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Demand Modeling with Overlapping Time Periods

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Abstract

It is common and also predominant in the inventory control literature that demand follows normal distribution, and according to central limit theorem, demand per period will also follow normal distribution. However, in many real life situations, demand does not necessary follow normal distribution, and therefore, use of expressions used to calculate demand parameters per period are not suitable. This research suggests that available demand data are grouped into periods of desired length by overlapping. Demand data obtained by this approach provide valuable information for risk study. Suggested approach is evaluated using periodic review inventory model where all unsatisfied demand is backordered, and the same inventory control model is used to control inventories of slow and fast moving items.

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Keywords: periodic review inventory control; stochastic demand; grouped demand; overlapping periods; risk

1. Introduction

Managing inventories under uncertainty is a topic that has received and continues to receive a lot of attention from academics and practitioners alike, because of the major consequences that the related decisions may have on the economic performance of a firm [1]. In order to manage firm's inventories, practitioners and academics usually use continuous and/or periodic review policies. Under an inventory continuous review policy, the inventory position of an item is monitored after every transaction and the policy is to order a lot of size Q when the inventory level drops to the reorder point, s. Under a periodic review policy, a review of the inventory level is made at fixed interval of time, once every R time-units, and ordering decision, regarding how much to raise the inventories for the next period is made.

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Nomenclature

A	fixed ordering cost, €/order
R	review period, days
L	lead time, days
S	maximum inventory level (base stock), item units
M	theoretical maximum inventory level (base stock), item units
x_{R+L}	demand over period $R+L$, item units
x_L	expected demand over lead time, item units
$f(x_{R+L})$	probability density function of demand over period $R+L$
$F_{R+L}(.)$	cumulative distribution function of demand over period $R+L$
h_d	unit holding cost, €/day
π	backorder cost, €/unit backordered

A base stock system is a special case of the fixed reorder quantity policy in which, at all times, the sum of the on-hand inventory and the on-order amount is equal to the target inventory [2]. Therefore, the two basic parameters to be controlled in a periodic review system are: how often to review inventories and how much to raise the inventories at each review period. The complexity of the task of determining the optimum replenishment decision at any period is also dependent on several factors that are out of control. One of these uncontrollable factors is the nature of demand, especially demand variability. When the fluctuations in demand are high, relatively larger safety stocks are required to avoid shortages, and as a result, holding costs are increased [3]. However, it is obvious that firms have to keep some safety inventory to mitigate the impact of changes in the external environment, in order to ensure production continuity and deliveries on time.

When modeling demand then it is usual to suppose that demand follows normal distribution if demand for an item is fast, while when demand for an item is slow, then it is usual to suppose Poisson distribution. Reason why normal distribution always enjoys wide applications in both research and practice is of course that many theoretical results are available. However, one important disadvantage of the normal demand assumption is that demand may take negative values, particularly for high coefficients of variation [4]. Also, when demand variability is very high, it may be enormously expensive and unnecessary to insist on a base-stock level of the form $\mu + z \cdot \sigma$ as suggested by the normal distribution [5]. If demand data are available, but sparse, it is recommended in [5] to fit the empirical distribution to one of the known non-negative distributions such as the Gamma, the Negative Binomial or the Lognormal, while when demand data are abundant, then using the empirical distribution itself may be the best thing to do. However, if normal distribution must be used then it should be used with upper bound proposed in [5].

When controlling inventories of slow moving items, especially of spare parts, suitable forecasting methods are often addressed in the inventory literature, such as publications of [6, 7, 8]. Besides forecasting models, there are also some publications that discuss development and application of effective inventory models for controlling of slow moving items. When demand for an item is slow or intermittent, then typical assumption is that demand follows a Poisson, compound Poisson, Geometric or Negative Binomial distributions [9]. In such situations, the usual inventory control policies applied are of $(S-1, S)$ type [10, 11].

Estimating demand parameters for periods longer than time unit can be very difficult, especialy in situations when demand data are sparse and available for limited time periods. In this paper an approach for demand modeling is proposed to overcome such situations. Proposed approach groups available demand data into periods of desired length that mutually overlaps. To analyse proposed approach for demand modeling periodic review inventory control model is used. Proposed periodic review inventory control model differs from similar ones in terms that define demand parameters. Demand parameters used in this research are obtained using proposed approach for demand modeling. Inventory control model developed in this research is general, and does not consider any special conditions or assumptions.

