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Fault Accommodation in Technical Systems Based on Logic-Dynamic Approach

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Abstract

The paper is devoted to the problem of fault accommodation in nonlinear dynamic systems related to constructing the control law which provides full decoupling with respect to fault effects. Existing conditions are formulated and calculating relations are given for the control law. The logic-dynamic approach is used to solve the problem whose features are consideration of the systems with non-smooth nonlinearities and the use of relatively simple linear methods which may be supported by existing programming systems, e.g. MatLab.

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1. Introduction

An increasing demand on reliability and safety for critical purpose control systems calls for the use of fault tolerant control (FTC) techniques. The goal of the FTC is to determine such a control law that preserves the main performances of the system in the faulty case while the minor performances may degrade. There are two principle approaches to the FTC [1, 4, 6, 7, 8, 11]. The first of them involves adaptive control techniques and assumes on-line fault detection and estimation followed by control law accommodation. The second approach is focused on such a control law determination which provides full decoupling with respect to fault effects in output space of the system.

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In contrast to the first approach, the second approach does not need in fault estimation. Therefore, such approach looks reasonable if on-line fault estimation is problematic.

The problem of fault accommodation in dynamic systems was solved in [5, 9] based on differential geometry and algebra of functions and demands complex analytical calculations. In this paper, we use the logic-dynamic approach proposed in [2, 10]. The main idea of this approach is replacing the nonlinear system under consideration by certain linear one, solving the problem for this linear system involving linear methods and, finally, taking into account nonlinear terms to correct the obtained solution. The main features of the proposed approach are: (1) it considers the systems with non-smooth nonlinearities in dynamics, (2) it involves known linear methods that results in possibility to solve the fault accommodation problem by existing programming systems without using the symbolic software, (3) it can be applied both to discrete-time and continuous-time systems.

The logic-dynamic approach to solve the fault accommodation problem was considered in [3]. The present paper takes more sophisticate analysis that allows to obtain simpler solution (in particular, static solution) and extend a class of systems which the fault accommodation problem can be solved for.

To apply the logic-dynamic approach and to take into account the faults, it is assumed that the initial system Σ is described by the following model

$$x(k+1) = Fx(k) + Gu(k) + C\begin{pmatrix} \varphi_1(A_1x(k), u(k)) \\ \cdots \\ \varphi_p(A_px(k), u(k)) \end{pmatrix} + Dd(k), \qquad y(k) = Hx(k)\,, \tag{1}$$

where F, G and H are the matrices of appropriate dimensions, describing a linear part of the system; D is known constant matrix, d is a vector, describing the faults: if a fault occurs, $d(k)$ becomes an unknown function of time, otherwise $d(k)=0$; C is $n \times p$ constant matrix; $\varphi_1,...,\varphi_p$ are nonlinear functions which maybe non-smooth; $A_1,...,A_p$ are row matrices.

It is assumed that the fault detection and isolation procedure is performed by known methods (see e.g. [1]). If a fault occurs, $d(k)$ becomes an unknown function, and a solution of the control problem based on the model (1) becomes impossible. To overcome this difficulty, one suggests to obtain the vector $u(t)$ according to the relation

$$u(k) = g(x_0(k), y(k), u_*(k)), \tag{2}$$

where g is the vector function to be determined, $u_*(k)$ is a new control vector, $x_0 \in R^{n_0}$, $n_0 \le n$, is a state vector of the system Σ_0 described by the model

$$x_0(k+1) = F_0x_0(k) + G_0u_*(k) + J_0y(k) + C_0\varphi(A_0\begin{pmatrix} x_0(k) \\ y(k) \end{pmatrix}, u_*(k)), \tag{3}$$

where F_0, G_0, J_0, C_0, A_0 are the matrices to be determined.

Assume that the control (2) exists and the fault occurred and was detected, then a solution of the control problem is performed on the basis of the additional system Σ_* described by the model

$$x_*(k+1) = F_*x_*(k) + G_*u_*(k) + C_*\varphi(A_*x_*(k), u_*(k)), \tag{4}$$

corresponding in a definite sense to the initial model (1); here F_*, G_*, C_*, A_* are the matrices to be determined. Note that (4) does not contain the unknown vector $d(k)$. Therefore, fault accommodation effect may be achieved by using the model (4) for control determination.

The problem under consideration is to determine the existing condition for the control (2) and to obtain the function g and the matrices, describing the systems Σ_0 and Σ_*.

