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Improving the Finite Element Simulation of Wear of Total Hip Prosthesis' Spherical Joint with the Polymeric Component

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Abstract

The advanced model of wear in the spherical joint of total hip prosthesis comprising an acetabular cup of ultra-high molecular weight polyethylene (UHMWPE) in combination with a metal or ceramic femoral head is developed. The wear model is based on the classical Archard-Lancaster equation in common with all other studies reported in literature. The finite element solution of the contact problem between the cup and the head was employed under the loading and angular motions conditions according to the ISO 14242-1 demands. The polymer wear in terms of cumulative linear and volume wear when the wear factor is chosen to be a function of contact pressure is first evaluated.

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Keywords: Total hip prosthesis; wear; ultra-high molecular weight polyethylene; finite element simulation; spherical joint

1. Introduction

As shown by the medical practice, the main factor contributing to prolonging the lifespan of artificial joint replacements is to reduce the wear and wear particles in the bearing couple [1]. This is especially important for modern total hip replacements (THR), which include the acetabular cup of ultra-high molecular weight polyethylene in combination with a metal or ceramic femoral head, because currently there is a serious trend towards younger patients who are on indications needed a hip replacement. It is obvious that a lifespan of such prosthesis must be extended up to a maximum of 20 years or more. Thus, the study of wear becomes essential important for the development of new bearing couples, evaluation of designs and function of the existing joint prostheses. In these

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clinical studies, the evaluation of retrieved implants are implemented in terms of penetration depth of the femoral head into the cup, using the X-ray technique, and experimental laboratory studies using simulators for the wear test under replaying more realistic conditions of loading and motion [2]. But such experimental studies are expensive and time consuming to carry them out, although necessary for the preclinical assessment of the wear parameters. To reduce the amount of laboratory experimental studies, there is a well-known theoretical approach to modelling of wear, first pioneered by Maxian et al [3], which was then developed by other researchers [4] and in recent works [5,6]. This approach allows to pre-select the designs, materials, highlight the different variables and to investigate the influence of individual factors on the parameters of wear and thus generate data to perform further only limited experimental tests. The basis of most theoretical studies reported in the literature, is the finite element method for definition of the contact pressure at the articulating surfaces [7], because the advantage of the method lies in its flexibility and the opportunity of studying a variety of design, technological and operational factors of the prosthesis elements, including elastic-plastic deformation of the UHMWPE material [8]. But in almost all known cases of further wear definition, the simulation model is simplified for reduction the complexity of the calculations, thus reducing the reliability of the results.

The aim of this study is to improve the existing method of wear simulation of the bearing couple in a spherical joint of total hip prosthesis comprising a cup of ultra-high molecular weight polyethylene (UHMWPE), which is based on the solution of the contact problem using finite element analysis, taking into account the parametric dependence of wear factor on the contact pressure that has not been studied before.

2. Materials and methods

2.1. Hip prosthesis, loading and motion

The present study examined the hip joint prosthesis with a solid femoral head of cobalt-chromium alloy or ceramics (alumina or zirconia) employed against a soft (UHMWPE) acetabular cup. The elasticity modulus E of titanium shell is about two orders of magnitude greater than that of the cup, so further the outer surface of the cup is assumed fully constrained for the finite element model. The radius of the femoral head is R_1 = 16 mm, which corresponds to the most frequently used its standard diameter of 32 mm for a given combination of materials in the couple. The inner radius of the acetabular cup is R_2 = 16.15 mm, so that the radial clearance equals to $R_2 - R_1$ = 0.15 mm [9]. The thickness of the cup wall was chosen as 8 mm to provide its necessary rigidity. The elasticity modulus E and Poisson's ratio ν were chosen as 1.4 GPa and 0.46 respectively [10] for the cup and 210 GPa and 0.3 respectively for the head.

Demands to the implant, its positioning, creation of the load and motions specified in the joint, correspond to ISO 14242-1 test standard [11]. The right hip joint, defined in anatomical fixed coordinates $x'y'z'$ and shown in Fig. 1, where the axis z' is directed superior, x' medial and y' posterior perpendicular to the plane of the drawing, was represented as a model.