2. Demand modeling

In inventory control it is often necessary to estimate demand parameters for time periods of certain length. For example, reorder point level of continuous review inventory models, at which system places order to supplier, is estimated using demand over lead-time, e.g. lead-time demand. In order to estimate maximum inventory level in periodic review inventory models it is necessary to know demand over time period equal to sum of review period R and lead time L, e.g. $R+L$.

When demand data are considered and analysed for unit time then demand parameters can be estimated using basic statistical tools. However, when demand data are considered and analysed for time period od certain length, over some long time horizon, where length of time period for which demand parameters needs to be estimated is longer than unit time, some problems can emerge. Generally, it is not stated in literature how demand parameters are estimated for certain time period length, except on the basis of daily demand parameters.

If expected daily demand is $\overline{x}$ and variance v, and time period length is T, then expected demand $\overline{x}_T$ and variance v_T over time period of length T, can be calculated using following expressions:

$$\overline{x}_T = \overline{x} \cdot T \tag{1}$$

$$v_T = v \cdot T \tag{2}$$

When time period length T is variable, with expected value τ and variance ϑ, and if expected daily demand is $\overline{x}$ and variance v, then expected demand $\overline{x}_T$ and variance v_T over time period of variable length T, can be calculated using following expressions [12]:

$$\overline{x}_T = \overline{x} \cdot \tau \tag{3}$$

$$v_T = \tau \cdot v + \overline{x}^2 \cdot \vartheta \tag{4}$$

Expressions shown above are appropriate and valid in situations when demand parameters are estimated for time periods of length T, where daily demand follows normal distribution, because demand for period T will also follow normal distribution. This comes from central limit theorem. Assumption of demand normality is appropriate in situations where coefficient of variation is relatively small. However, there are many realistic situations where demand distribution does not follow normal distribution and application of normal distribution in such situation is questionable [13]. Therefore, in situations when demand distribution does not follow normal distribution, expressions presented above, generally are not appropriate for estimation of demand parameters for time periods of length T, and it is necessary to estimate demand parameters using some other methods.

If demand data are available for some long time horizon then approach presented below can be used. Typical stochastic time series of realised daily demand, over some time horizon, is shown in upper chart of Fig. 1. For given time horizon and realised daily demand, we divide time horizon into periods of equal length T, and then sum all daily demands in every period, and then estimate total demand for every such period. Division of time horizon into periods and total demand for every period are shown in lower chart of Fig. 1. Using statistical tools it is possible to estimate required statistical demand parameters for time period of length T, such as expected demand, variance and distribution of demand.

This approach has two very important disadvantages: time horizon must be long, while review period and lead time must be short enough, and drastic sample demand data size reduction used to estimate demand parameters, because new sample demand data size is reduced by factor equal to review period length.

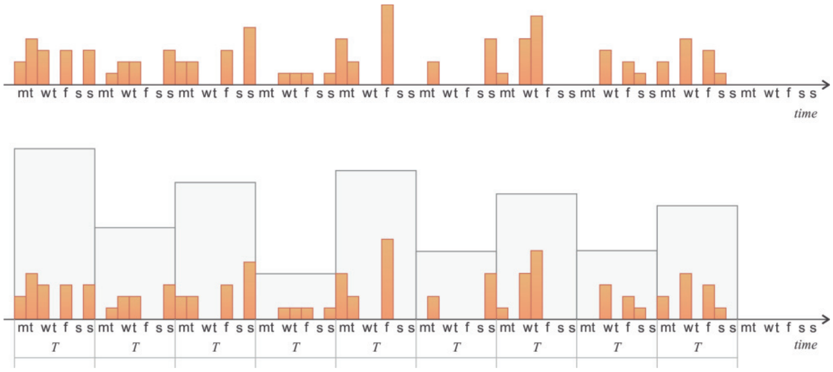

Fig. 1. Division of time horizon into periods of length T and grouping of demand data.