2. Problem solution

2.1. Auxiliary system Σ' design

Consider at first for simplicity the case when the initial system contains the single type of nonlinearity, i.e. $p = 1$. To solve the problem, introduce the auxiliary system Σ' of maximal dimension described by the model

$$x'(k+1) = F'x'(k) + G'u(k) + J'y(k) + C'\varphi(A'\begin{pmatrix} x'(k) \\ y(k) \end{pmatrix}, u(k)), \tag{5}$$

where the state vector x' satisfies the condition

$$x'(k) = \Phi x(k) \tag{6}$$

for some matrix Φ. The model (5) does not depend on the unknown vector $d(k)$ and can be used to design the observer for estimating the initial system state vector when the fault occurred. In the paper, this system will be used to construct the systems Σ_0 and Σ_* and the control law (2).

The logic-dynamic approach assumes [2, 10] that on the first and second steps, the linear part of the system Σ is considered and the linear part of the system Σ' is constructed by linear methods with some additional restriction. It is known [10] that the following relationships are between matrices, describing the systems Σ and Σ':

$$\Phi F = F'\Phi + J'H, \qquad G' = \Phi G, \qquad \Phi D = 0, \tag{7}$$

$$C' = \Phi C, \qquad A = A'\begin{pmatrix} \Phi \\ H \end{pmatrix}. \tag{8}$$

The last expression in (8) is an additional restriction on the matrix Φ which is taken into account on the second step. The second equality in (8) holds if and only if rows of the matrix A are linearly dependent on rows of the matrices Φ and H that is equivalent to the condition

$$rank\begin{pmatrix} \Phi \\ H \end{pmatrix} = rank\begin{pmatrix} \Phi \\ H \\ A \end{pmatrix}. \tag{9}$$

If the model (1) contains several nonlinearities, the matrix A in (8) and (9) is replaced by A_i, $i = 1,...,p$.

To take into account the condition $\Phi D = 0$, introduce the matrix D^0 of maximal rank such that $D^0 D = 0$. Then $\Phi = ND^0$ for some matrix N. Rewrite the first equation in (7) with $\Phi = ND^0$ by separating known matrices from the unknown ones:

$$(N \quad -F'N \quad -J')\begin{pmatrix} D^0 F \\ D^0 \\ H \end{pmatrix} = 0. \tag{10}$$

Let $(V \quad W \quad Z)$ be a solution of (10), therefore the relation $W = -F'V$ is true that is rows of the matrix V are linearly dependent on the matrix Z rows. To take into account the last equality, one has to remove from the matrix $(V \quad W \quad Z)$ all rows where the corresponding row of the matrix V is independent of the rows of Z. Algorithm below finds the matrix satisfying the condition $W = -F'V$. Denote the i-th row of the matrix W by W_i and the number of rows of W by w.

Algorithm.

Step 1. Set $i := 1$.

Step 2. If $rank(V) = rank\begin{pmatrix} V \\ W_i \end{pmatrix}$, go to Step 4.

Step 3. Remove the i-th row from the matrix $(V \quad W \quad Z)$, denote the obtained matrix by $(V' \quad W' \quad Z')$, set $(V \quad W \quad Z) := (V' \quad W' \quad Z')$, $w := w - 1$ and go to Step 1.

Step 4. Set $i := i + 1$. If $i < w$, go to Step 2.

Step 5. Find in the matrix Z' the maximal number of linearly independent rows, remove the rest rows from this matrix and remove the corresponding rows of the matrices V' and W'. Denote the matrix obtained by $(V^0 \quad W^0 \quad Z^0)$.

It follows from (14) that $J' = -Z^0$ and $\Phi = V^0 D^0$; besides the matrix F' can be found from the algebraic equation $F'V^0 = -W^0$.

To finish the second step of the logic-dynamic approach, check whether or not the matrix Φ satisfies the condition (9). If not, the system (5) invariant with respect to the unknown function $d(t)$ does not exist and fault accommodation problem is not solvable. Otherwise, Step 3 of the logic-dynamic approach can be performed. To do this, one can find the matrix A' from the algebraic equation (8). Suppose that the matrix Φ satisfies the condition (9); set $G' = \Phi G$ and $C' = \Phi C$ according to (7) and (8), respectively. Thus, the system Σ' described by (5) has been constructed. To simplify the references, denote this system by

$$x'(k+1) = f'(x'(k), y(k), u(k)). \tag{11}$$

To simplify a solution, assume that when some component of the function f' contains the output y, then it contains the control u.