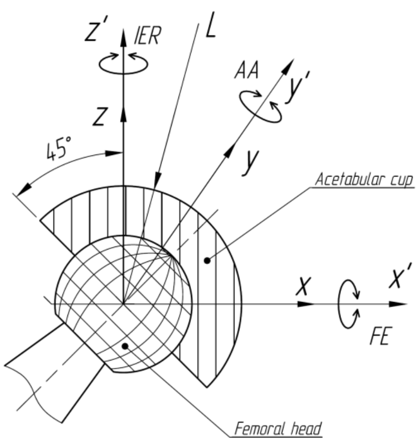


Fig. 1. Front view of the right hip joint with the specified directions of rotation (**L** is the resultant load vector).

The sum of inclination angle of cup and the angle between the load vector and the vertical is on average 60° to a base plane of the cup. Accordingly, inclination of the polar axis of the acetabular cup to the load line corresponds to 30°. The movable coordinate system used for the Euler angles coincides with the center of the cup, is placed in the center of the head and fixed to the head. At the same time the head has three rotational degrees of freedom, known as FE (flexion-extension), AA (abduction-adduction), and IOR (inward-outward rotation), which correspond to the angular movements profiles in Fig. 2 according to the demands of the specified standard. The profile of the resultant load vector in the anatomical coordinate system within one gait cycle, which corresponds to the period of time of 1s, it is also consistent with the pattern in ISO 14242-1. In the simulation of wear, it is proposed to use a simplified coordinate system XYZ fixed to the cup and placed in its center, see Fig. 3.

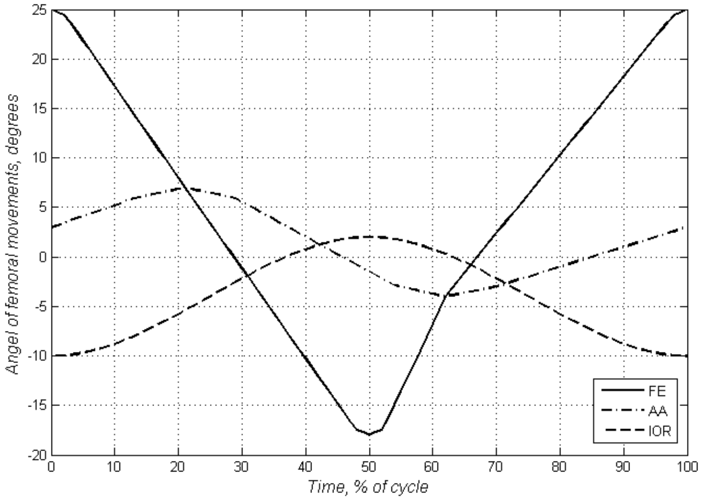


Fig. 2. Graphs of angular movements of the femoral head according to ISO 14242-1.

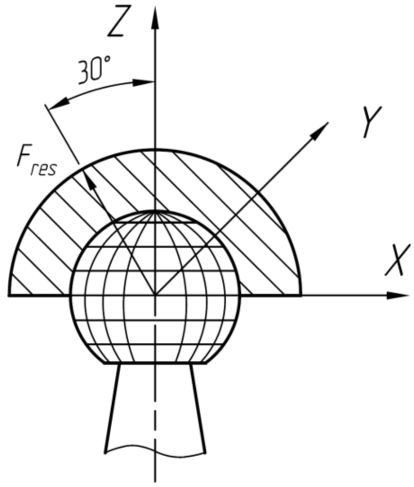


Fig. 3. A simplified coordinate system XYZ .

Graphs of the resultant load vector F_{res} and of its components F_X, F_Z as projections of two coordinate axes X and Z are shown in Fig. 4. The computational model uses inversion method, i.e. the load is applied to the center of the head and acts in the direction of the cup, since the cup is constrained over its outer surface. In this case, the load component F_Z is positive and F_X becomes negative.

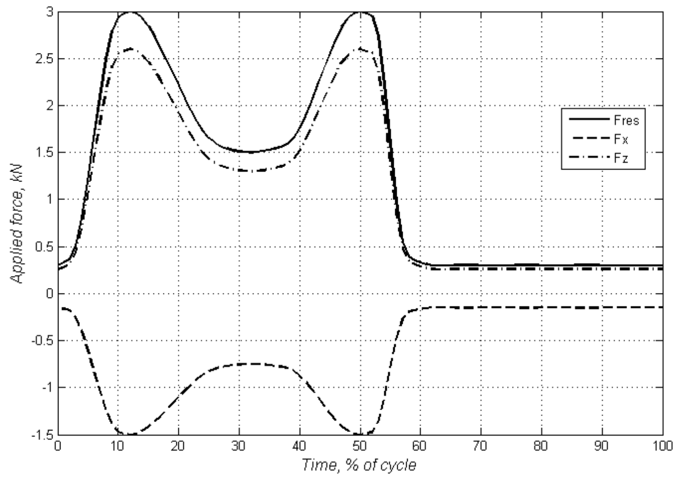


Fig. 4. Graphs of the resultant load vector F_{res} and of its components F_X, F_Z, as projections of two coordinate axes according to ISO 14242-1.

