \beta, v_k) = x_a - (l - Z(t_N, f, \beta, v_k)^{<2>}) \cdot cos\,\beta, \tag{5}$$

$$y_t(t_N, f, \beta, v_k) = y_a - (l - Z(t_N, f, \beta, v_k)^{<2>}) \cdot sin\,\beta. \tag{6}$$

Point B (Fig. 2) coordinates are

$$x_b = x_a - l \cdot cos\,\beta, \tag{7}$$

$$y_b = y_a + l \cdot \cos\beta. \tag{8}$$

2.2. Berry falling in the air

After berry has left the belt's lower point A at the moment of the time t_N, it starts free fall in the air. This motion is described by the following equations

$$m \cdot y'' = -m \cdot g + \frac{1}{2} \cdot \rho \cdot y'^2 \cdot c_d \cdot A, \tag{9}$$

$$m \cdot x'' = -\frac{1}{2} \cdot \rho \cdot x'^2 \cdot c_d \cdot A, \tag{10}$$

where x'' and y'' are accelerations, x' and y' ´are velocities in the direction of x- and y-axis. Initial conditions for the equations (9) and (10) are

$$x(t_N, f, \beta, v_k) = x_t(t_N, f, \beta, v_k),\ x'(t_N, f, \beta, v_k) = x'_t(t_N, f, \beta, v_k),\ y(t_N, f, \beta, v_k) = y_t(t_N, f, \beta, v_k),$$
$$y'(t_N, f, \beta, v_k) = y'_t(t_N, f, \beta, v_k). \tag{11}$$

To determinate the coordinates x and y during free fall, equations (9) and (10) were solved under initial conditions (11) numerically similar to the solution of equation (1).

3. Results and discussion

3.1. Berry displacement and velocity on belt

Let's presume that berry and belt conveyor have the following values: berry mass $m = 0.02kg$, belt velocity $v_k = 1m/s$, friction coefficient between berry particle and conveyor belt $f = 0.01$, air density $\rho = 1.2\ kg/m^3$, air resistance constant $c_d = 0.47$ (considering that sea buckthorn berry is almost a sphere), surface frontal area $A = \pi \cdot r^2 = 0.00031m^2$, width of the belt (distance between points AB) $l = 0.6m$, point A coordinates $x_a = 0.8m$ and $x_a = 0.5m$. Under initial conditions (2) $s_0 = 0.3m$, it means that the berry touches the belt in the center, because $AB = l = 0.6m$.

The analysis of the separator's construction (Fig. 1) shows that there are mainly three possible values used for adjusting the separation process of the conveyor belt: inclination angle, velocities and friction coefficient (Fig. 3 – Fig. 5).

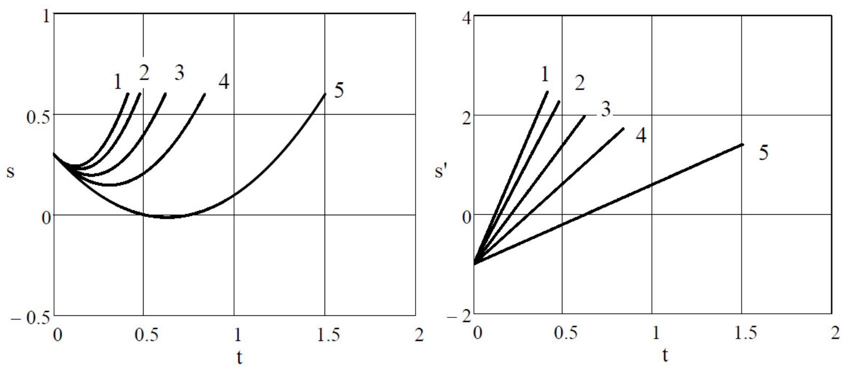

Fig. 3. Left: berry displacement along conveyor belt. Right: berry velocity along conveyor belt. Corresponding different inclination angles of the conveyor belt: 1) $\beta = 60°$, 2) $\beta = 45°$, 3) $\beta = 30°$, 4) $\beta = 20°$, 5) $\beta = 10°$; $v_k = 5m/s$ and $f = 0.01$.