2.2. Control law design

Find in the function $f'(x', y, u)$ all terms of the form $\alpha_j(x', y, u)$, $j = 1, \ldots, r$, , containing the output y and the control u; note that by the above assumption, such terms exist. It is assumed that each term contains minimal number of variables; note that in some cases the equality $\alpha_j(x', y, u) = f_i'(x', y, u)$ is possible. Denote $\alpha = (\alpha_1, \ldots, \alpha_r)^{\mathrm{T}}$ and assume that $rank(\partial\alpha / \partial u) = s$ for some integer s for all x', y, and u except on a set of measure zero. Set

$$\begin{aligned} u_{*1} &:= \alpha_1(x', y, u), \\ &\ldots \\ u_{*r} &:= \alpha_r(x', y, u). \end{aligned} \tag{12}$$

It is assumed that the function $\alpha(x', y, u)$ contains m' components of the control vector $u(k)$. Clearly, $m' \geq s$ and $r \geq s$ by definition. Consider four cases; assume that the function α does not contain functionally dependent components.

1. $m' = r = s$. In this case equations (12) are solvable (generically) uniquely for some m' components of the control vector; without loss of generality assume that they are the first m' components $u_1, \ldots, u_{m'}$:

$$u_i = g_i(x', y, u_*), \qquad i = 1, \ldots, m', \tag{13}$$

g_i is some function. One can set

$$u_i := u_{*i}, \qquad i = m' + 1, \ldots, m, \tag{14}$$

for the rest components.

2. $m' > r = s$. In this case the function γ contains $m' - r$ redundant components of control; without loss of generality assume that they are the last $m' - r$ components $u_{r+1},\ldots,u_{m'}$. One can set

$$u_i := u_{*i}, \qquad i = r+1,\ldots,m,$$

for these and the rest components and solve equations (12) in the form (13) for $i = 1,\ldots,r$.

3. $m' \geq r > s$. We need to find the matrix P with s rows such that

$$rank\left(P\frac{\partial\alpha}{\partial u}\right) = s$$

for all x', y, and u except on a set of measure zero. The matrix P collects s functionally independent components from all components of the function α; redundant components of the vector u (they exist when $m' \geq r$) are now in the function $P\alpha$. Set $u_i = u_{*i}$, for $i = s+1,\ldots,m$, then the equation $u_* = P\alpha$ is solvable uniquely for $i = 1,\ldots,s$ in the form (13).

4. $r > m'$. In this case equations (12) are incompatible; to solve the problem, one needs to use the matrix P collecting s functionally independent components from all components of the function α by analogy with the third case.

To check a quality of the obtained solution (if it exists), replace in (11) the first m' components of the vector u by the expressions $g_i(x', y, u_*)$ according to (13) (s components are used in cases 3 and 4). Note that such a replacement corresponds to the feedback for fault accommodation. Denote the function f' after the replacement by f'_*. If the function f'_* does not contain the vector y, then the control in the form (13) and (14) has been constructed. Otherwise, one has to analyze the function f'_* by analogy with f' and correct the control law g. Consider for simplicity the case when f'_* does not contain the vector y.

2.3. Systems Σ_0 and Σ_* design

To construct the system Σ_0, the vector u in the function f' is replaced by u_* according to (13) and (14) and the components of the vector x' are formally replaced by the components of the vector x_0. In some cases the dimension of the obtained system may be reduced, see for details [5]. Recall that in the static case the system Σ_0 is absent.

To construct the system Σ_*, the vector u in the function f' is replaced by u_* according to (13) and (14) analogously to the system Σ_0. Note that such a replacement results in the equations, containing the components of the output vector in the form $y_j = h'_j(x')$ for some function h'_j. Replacing y_j by $h'_j(x')$ and then x' by x_*, one obtains the system Σ_*.

3. Static solution

If the function α (or $P\alpha$) does not contain the components of the state vector x', then the control law g is free from x' as well and (2) takes the form $u = g(y, u_*)$. This corresponds to the static solution where the system Σ_0 is absent. The sufficient condition of such a solution is given by the following proposition.

Proposition. If

$$rank\begin{pmatrix} D^0F \\ H \end{pmatrix} < rank(D^0F) + rank(H) \tag{15}$$

and

$$rank\begin{pmatrix} A \\ H \end{pmatrix} = rank(H), \tag{16}$$

the fault accommodation problem has the static solution.

Proof. Clearly, (15) is equivalent to existence of the nontrivial solution of the equation

$$Z\begin{pmatrix} D^0F \\ H \end{pmatrix} = 0 \cdot \tag{17}$$

Let the matrix $Z = (N \quad -J')$ represents all linearly independent solution of (17). Set $\Phi := ND^0$, then $\Phi F = J'H$. Next, if (16) holds, then $A = A'H$ for some matrix A'. The last two equations allow constructing the system Σ' in the form

$$x'(k+1) = G'u(k) + J'y(k) + C'\varphi(A'y(k), u(k)),$$

where $C' = \Phi C$ and $G' = \Phi G$. Since the right-hand side of this equation does not contain the state vector x', then the function α (or $P\alpha$) does not contain x' as well, and the fault accommodation problem has the static solution.