2.2. Wear simulation

The wear simulation was based on the classical Archard-Lancaster equation as used in all the previous theoretical studies discussed above. According to these studies, the adhesive-abrasive wear mechanism prevails in total hip replacement with UHMWPE component [12]. In this mechanism, the generation of wear debris depends on the sum interaction of the contact pressure, kinematics, and tribological properties of the couple. Until now, the total interaction for ideal uniformly loaded isotropic surfaces with a nominal contact pressure in the linear elastic condition is usually adopted to describe by the relation

$$H = kpS \tag{1}$$

where H is the wear depth, k the constant empirical wear factor depending on the material and the nature of the surface, p the contact pressure, S the sliding distance.

Equation (1) proposed in a discrete kind in the parametric form for the evaluation of variables of THR mechanical design, in the work [3] is described as following

$$\Delta H(\theta,\varphi) = \sum_{i=1}^{n} k\sigma(\theta,\varphi,t_i)\Delta S(\theta,\varphi,t_i) \tag{2}$$

where $\Delta H(\theta,\varphi)$ is the accumulative local linear wear depth at the contact surface in a spherical coordinate system, $\sigma(\theta,\varphi,t_i)$ the normal contact pressure between the counter-face surfaces at the same point of the time instant t_i of the gait cycle, $\Delta S(\theta,\varphi,t_i)$ the increment of the arc sliding distance between the adjacent measuring points under the same conditions.

There are three key parameters in the above equations (1) and (2): the wear factor, the normal contact pressure, and the kinematic parameters, i.e. the sliding distance on the surface of a sphere. Both the normal contact pressure and the sliding distance are functions not only of time, but also the spherical coordinates (θ,φ).

The wear factor is often determined from simple wear tests for a given combination of materials under a given set of input conditions. For the combination of UHMWPE-on-metal materials considered in this study, the wear factor have been reported to be in a wide range of values and depends on the molecular weight of UHMWPE, the lubricant and the surface roughness of the interacting bodies, and the sterilization method [4]. In all the previous wear calculations the wear factor was assumed to be constant for a given material combination ($1.066 \cdot 10^{-6}$ mm^3/Nm [3,13], $0.8 \cdot 10^{-6}$ mm^3/Nm [7] and others). There were sometimes attempts to correlate it with the value of maximum contact pressure, and in all the experimental studies the wear rate tended to decrease with increasing the contact pressure. Guided by these studies there was obtained the experimental dependence for wear factor [14] (in mm^3/Nm)

$$k = 7{,}99 \cdot 10^{-6} \sigma_0^{-0{,}653} \tag{3}$$

where σ_0 is the maximum contact pressure in MPa.

Equation (3) represents a power function with a negative power, which implies that with increasing the contact pressure the wear rate is reduced, and on the contrary, with its decreasing – increased. Other experimental studies on the dependence of wear on the contact pressure, load and nominal area of the contact surface [15] conducted with cylindrical pins of 5.9 mm diameter at the pin-on-plate simulator, showed a decisive influence at the wear factor the contact pressure and the negligible effect of other specified parameters. Tests were carried out to $1.5 \cdot 10^6$ cycles with evaluation of the nominal contact pressure in the range of 0.56 – 12.73 MPa. As a result, the relationship which gives the value approximately 6 times lower than the formula (3), was obtained

$$k = 2 \cdot 10^{-6} \sigma^{-0,84} \tag{4}$$

where σ is the nominal contact pressure in MPa, i.e. the load in relation to the area of the wear surface. Moreover, the contact surface of all the samples of pins, tested at a given rig, results in a central nipple on the pin. One can therefore assume that the contact pressure has some irregularity on the contact surface of the pin, decreasing to its edges, which increased the wear factor in this site according to (4). Since there is such a phenomenon, then it may be reasonable to use the formula (4) as a variable parameter during the calculation of wear increment at determining the accumulative local linear wear depth by the relationship (2). Also known another experimental study carried out at the same type of setup with pins of the modern cross-linked UHMWPE of 9 mm diameter (resin GUR 1020, γ-sterilized in nitrogen), which has well-known for this material mechanical properties of E and ν equal to $0.8 \cdot 10^3$ MPa and 0.46, respectively. It was carried out on average to 85000 cycles in the range of nominal contact pressure of 0.25 – 11.0 MPa. Thus were obtained the following relations for the wear factor [16] (in mm^3/Nm)