Problem of estimation of demand parameters mentioned above can be exceeded using approach presented below. If historical demand data are available, then using approach presented above, we divide time horizon into periods and then estimate demand parameters for time period of length T. In the next step we create *new* demand time series, which is modified original time series. Modification is done in such a way that first daily demand from original time series is set to the end of the original time series, and this is how we get new time series. New time series is then again divided into periods of length T and then demand parameters for time period are estimated. This procedure is repeated $T-1$ times, wherein for every new time series, original is the previous one. Presented approach of grouping demand data by overlapping periods, is graphically presented on Fig. 2.

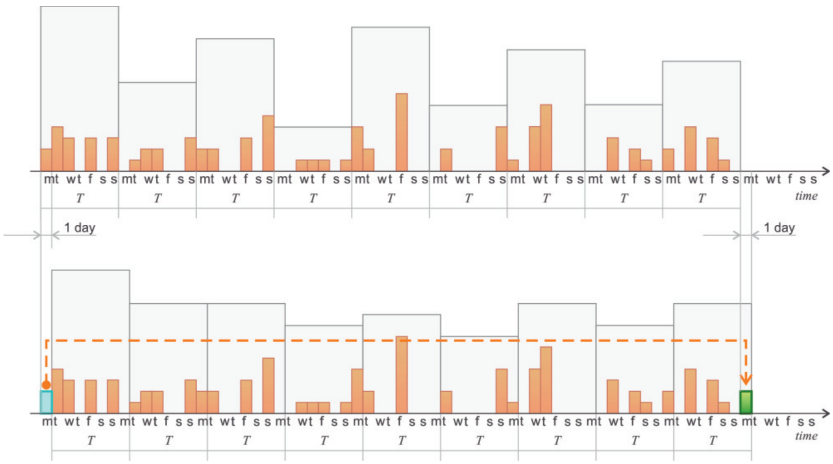

Fig. 2. Graphical presentation of demand grouping into periods that overlaps.

Using this approach we get T time series of demand data grouped into periods of length T. When analysing demand data obtained using proposed approach, we can consider two different approaches in dealing with obtained data: using all the T time series or using just one of T time series.

If all the T time series are used in analysis, than expected value of demand will be close to that estimated using (1), while variance of demand will be higher than that estimated using (2). Even though proposed approach results in higher variability of demand data, it has benefit in risk reduction. Optimal policy is less affected by sudden or expected changes in demand and/or supplier's response, because values of optimal control parameters will be higher (overestimated) than those estimated for stable and certain environments. Values of control parameters are directly dependant on variability of demand data, and when variability is higher than values of control parameters will be higher too. Such policy will have higher total average cost, but one have to find compromise (trade-off) between costs, on one side, and risk reduction and simplicity of proposed approach, on the other side. Finally, this approach can be used in uncertain environments, where demand is volatile and suppliers unreliable.

It is possible among T time series to find one with smallest demand variance that corresponds to certain time when periods are started to create. For example, it can be that demand variance is lowest when periods start at Tuesday (see Fig. 2.). When variance is lower than we can expect that policy control parameters will have lower values, such as lower maximum inventory level, safety stock or reorder point, and consequently will lead to lower expected total cost. Information about demand behaviour can be very helpful in negotiations with suppliers about order timing. However, this approach is very prone to risk, because small changes in demand can lead to variance increase, and consequently, out-of-stock situations and appearance of backorders. This will decrease service levels and increase expected total cost. Therefore, this approach is suitable for systems that operate in certain and stable environments, and can be useful in finding demand patterns with smallest variance that will help in expected total cost decrease.

3. Inventory model formulation

To analyse proposed approach to demand modeling by grouping demand data into periods that overlap, we use periodic review inventory model where all unsatisfied demand is backordered. We consider single item, single location, and periodic review inventory control problem.

We suppose that demand in successive time periods is positive, random, independent and identically distributed variable. When system has positive inventory level, all customer demands are satisfied immediately from inventories on stock. When system is out of stock, we suppose that customers are willing to wait for next order delivery to satisfy their demand, so all unsatisfied demand is backordered. When next order arrives, backorders are satisfied first and then regular demands. When system has positive inventory levels, then for every item unit on-hand per time unit, holding cost is charged. As exact number of item units on-hand is not known in advance, holding cost is charged on expected number of item units on-hand. For every item unit, system is charged h_d currency units per time unit. When system is out-of-stock than for every backordered unit, system is charged π currency units per item unit. We suppose that lead-time L, the time between placing and receiving an order, is constant. Lead-time is shorter than review period R, meaning there is no order overlapping, and in any time period, system has the most one outstanding order.