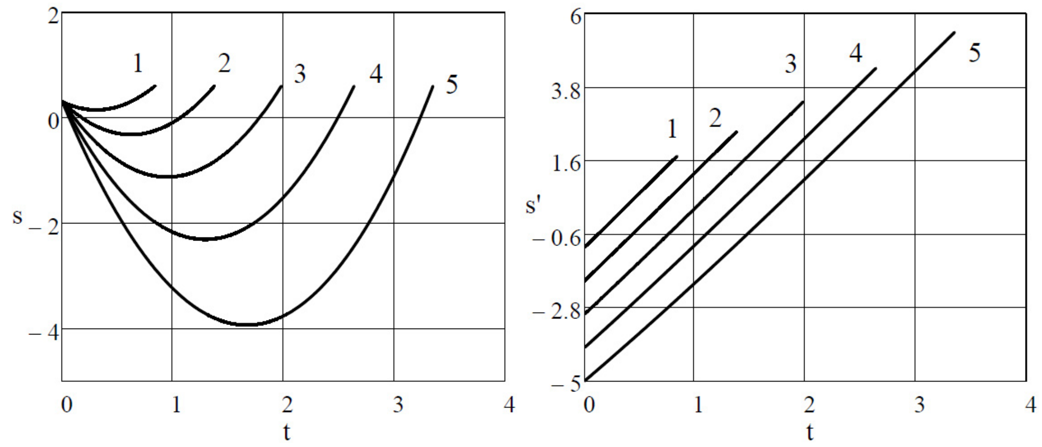

Fig. 4. Left: berry displacement along conveyor belt. Right: berry velocity along conveyor belt. Corresponding different velocities of the conveyor belt: 1) $v_k = 1m/s$, 2) $v_k = 2m/s$, 3)$v_k = 3m/s$, 4) $v_k = 4m/s$, 5) $v_k = 5m/s$; $\beta = 20°$ and $f = 0.01$.

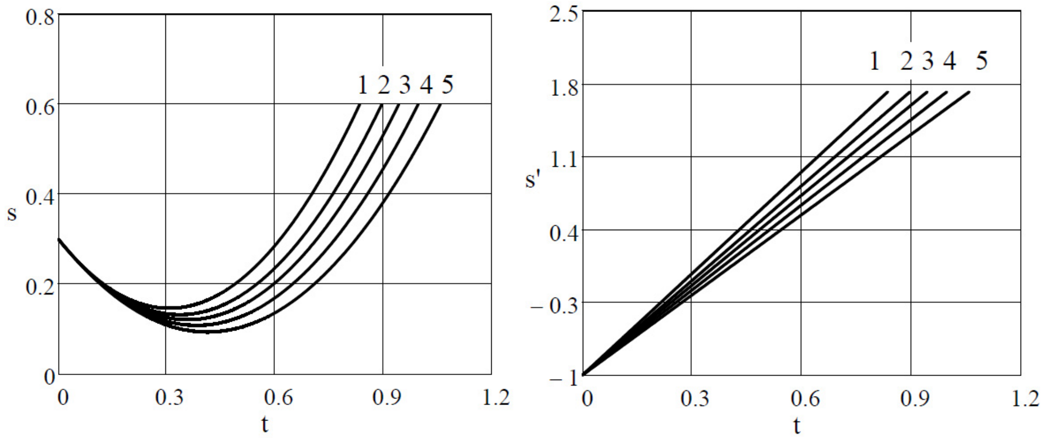

Fig. 5. Left: berry displacement along conveyor belt. Right: berry velocity along conveyor belt. Corresponding different friction coefficient between berry particle and conveyor belt: 1) $f = 0.01$, 2) $f = 0.05$, 3) $f = 0.1$, 4) $f = 0.15$, 5) $f = 0.2$; $v_k = 5m/s$ and $\beta = 20°$.

Figures 3-5 show that after contacting the belt, the berry first starts to move in the direction of belt with velocity v_k and then, due to inclination angle and gravity, moves down along the belt until it reaches point A (Fig. 2). Displacement decrease and negative velocity indicate berry movement along the belt with velocity v_k. Negative displacement means that berry moves over the point B (Fig. 2). In reality, it means that berry will fall over point B along with leaves and small branches (Fig. 1).

The calculation of berry displacement and velocities indicates that there are possible values, in which case the berry moves along with velocity v_k and will not move in the correct direction at all. This can be adjusted with all three parameters (v_k, f, β) and also initial values s_0 and s'_0.