$$\begin{aligned} k &= 2.7 \cdot 10^{-6} \left(p / p_{ref}\right)^{-0.57}, \text{when } p / p_{ref} \le 2.53 \\ k &= 6.0 \cdot 10^{-6} \left(p / p_{ref}\right)^{-1.44}, \text{when } p / p_{ref} > 2.53 \end{aligned} \tag{5}$$

where p and $p_{ref} = 1.1$ MPa are the actual nominal contact pressure and the same for the running-in condition, respectively.

An attempt to explain the effect of the increase in the wear factor at lower contact pressure was carried out in [16], where as a result of experimental studies was found the following. At contact pressures below 2.0 MPa, the contact surface of the polymer becomes a polished, thereby increasing its adhesive wear due to seizure between the interacting surfaces. When the pressure rises above a specified value the contact surface experiences a local overheating, leading to the toughening of the material, resulting in the wear surface arise protuberances, which create a surface texturing, where is easier to hold the lubricant. This reduces the actual contact surface to decrease the adhesive wear and thereby to improve its tribological properties. In all the above experimental studies of wear factor, the lubrication conditions were nearly the same with the use of diluted serum and the relative rotational reciprocating movement of the pin and the plane. The mechanical characteristics of the UHMWPE material at obtaining the formulae (3) and (4) correspond to the characteristics of the polymer adopted in this study. The analysis of values k, obtained by the relations (4) and (5), shows that they are essentially the same at the nominal contact pressure of 0.25 MPa and 9 MPa, and has the difference of more than 70% to the higher values in the range of from 2 to 3 MPa, obtained from the formula (5). Despite this difference, the relationship (5) together with (4) can also be used in calculating the wear depth with k as a variable value for the comparative evaluation. In this study, the following two kinds of wear calculation are carried out: by the equation (2) with a constant wear factor of $1.066 \cdot 10^{-6}$ mm^3/Nm and with that as a variable parameter, which is calculated to be defined on each considered point of the contact surface in the course of solving the contact problem by the expressions (3), (4), and (5); and the results of these calculations are compared. The results will be also compared with known wear calculations parameters for the constant wear factors of $1.066 \cdot 10^{-6}$ mm^3/Nm [3,13] and $0.8 \cdot 10^{-6}$ mm^3/Nm [7].

2.3. Sliding distance analysis

Determining the sliding distance of any marker point on the femoral head surface over the surface of the cup and vice versa are sufficiently described in [17]. Since in this study the wear of soft cup occurs, it is necessary to determine the sliding distance of marker points on the head surface over the surface of the cup. The calculation method is based on the Euler angles, allowing determining the position of each point on the sphere from the initial position. Previous point as the initial is not used to avoid the accumulation of errors in calculation. The directions of axes of the moving coordinate system associated with the head in relation to the fixed coordinate system associated with the cup are determined by unit vectors $\mathbf{u}^1$, $\mathbf{u}^2$ and $\mathbf{u}^3$. The directions of the axes of the fixed coordinate system are: $\mathbf{U}^1=[1,0,0]^T$, $\mathbf{U}^2=[0,1,0]^T$, $\mathbf{U}^3=[0,0,1]^T$. Respect to the axes x, y and z, the rotation according to the Euler angles was carried out in the form of an anatomical sequence of FE → AA → IER (Fig. 1). The angles corresponding to discrete values of profiles FE, AA and IER (Fig. 2), denoted by $\alpha_i, \beta_i, \gamma_i$, $i = 1,2,3,...,n$, where n is the number of discrete points or sets of rotation angles. The marker point K fixed to the head and having one set of rotation angles ($\alpha_i, \beta_i, \gamma_i$), is sequentially rotated from the initial position to a new position K_0 along the sliding track on the cup surface according to the relationship

$$\mathbf{r}_\mathrm{i} = \mathbf{R}_{xyz}(\alpha_i, \beta_i, \gamma_i)\mathbf{r}_0$$