Proposed mathematical model of expected total cost function is defined by two parameters: review period length R, and maximum inventory level S in inventory control system. Total cost function has three component costs: ordering cost, holding cost and backorder cost (penalty cost). The expected total cost $C(R,S)$ per unit time for proposed periodic review inventory model, is given by expression:

$$C(R,S) = \frac{A}{R} + h_d \cdot \left[\int_0^S \left(S - \frac{x_{R+L}}{2} \right) \cdot f(x_{R+L}) \, dx_{R+L} - \frac{1}{2} \bar{x}_L \right] + \frac{\pi}{R} \cdot \int_S^{+\infty} (x_{R+L} - S) \cdot f(x_{R+L}) \, dx_{R+L} \tag{5}$$

The derivation of expected total cost function $C(R,S)$ (5) over parameters R and S, and equaling derivatives to zero, gives the first-order condition for optimality, i.e. optimal values of control parameters R^* and S^*, which minimize expected total cost function $C(R^*,S^*)$.

$$\begin{aligned} \frac{\partial C(R,S)}{\partial R} &= -\frac{A}{R^2} + h_d \cdot \left[\int_0^S \left(S - \frac{x_{R+L}}{2} \right) \cdot \frac{\partial f(x_{R+L})}{\partial R} \, dx_{R+L} \right] - \\ &- \frac{\pi}{R^2} \cdot \int_S^{+\infty} (x_{R+L} - S) \cdot f(x_{R+L}) \, dx_{R+L} + \frac{\pi}{R} \cdot \int_S^{+\infty} (x_{R+L} - S) \cdot \frac{\partial f(x_{R+L})}{\partial R} \, dx_{R+L} = 0 \end{aligned} \tag{6}$$

$$\frac{\partial C(R,S)}{\partial S} = h_d \cdot \int_0^S f(x_{R+L})\,dx_{R+L} - \frac{\pi}{R} \cdot \int_S^{+\infty} f(x_{R+L})\,dx_{R+L} = 0 \tag{7}$$

Replacing $\int_0^S f(x_{R+L})\,dx_{R+L}$ with $F_{R+L}(S)$, and $\int_S^{+\infty} f(x_{R+L})\,dx_{R+L}$ with 1- $F_{R+L}(S)$, from (7) follows:

$$F_{R+L}(S) = \frac{\pi}{\pi + h_d \cdot R} \tag{8}$$

Optimal value of cumulative distribution function of demand over period $R+L$, for which total cost function is minimized, can be obtained using (8). As can be seen from (8), optimal value of cumulative distribution function can be only obtained if value of R is known. Using (6) to find optimal value of control parameter R, i.e. optimal review period length, is very complex, because of $\partial f(x_{R+L})/\partial R$ terms. These terms are not trivial, because with changes in R, expected value and variance of demand will change too. Also, with changes in R, it is likely that shape of demand distribution will change, and total cost function can be only written in general form, making it very complex to solve analytically. So, if R is known in advance, using inverse cumulative distribution function of demand or numerical methods with known cumulative distribution function of demand, it is possible to find optimal value of maximum inventory level S^*, which minimizes total cost function.

4. Numerical results

In this part of work, we conduct verification and testing of proposed inventory model and approach to demand modeling, with aim to: estimate accuracy at which optimal control parameter value is estimated, and reliability of proposed model and approach to demand modeling, in order to estimate possibilities of their application in theoretical and real life contexts. Verification and testing are conducted on real life examples, on items with different but important characteristics such as: costs, demand parameters and shapes, and also for different review period lengths. Items used in analysis significantly differ in parameters such as: purchase and selling price, demand frequency, value and variance. Selected items have relatively low and high values of these parameters.

Item 1 is from grocery group and is widely available in the most grocery stores. Surrogate items for Item 1 are also available. Data for Item 1, used in analysis, are as follows: time horizon length is 355 days, total demand in time horizon is 16424 units, daily demand $\bar{x}$ = 46.20, standard deviation is 29.87, purchase price c = 1.64 €/unit, fixed ordering cost A = 10 €/order, backorder cost π = 0.70 €/unit backordered, holding cost h_d = 0.001348 €/unit/day. Frequency, demand value and variance for Item 1 are high, so it can be classified as fast moving item.