3.2. Berry trajectory on the belt and in the air

For visualization of the belt according to equations (5) and (6) (Fig. 2, line AB), the coordinates are defined as matrixes on the Mathcad worksheet

$$c_x = \begin{pmatrix} x_b \\ x_a \end{pmatrix},\ c_y = \begin{pmatrix} y_b \\ y_a \end{pmatrix}.$$

Berry trajectory can be visualized with the following matrixes according to equations (3) and (4), where $min\ (x_t)$ and $max(y_t)$ are built-in Mathcad functions for finding minimum and maximum values

$$b_x = \begin{pmatrix} min\ (x_t) \\ x_t \end{pmatrix},\ b_y = \begin{pmatrix} max(y_t) \\ y_t \end{pmatrix}.$$

On Fig. 6 we can see that if the berry touches the belt in point $s_0 = 0.3m$, it will first move in the direction of belt with velocity v_k, like we saw on Fig. 3 - 5. On Fig. 6, when $\beta = 10°$, the berry will move close to point B, i.e. berry can fall over the belt if point B is on the edge of the belt.

On Fig. 6, when $\beta = 30°$ and $\beta = 60°$, the berry will not reach point B and will stay on the belt until point A.

For example, if $s_0 = 0.3m$ and $s'_0 = 0.5m/s$, then, provided that $\beta = 60°$, the berry will move towards point A instantly after touching the belt (Fig. 7). When $\beta = 10°$ and $\beta = 30°$, it will still show movement along the belt with velocity v_k in the beginning (Fig. 7).

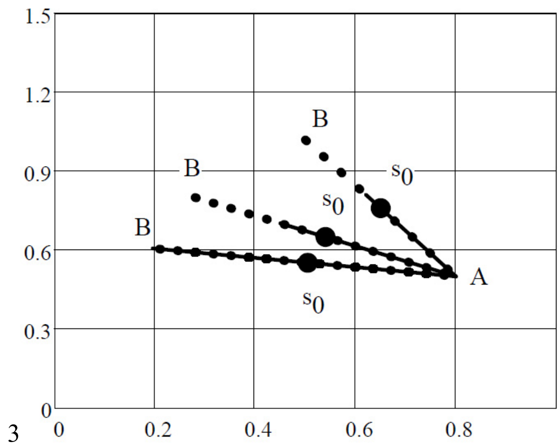

Fig. 6. The berry (continuous line) and the belt (dotted line) trajectories, s_0 location (large dot): 1) $\beta = 10°$, 2) $\beta = 30°$, 3) $\beta = 60°$; $f = 0.01$, $v_k = 1m/s$, $s_0 = 0.3m$, $s'_0 = 0$.

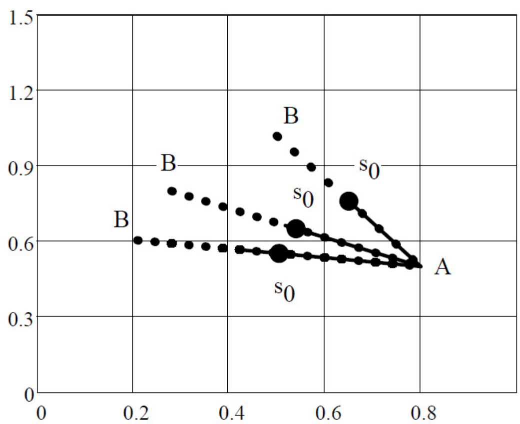

Fig. 7. The berry (continuous line) and the belt (dotted line) trajectories, s_0 location (large dot): 1) $\beta = 10°$, 2) $\beta = 30°$, 3) $\beta = 60°$; $f = 0.01$, $v_k = 1m/s$, $s_0 = 0.3m$, $s'_0 = 0.5m/s$.

When summing up the solutions of equations (5), (6), (7), (8), (9) and (10), we get the entire trajectory of the berry on the belt and through the air (Fig 8). Based on the air trajectories, it is possible to determine the location of the collector or measuring box under the sea buckthorn berries separator.

The motion of the berry particle according to calculations, with different conveyor inclination angle, is visualized in video clip [10].

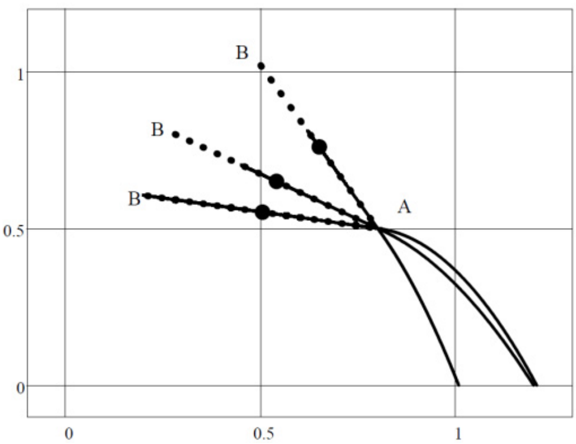

Fig. 8. Berry's entire trajectory on the belt and through the air, when 1) $\beta = 10°$, 2) $\beta = 30°$, 3) $\beta = 60°$; $v_k = 1m/s$, $f = 0.01$, $s_0 = 0.3m$, $s'_0 = 0$.