where $\mathbf{r}_0$ is the position vector of marker point in initial position, $\mathbf{r}_\mathrm{i}$ the position vector of marker point after rotations according to $\alpha_i, \beta_i, \gamma_i$, $\mathbf{R}_{xyz}(\alpha_i, \beta_i, \gamma_i)$ the rotation matrix determined as [18]

$$\mathbf{R}_{xyz}(\alpha_i, \beta_i \gamma_i) = \begin{bmatrix} \cos\beta\cos\gamma & -\cos\beta\sin\gamma & \sin\beta \\ \sin\alpha\sin\beta\cos\gamma + \cos\alpha\sin\gamma & -\sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & -\sin\alpha\cos\beta \\ -\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \cos\alpha\sin\beta\sin\gamma + \sin\alpha\cos\gamma & \cos\alpha\cos\beta \end{bmatrix}$$

To obtain the distance on the sphere surface of the head between the two adjacent positions of the point on the track, it is necessary to determine the angle between its two positions, whose cosine is the scalar product of vectors divided by the product of the moduli of these vectors, and multiply it by the radius of the sphere R_1, i.e.:

$$\Delta S(\theta, \varphi, t_i) = R_1 \cdot \arccos\left(\frac{\mathbf{r}_{\mathrm{i}+1}\mathbf{r}_\mathrm{i}}{|\mathbf{r}_{\mathrm{i}+1}| \cdot |\mathbf{r}_\mathrm{i}|} \right) \tag{6}$$

To reduce the computation time, the number of discrete sets n of rotation angles (α_i, β_i, γ_i) is chosen to be as 25 intervals in one gait cycle. In order to improve the accuracy of defining the sliding distance of point K as it moves from one discrete position to an adjacent one, this movement is divided into 4 intervals, each of which is determined by its sliding distance by the formula (6). Then the total sliding distance for a single discrete sliding movement is determined by the summation of these incremental distances.

2.4. Finite element wear simulation

The problem solving process is carried out jointly in the ANSYS and MathLab software packages based on a series of sequential operations. At first, full-scale conjugate components of the couple are created in the form of 3-D solid-to-solid model with parameters mentioned above of the head and the cup, and the existing clearance between them. To improve the accuracy of the calculations, the finite element mesh in the form of bricks and wedges is used. Thus, the number of finite elements is chosen so that the solution results were not different hereinafter more than 1%. One gait cycle corresponding to the period of time of 1s, is divided into 25 equal intervals, in contrast to the 16 and 21 ranges in [3] and [13], respectively. Thereby the calculation accuracy is increased because of the presence of sharp peaks at the load profiles of Fig. 4. The end of each interval corresponds to a set of rotation angles (α_i, β_i, γ_i) from the kinematic waveforms in Fig. 2 and to corresponding values of load components F_{Zi} and F_{Xi} in Fig. 4. At each interval of a spatial domain solution when changing θ and φ from 0 to π, the number of mesh grids is chosen of 24×24 in view of satisfying the above condition of accuracy, that it was checked by preliminary studies. Then for the model, the contact surface is generated on the softer inner surface of the cup according to ANSYS guidelines. The nodes' numbers on the head surface and their Cartesian coordinates are outputted in an array to a text file. This text file is read hereinafter in the MathLab software, and the computation of sliding distances of each of the specified nodes for all the 25 intervals, mentioned in one gait cycle, by the formula (4), is carried out, the results of which are then outputted as the corresponding arrays into the same text file for later reading in ANSYS.

Further computational operations are conducted only in the ANSYS software for all of the 25 time intervals in a loop mode than the computational complexity is reduced in the simulation of wear over a long period of time. At the beginning of each cycle by solving the contact problem of 3-D surface-to-surface, the values of normal contact pressure $\sigma(\theta, \varphi, t_i)$ at the nodes on the contact surface are defined by the corresponding load components values F_{Zi} and F_{Xi} for this interval. For boundary conditions, the outer surface of the cup is fully constrained. Then the file with array $\Delta S(\theta,\varphi,t_i)$ is read, and the matching of the node on head to the nodes on the contact surface of the cup is checked, and the nearest node on the contact surface after moving the node on the head at the appropriate distance $\Delta S(\theta,\varphi,t_i)$ is found. Thereafter, according to the formula (2), the local wear depth increment $\Delta H(\theta,\varphi)$ is determined in each of the nodes found at the contact surface by multiplying the sliding distance $\Delta S(\theta,\varphi,t_i)$ passed by the node, by the amount of contact pressure $\sigma(\theta,\varphi,t_i)$ applied in the preceding position of the node, and by the wear factor. By summing the local depth increments of wear, determined at each interval, it can obtain a distribution of the cumulative linear wear of the cup surface at one gait cycle and the volumetric wear, as the volume of part of the material removed out of the cup. The results obtained are placed in a text file. Then the actual geometry of the bearing surface of the cup is adjusted by moving the nodes by the amount of linear wear derived from the specified text file and a new contact surface is generated. After that the specified computing sequence is repeated in a loop mode to perform the required number of cycles corresponding to a predetermined number of the gait cycles.