4. Comparison with known approach

Remind that another approach to solve the fault accommodation problem was considered in [3, 5, 9]. The main steps of this approach are as follows. The system Σ' is designed and the following equations

$$\begin{aligned} u_{*1} &= f_1'(x', y, u), \\ &\cdots \\ u_{*r} &= f_r'(x', y, u), \end{aligned} \tag{18}$$

are considered. The problem of solvability of these equations for the control u is analyzed; when $m' \geq r > s$, the equation $u_* = Pf'$ similar to $u_* = P\alpha$ is considered. The results of analysis are represented in the form (13) and (14). The system Σ_0 in [3, 5, 9] coincides with Σ'; the system Σ_* is found in the form of composition of the two subsystems, the first has the simplest form

$$\begin{aligned} x_{*1}(k+1) &= u_{*1}(k), \\ &\cdots \\ x_{*r}(k+1) &= u_{*r}(k). \end{aligned} \tag{19}$$

The second subsystem is autonomous; the special procedure to design this subsystem was developed in [3, 5, 9]. Comparing similar equations (12) and (18), one concludes that each equation in (12) contains minimal number of variables that gives a possibility to obtain the static solution (if it exists). Unlike, the right hand sides of equations in (18) coincides with that describing the system Σ'. The approach suggested in [3, 5, 9] cannot yield the static solution (even if it exists).

The main disadvantage of (18) is redundant variables which may result in insolvability of (18) for the components of the control vector u; consider the example. Let $r = 1$ and

$$x_1'(k+1) = f_1'(x'(k), y(k), u(k)) = u_1 y_2 + x_1' sign(u_1 + u_2)\,. \tag{20}$$

Clearly, the equation $u_{*1} = u_1 y_2 + x_1' sign(u_1 + u_2)$ is unsolvable both for u_1 or u_2 If however one takes in (20) minimal number of variables, this gives the function $u_{*1} = u_1 y_2$ and the static solution $u_1 = u_{*1} / y_2$. Note that replacement of u_1 by u_{*1} / y_2 in (20) yields

$$x_1'(k+1) = f_1'(x'(k), y(k), u(k)) = u_{*1} + x_1' sign(u_{*1} / y_2 + u_2)$$

containing y_2 and the control u_2. This means that there is need to repeat the correction of the control. Setting $u_2 := u_{*1} / y_2 + u_2$, one obtains the equation solvable for u_2. Therefore the approach suggested in [3, 5, 9] does not yield the solution in comparison with the present paper. Thus, the suggested approach allows extending a class of system which the fault accommodation problem can be solved for. Besides, it allows obtaining simpler solution generally.

5. Implementation and future researches

The systems Σ_0, Σ_* and the control law (2) can be used to achieve the fault accommodation effect by the following way. Assume that the task consists in finding a control transferring the faulty system Σ described by (1) from the state $x^{(1)} = x(k_1)$ to $x^{(2)} = x(k_2)$. To solve this task, find the states $x_*(k_1) = \Phi_* x(k_1)$ and $x_*(k_2) = \Phi_* x(k_2)$, corresponding to the states $x^{(1)}$ and $x^{(2)}$. Appropriate control $u_*(k)$, $k_1 \le k \le k_2$, solving this task for the system Σ_* is found, and then the expression (2) is used to find a control $u(k)$, $k_1 \le k \le k_2$, solving the task for the initial system with a range of accuracy given by matrix Φ_*.

It is known that in some cases the auxiliary system (5) invariant with respect to the unknown function $d(k)$ does not exist. This means that there is no way to provide full decoupling with respect to faults effects, therefore the fault accommodation problem is not solvable. The plan of future researches is to overcome this difficulty by developing the method to find optimal partial decoupling minimizing the faults effects.

Conclusion

In the paper, the fault accommodation problem in the systems, described by model (1), has been considered. To solve this problem, the logic-dynamic approach has been used. The feature of this approach is that it allows to avoid complex analytical calculations and use the standard mathematical packages (e.g., Mathlab) to perform the necessary linear operations. Besides, it can be applied both to discrete-time and continuous-time systems. Existing conditions have been formulated and calculating relations have been given for the control law guaranteeing full decoupling with respect to faults effects. The future plan is developing the method to find optimal partial decoupling with respect to faults effects when full decoupling is impossible.

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