Item 2 is household air-conditioning device and is available in the most home appliances stores. Data for Item 2, used in analysis, are as follows: time horizon length is 247 days, total demand in time horizon is 18 units, daily demand $\bar{x}$ = 0.07, standard deviation is 0.25, purchase price c = 376.72 €/unit, fixed ordering cost A = 50 €/order, backorder cost π = 100 €/unit backordered, holding cost h_d = 0.31 €/unit/day. Frequency, demand value and variance for Item 2 are low, so it can be classified as slow moving item.

In order to verify and test proposed inventory model and approach to demand modeling, it is necessary to compare values of model performance indicators to those of simulation. Values of simulation performance indicators are considered exact ones. Simulation is performed using proposed periodic review inventory model and its input and control parameters on real demand data. Performance indicators used in this experimental setting are: P_1 and P_2 service levels [11], average inventory level, ratio of average to maximum inventory level, expected value of backordered units, total and component costs. Ratio of average to maximum inventory level I/M is one of performance indicators used to estimate optimality of control parameter value. For example, when value of this ratio is close to 1, then it indicates that value of control parameter is overestimated, and consequently, system has excessive inventories. Values of this ratio close to 0 indicate that control parameter value is underestimated meaning system operates without inventories. Values of this ratio close to 0.5 indicate that value of control parameter is optimal or near optimal, because in ideal situation value of this ratio should be 0.5.

To estimate accuracy and reliability of proposed inventory model we have used two measures: error or deviation of model performance indicators to those simulated, and relative error used to estimate relative deviations of performance indicators. Error Δ_j is difference between values of j -th model and simulation performance indicator and can be calculated using following expression:

$$\Delta_j = Model_j - Simulation_j \tag{9}$$

where: $Model_j$ – value of j -th model performance indicator, $Simulation_j$ – value of j -th simulation performance indicator. If error Δ_j is negative, than it means that model underestimates j -th performance indicator value, meaning real value is greater, and if error Δ_j is positive, than it means that model overestimates j -th performance indicator value, hence real value is smaller. Error shows how much expected value differs from exact one, but it does not show degree of sensitivity. To estimate sensitivity of differences we have used relative error. Relative error $\Delta_j\%$ of j -th performance indicator, can be calculated using following expression:

$$\Delta_j\% = \frac{\Delta_j}{Simulation_j} \cdot 100\% \tag{10}$$

Error and relative error of total cost, as one of the performance indicators, are especially important, because control parameters of proposed inventory model are obtained using this indicator. Errors and relative errors of other performance indicators are also important in analysis of trade-offs between average inventory levels and backorders, i.e. holding and backorder costs, and can be used to further improve inventory control models.

Numerical experiments are performed on Items 1 and 2 using proposed periodic review inventory model and demand parameters obtained using method of grouping demand data into periods that overlap. Optimal values of maximum inventory levels S , for fixed and known values of review period R and lead-time L lengths, and also values of performance indicators, are shown in tables below. Relative errors of performance indicators for which simulation value is equal to 0 could not be calculated and are represented with hyphen (-).

Table 1. Comparative view of optimal control values and values of relative errors of performance indicators of (R,S) model, for Item 1, review period lengths R = 7, 14 and 30 days, and lead time length L = 2 days.

Review period length R	7	14	30
Maximum inventory level S	643	1053	1874
Performance indicators	Relative error, %		
P_1 service level	0.0000	3.8613	4.9279
P_2 service level	0.0000	-0.0100	1.2051
Maximum inventory level M	0.0000	4.2574	0.0000
Average inventory level I	-8.7993	-6.4543	-8.6980
Ratio I / M	-8.7993	-10.2743	-8.6980
Backorders	-	-	-41.4634
Ordering cost	0.0000	0.0000	0.0000
Holding cost	-8.7991	-6.4545	-8.6980
Backorder cost	-	0.0000	-41.4634
Total cost	-1.0986	1.0561	-13.2350

Table 1 shows that model has excellent results for review period lengths of 7 and 14 days. It comes to support decision for more frequent ordering of observed item. However, results for review period length of 30 days shows very poor results. Because of very high values of demand and its variance, and long review period, it can be noticed that significant relative error of backorder and total cost are present. Precisely, model underestimates number of backordered units. In case, number of backordered units estimated by model is equal to that exact, relative error of total cost would be -5.8%.