Ideally, the bearing surface geometry and the contact pressure should be updated after each cycle. But because of the small depth of wear in one cycle, such a process would be time-consuming in terms of computational operations for a large number of cycles. Since obviously it can be assumed that the implementation of appreciable wear requires a certain number of cycles or intervals of modification N_0 and therefore it is probably only necessary to update the contact pressure calculation once per specified number of cycles. According to the demands of ISO 14242-1, the number of cycles in the simulation of wear should be of $3{\cdot}10^6$. On the other hand, considering the needs of the 20-year operation of the prosthesis with the average number of steps per year of $1{\cdot}10^6$, this may corresponds to cycles of $20{\cdot}10^6$ [13]. The linear and volumetric wear, obtained at the end of the modification interval, are determined by simply multiplying the wear for one cycle by N_0. Herewith, this modification interval which does not affect the end result has been chosen in the range from 1.5 to 3 months, which corresponds approximately to $0.123{\cdot}10^6$ and $0.246{\cdot}10^6$ cycles, respectively and satisfies to recommendations of [13].

3. Results

To substantiate the correctness of the techniques of numerical studies have been carried out a number of checks. The first check is a mesh grids density. Here, the spatial solution domain for both θ and φ in the range from 0 to π was divided into a number of uniform grids of 15x15, 24x24, 30x30 and 50x50. The results showed that the grid of 24x24 gives an accuracy of the contact problem solutions within 1% over the mesh of 50x50. Therefore, to reduce the computational complexity, the grid of 24x24 is chosen. The second check is an evaluation of the accuracy of determining the sliding distance. For this purpose, the results were compared with calculations of the parameter according to the method of this study and presented in [19]. The differences were in the range of 4%, since this technique is more accurate for determination of the sliding distance. The third check, the time interval of upgrading the geometry of the cup surface or so-called modification interval N_0 has been selected in the range from 1.5 to 3 months, which has an effect on the result of wear calculation within 0.5%.

Also the magnitude of the product of $k \cdot \sigma$ in the formula (2) in the case of a variable wear factor has been further evaluated. It turned out that when the contact pressure σ is changed from its maximum value to the value of 0.01%, the specified product is also reduced but not significantly, by about 5 times only. To avoid uncertainties at the calculation of the wear factor when the contact pressure approaches to zero value, the lower limit of pressure below which the wear factor was assumed a constant, equals to its value at the established boundary, was set. The varying of contact pressure boundary in a range from 5% to 0.01% of its maximum value has a negligible effect on the results of wear calculation. Therefore, in the present study, this boundary was set at 0.1% of the maximum contact pressure. The calculation results of the linear and volumetric wear parameters in this study are presented at the graphs of Fig. 5 and 6 and for comparison with the most well-known results from the literature [3,13,7] are listed in Table 1.

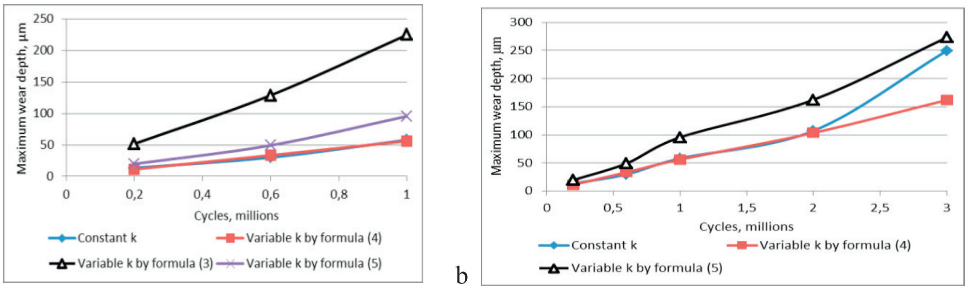
a 

Fig. 5. Variations with time in terms of the walking cycles of the maximum cumulative linear wear (the maximum wear depth) (a) in a narrow range of up to 1 million and (b) up to 3 million according to ISO 14242-1, when the wear factor k is chosen to be a constant of $1.066 \cdot 10^{-6}$ mm^3/Nm and a variable parameter defined by formulae (3), (4) and (5).