3.3. Total time of berry motion

The total time of berry motion depending on inclination angle β, velocity of conveyer belt and friction coefficient was also determined.

Fig. 9 shows total time of berry motion according to the inclination angle (Fig. 2) of the conveyor belt. The total time decreases when angle increases.

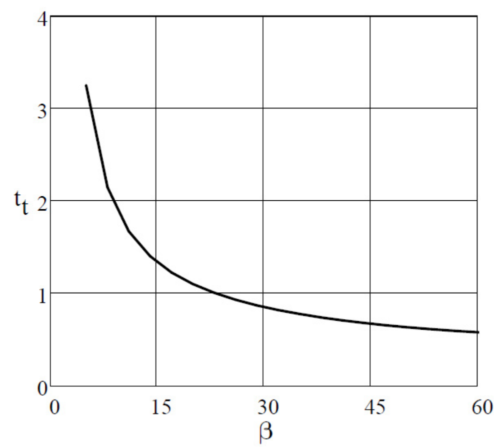

Fig. 9. Total time of berry motion depending on inclination angle β of conveyor belt if $v_k = 1m/s$, $f = 0.01$, $s_0 = 0.3m$, $s'_0 = 0$.

Fig. 10 shows total time of berry motion according to different velocity of the conveyor belt (Fig. 2). The total time increases when velocity increases. The relation of total time and conveyor belt velocity increase are almost linear.

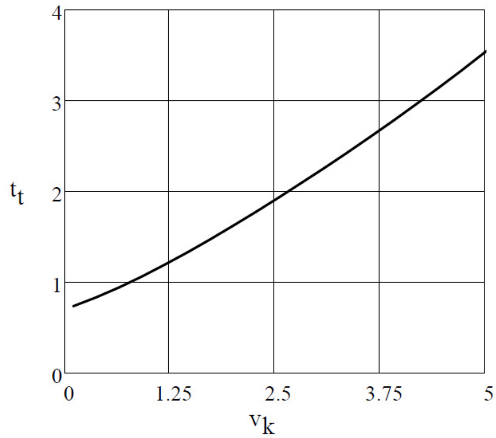

Fig. 10. Total time of berry motion depending on velocity of the conveyor belt if $f = 0.01$, $\beta = 20°$, $s_0 = 0.3m$, $s'_0 = 0$.

Fig. 11 shows total time of berry motion depending on the friction coefficient. The total time increases when friction coefficient increases.

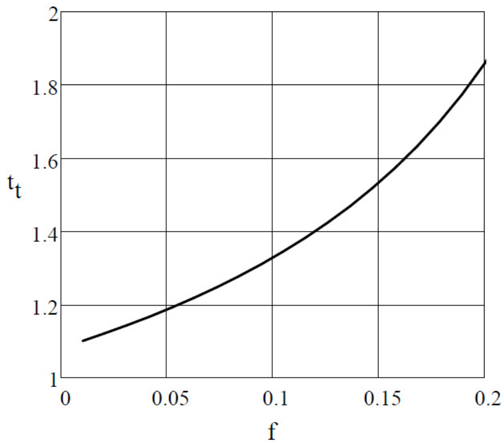

Fig. 11. Total time of berry motion according to different friction coefficients, berry particle and conveyor belt: $v_k = 1m/s$, $\beta = 20°$, $s_0 = 0.3m$, $s'_0 = 0$.

The outcomes indicated in the results and discussion sections are theoretical and must be validated with experimental data. First steps are made to build a prototype of the sea buckthorn berries separator according to patent EE05717B1. Results of experimental data will be discussed in future papers.

Conclusions

This paper shows that: 1) From the initial position at beginning of motion, a berry moves along the belt in the direction of the conveyer belt at certain velocity, stops and then moves along the belt to the lower point of the belt and falls down. 2) There are initial berry values s_0 and s'_0, in which case the berry does not reach the upper edge of the belt. 3) Total time of berry motion depends substantially on the inclination angle of the belt, coefficient of friction between berry and belt and velocity of the conveyer belt. 4) Composed video clip provides visual representation of full motion of the berry in separator under different angles of inclination of conveyer belt. Mathematical program Mathcad turned out to be a convenient tool for solving problems related to mechanics.

Results of this paper can be used by engineers for designing the conveyors and by educators for creative teaching of mechanics and engineering mechanics [11] of agricultural machinery.

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