Table 2. Comparative view of optimal control values and values of relative errors of performance indicators of (R,S) model, for Item 2, review period lengths R = 7, 14 and 30 days, and lead time length L = 2 days.

Review period length R	7	14	30
Maximum inventory level S	3	3	4
Performance indicators	Relative error, %		
P_1 service level	-2.12	-2.6310	12.0774
P_2 service level	0.0000	5.7285	11.7224
Maximum inventory level M	0.0000	0.0000	0.0000
Average inventory level I	-1.1407	-2.5000	-3.0100
Ratio I / M	-1.1407	-2.5000	-3.0100
Backorders	-	0.0000	0.0000
Ordering cost	176.9231	63.6364	12.5000
Holding cost	-0.8434	-2.4209	-2.9029
Backorder cost	-	0.0000	0.0000
Total cost	134.8451	41.4202	5.2253

Table 2 shows that relative errors of expected total cost have significantly high values. Reason for such high values of relative errors lay in very high relative errors of ordering cost. According to proposed model, number of orders is calculated as a ratio between time horizon and review period lengths. This ratio, i.e. number of orders, is very high especially for short review periods R, and therefore we can expect very high ordering cost. In reality, because of very small and rare demand for Item 2, number of realized orders is significantly lower, and system does not place an order to supplier every time when review is done, because inventory level at review, is already equal to maximum.

Because of mentioned problem, we have analysed situations when system does not charge ordering cost, while all other parameters are unchanged. In Table 2a, relative errors of costs are shown, while all other relative errors of performance indicator are equal to those in Table 2.

Table 2a. Comparative view of optimal control values and values of relative errors of performance indicators of (R,S) model, for Item 2, review period lengths R = 7, 14 and 30 days, lead time length L = 2 days, and fixed ordering cost A = 0.

Review period length R	7	14	30
Maximum inventory level S	3	3	4
Performance indicators	Relative error, %		
Ordering cost	0.0000	0.0000	0.0000
Holding cost	-0.8434	-2.4209	-2.9029
Backorder cost	-	0.0000	0.0000
Total cost	-0.8434	-1.5692	-1.5509

Relative error of total cost, where relative error of ordering cost is omitted, shows excellent matching with exact ones. Also, all other model performance indicators show excellent matching with exact ones.

Summary and conclusion

We analyzed new approach to demand modeling for time periods of desired length. After demand is modeled and its statistical parameters are estimated, we have used them to describe demand parameters in proposed periodic review inventory control model. Proposed approach to demand modeling enables significant improvements of demand parameters estimation with higher reliability and robustness. This was achieved through demand data grouping into periods of desired lengths that overlap. Achieved improvements of demand data modeling can be summarized as follows:

- To ensure large enough sample of demand data per period, used for estimation of reliable and robust demand parameters, even in situations when limited demand data are available. Proposed approach helps in dealing with limited demand data availability, and consequently, possibility of use of inventory control models at all.
- Demand data obtained using proposed approach are more robust, i.e. less sensitive to sudden changes in demand, such as unexpected increase or decrease of demand during some periods.
- Demand data contain all the possible realizations of demand per period that can occur in the considered time horizon, regardless of time when periods start to create, and thus helping in risk decrease.
- Possibility to find one time series with lowest variance among T time series of demand per period and thus to reduce total cost.

Testing and verification of proposed periodic review inventory control model have shown very good results, even in the optimization of items with very different demand and cost properties. Comparing performance indicators of proposed model to that simulated, we have determined acceptable deviations. Optimization results are especially good for short review periods R . Optimization results for Item 1 (classified as fast moving item) are very good for short review periods, with relative errors less than 2%, and poorer for longer review periods. Optimization results for Item 2 (classified as slow moving item) and short review periods are very poor, in case ordering cost is charged. Results are better for longer review periods, with relative error of total cost about 5.25%. However, if ordering cost is not charged, for any review period length, results are very good, with relative error of total cost less than 1.6%.

Future research should take into consideration analysis of proposed approach for demand modeling with continuous review inventory models and optimization of a group of items.

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