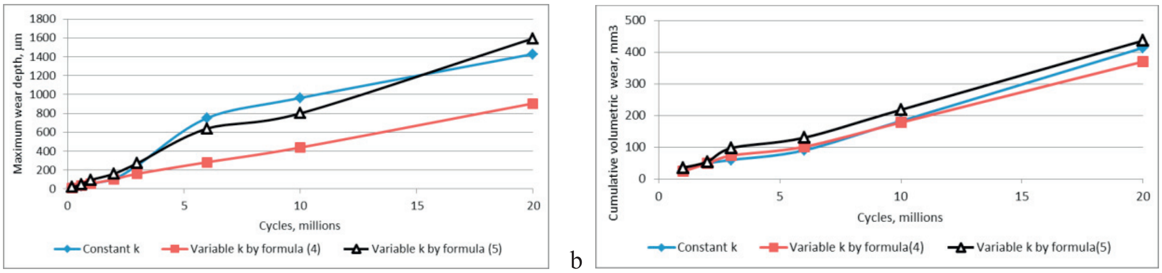
a 

Fig. 6. Variations with time in terms of the walking cycles (a) of the maximum cumulative linear wear (the maximum wear depth) and (b) the cumulative volumetric wear, when the wear factor is chosen to be a constant of $1.066 \cdot 10^{-6}$ mm^3/Nm and a variable parameter defined by formulae (4) and (5).

4. Discussion

The good agreement between the locations of area with the maximum contact pressure on the contact surface of the cup with the direction of the resultant load F_{res} as shown in Fig. 3 is found. Herewith the maximum value of the pressure for a given contact parameters at F_{res} = 3kN not exceed 5.75 MPa and is in the elastic range for the UHMWPE polymer.

Table 1. Comparative results of the linear and volumetric wear parameters for different methods of calculation.

Parameters	Average linear wear rate (mm/year)	Cumulative maximum linear wear (mm)		Average volumetric wear rate (mm^3/year)	Cumulative volumetric wear (mm^3)		Literature references and present study
		1 million cycles	20 million cycles		1 million cycles	20 million cycles	
Constant wear factor	0,041	0,041	0,82	27,1	27,1	541	Maxian TA et al [3]
	0,042	0,042	0,93	32,3	32,3	633	Kang L et al [13]
	0,049	0,049	0,98	38,9	38,9	778	Wu JSS et al [7]
	0,071	0,058	1,43	20,7	24,0	414	Present study
Variable wear factor	0,046	0,056	0,91	18,5	23,6	370	Formula (4)
(present study)	0,079	0,096	1,59	21,8	35,2	436	Formula (5)

Based on the Fig. 5a, it is clear that the use of formula (3) for the calculation with the variable wear factor, where acts as a variable the nominal contact pressure is totally unacceptable because of vastly superior wear parameter values, which are not consistent with any known results. Thus it is possible to eliminate the calculation by formula (3) from the further research. Analysis of the graphs in Fig. 5b, obtained in the range of test duration according to the ISO 14242-1 demands (3 million cycles), shows virtually identical maximum wear depth of up to 2 million cycles for the two variants of wear factor computations, where it is adopted to be a constant of $1.066 \cdot 10^{-6}$ mm^3/Nm, which chosen, by the way, at random from the previous studies [3,13] and a variable defined by formula (4). The using of wear factor as a variable value according to the formula (5) in the indicated range of tests gives a 50% excess of the wear parameters compared with previous two variants. Impact on wear parameters the extension of the range of test duration up to 20 million cycles can be evaluated from Fig. 6. The approximate coincidence of the maximum wear depth for the calculation variants with a constant wear factor of $1.066 \cdot 10^{-6}$ mm^3/Nm and as a variable value according to the formula (5) that exceed approximately 50% of the value obtained for the calculation based on the formula (4), are seen in Fig. 6a. Fig. 6b also illustrates that, in spite of this difference in the maximum depth of wear, the volumetric wear does not change very much. In this case, for a constant wear factor and as a variable value using the formula (4) the volumetric wear of up to 10 million cycles is almost identical, and only at 20 million cycles increases by about 12% at a constant wear factor. Computations for the same volumetric wear with a variable wear factor using the formula (5) gives the excess of 18% to 30% compared to the same variant using the formula (4). The phenomenon of the differences between the nature of the variation of linear and volumetric wear can be explained likely that there is some difference in the topology and shape of the worn surface of the polymer for various methods of calculation, but the difference in the worn volume is very small. Increased wear parameters obtained for the variant of using the formula (5) as compared with formula (4) were the result, likely, of the reduced elasticity modulus of the UHMWPE material $E = 0.8 \cdot 10^3$ MPa, compared with the chosen in this study, which had a practical effect on the derivation of the formula (5). The mechanical characteristics of the polymer in this study, referred to above, are chosen to be about corresponded the same of studies [3] and [13] to provide a comparison of the results obtained.

Regarding the results which are reflected also in Table 1, they can be analysed by such way. Here under the average wear rate, the value of cumulative wear at 20 million cycles, divided by 20, is meant and it can not match the value of the corresponding parameter of wear resulting from the numerical computation for 1 million cycles. With obviousness it can be noted fairly good agreement between the results of this study with the above results from the literature. But the existing difference between the wear parameters in [3,13] and the obtained in the present study, using the same constant wear factor of $1.066 \cdot 10^{-6}$ mm^3/Nm can be explained, most likely as next. In studies of [3] and [13], the “two-peak” load profile, measured by Paul, and the angular displacement profiles measured by Smidt and Johnston, which correspond to the actual conditions of the gait cycle and sufficiently different in a magnitude and shape from the corresponding profiles in accordance with ISO 14242-1, are applied. In formula (2), this affects both the value of nominal contact pressure and the length and shape of the sliding track. Application of the conditions of the specified standard in present study is determined by the fact that the simulator for tests on wear of total hip replacements are made in the laboratory of biomechanics [20]. In its design, the demands of this standard are implemented to the greatest extent, and the experimental study of these implants for comparison with the results of numerical simulation is intended to carry out in the nearest future. Table 1 also shows that the highest values of the wear parameters were represented in the study, according to [7]. This is most likely can be attributed to the following two factors. The load profile in the study is chosen to be a simplified form, in which the impact of a constant maximum value of the force in 3.5kN is a significant amount of time on the duration of the gait cycle. With such a load, the contact pressure must be increased compared with the pressure, obtained in the present study, and to act for a longer period of time. This should also lead to a decrease in the wear factor, but not as much as compared to the chosen value of $0.8 \cdot 10^6$ mm^3/Nm. Another factor is the lower mechanical properties of UHMWPE material, wherein the elasticity modulus $E = 0.8 \cdot 10^3$ MPa and Poisson's ratio $\nu = 0.47$.

Conclusion

The existing method of wear modeling the bearing couple in a spherical joint of total hip prosthesis comprising a polymeric cup of UHMWPE material in combination with a metal or ceramic femoral head, based on the solution of the contact problem between them with using the finite element analysis, was improved. Such the analysis has been employed only with a constant wear factor previously. Herewith, the use in numerical studies the constant wear factors requires a certain art in the choice of the value of this factor that can always not be achieved, to obtain the reliable results. The method of this study, taking into account the parametric dependence the wear factor on the contact pressure which reflects a more realistic contact conditions was first proposed and implemented. Comparison of the computation results in terms of the cumulative linear and volumetric wear at a constant wear factor, chosen from the known literature, and as a variable parameter, defined by various expressions, found them quite close agreement. As well as there was found a good agreement with the well-known previous studies [3,7,13], taking into account the differences of mechanical properties of the polymeric materials, nature of the load and their test methods. At the same time, in the case of variable wear factor, the results are closest, especially the cumulative volumetric wear by applying the formula (4) obtained in testing the polymers with the same mechanical properties as in the present study. This is the strong proof of the proposed method application, which takes into account the variable wear factor, in all similar future studies. The developed method is a serious tool for the implementation of the more accurate initial qualifying analysis of the design, materials and manufacturing process of THR, and thus allows reducing the use of expensive experimental studies using simulators. Further areas of research are the study of the effect of the THR design parameters and loading history (fast walking, up and down stairs, standing up and sitting down, standing on 2-1-2 legs, etc.) on the amount of wear of the prosthesis' elements using the presented techniques. Furthermore, in the laboratory of biomechanics, it is supposed to perform an experimental study of wear of the polymeric cup in a spherical joint of total hip prosthesis at the simulator, to compare the results obtained.

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