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Application of the Improved Method of Grids with the Estimation of Accuracy

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Abstract

The original method of digital differentiation of the approximation of function is synthesized. The method is based on generalize the improved method of grids. The question of accuracy of the obtained solution is examined. The numerical results are presented. The use of overlapping of interpolation intervals allows increasing an accuracy of the solution. The calculation results show that it is possible to adjust the accuracy of the solution either by changing the degree of the interpolation polynomial or with the help of overlapping of intervals.

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Key words: Matrix Equations, Method of Grids, Method of Uncertain Coefficients

1. Introduction

We shall confine ourselves to consider only the differential equation with zero regional conditions. In the case of general boundary conditions it is required to apply the matrixes, which are interpolated on Ermit. These matrixes turn out less elegant and more cumbersome, than the Vandermond matrixes, as both values of function and their derivative will be defined in this case. The method of uncertain coefficient can be used as for the traditional method of grids as for the “improved method of grids", which has been developed Kollats [1] for solution of partial differential equations especially. The method of grids allows to reduce a task of continuous analysis to a problem of solution of system of the algebraic equations [2]. The accuracy of the used interpolation polynomials is established by the well-known formulas from literature .

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2. Approximation of function and their derivatives

For an illustration we consider some elementary examples. Euler problem about a longitudinal bend. The differential equation of deflection curve in bend of beam (Fig.1), loaded by the longitudinal force F and using a dimensionless coordinate ξ, can be written as

$$[g(\xi)]y'' + [r(\xi)]y' + \lambda[s(\xi)]y = \{f(\xi)\} \tag{1}$$

With y (0) = 0 and y (n) = 0. Where $[g(\xi)]$, $[r(\xi)]$ and $[s(\xi)]$ are diagonal matrixes with the corresponding values of functions $g(\xi)$, $r(\xi)$ and $s(\xi)$ in points or nodes of interpolation, $f(\xi)$ is free function in the right part in the same points or nodes.

It is possible with the help of the formula [3-5] to calculate derivative value from interpolation polynomial in any point ξ of the closed interval. For example, at ξ = 1 the third derivative value is equal

$$\left.\frac{d^3P^*(\xi)}{d\xi^3}\right|_{\xi=1} = [0\,0\,0\,6\;\;24\;\;120\;\;\ldots n(n-1)(n-2)]^T\,[W_n]^{-1}\{y\} \tag{2}$$

The derivative values in nodes of interpolation are easy obtained from the formulae (2) if suppose, that ξ accepts consistently the values as: ξ = 0, 1, 2,…, n. Then the line of mth derivative from $[\,\xi\,]^T$ becomes a square matrix. At n = 4 matrixes - columns or the vector - column second derivative from $P_n^*(\xi)$ will be

$$P_4^{*''}(\xi) = \frac{d^2P_4^*(\xi)}{d\xi^2} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 6 & 12 \\ 0 & 0 & 2 & 12 & 48 \\ 0 & 0 & 2 & 18 & 108 \\ 0 & 0 & 2 & 24 & 192 \end{bmatrix} [W_4]^{-1}\{y\} = [-]_4''\,[W_4]^{-1}\{y\} \tag{3}$$

Where $[-]_4''$ is a matrix, which turns out as a result of the given operation. The lower index of the matrix specifies the polynomial order, and upper index - the derivative order. Vandermonde matrix$-\lfloor W_n \rfloor$, which elements is degrees of a natural line numbers. The foregoing formulae for differentiation of functions, which are given in discrete points, are generalization of the classical formulae of digital differentiation. Their error can be appreciated similarly, as it is carried out in the classical methods [6, 7]. Let's introduce a matrix $\left[O_n^{(m)}\right]$, which simultaneously carries out both operations of interpolation and differentiation of function given by a vector $\{y\}$

$$\left[O_n^{(m)}\right] = [-]_n^{(m)}[W_n]^{-1} \tag{4}$$

Taking into account (3) and (4) the system of the linear algebraic equations in a matrix form will be

$$[g(\xi)]\lfloor O_n'' \rfloor\{y\} + [r(\xi)]\lfloor O_n' \rfloor\{y\} + \lambda[s(\xi)]\{y\} = \{f(\xi)\} \tag{5}$$

This system of equations (5) according to the distributive operation can be written down in more convenient form

$$\left([g(\xi)]\left[O''_n\right]+[r(\xi)]\left[O'_n\right]+\lambda[s(\xi)]\right)\{y\}=\{f(\xi)\} \tag{6}$$

or

$$[D]\{y\}=\{f(\xi)\} \tag{7}$$

where $[D]=[g(\xi)]\left[O''_n\right]+[r(\xi)]\left[O'_n\right]+\lambda[s(\xi)]$ is a matrix operator of given differential equation. In case of constant coefficients the matrix operator becomes simpler. As all m matrixes, included in equation (7) can be calculated beforehand, the inferring (composing) of the equations becomes considerably simpler. The solution of system of the algebraic equations (7) can be carried out with the help of the inverse matrix

$$\{y\}=[D]^{-1}\{f(\xi)\} \tag{8}$$

Such solution is especially convenient in case of a large number of the right parts $\{f(\xi)\}$. In this case the inverse matrix $[D]^{-1}$ will carry out a role of resolving equation.

For an illustration we consider the numerical results, see fig.1 and equations 9-11.

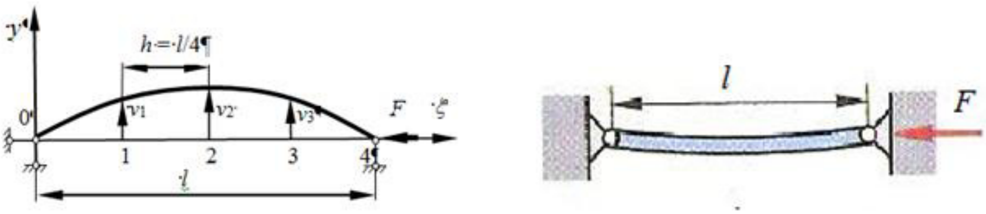


Fig. 1. Euler problem.

$$x=\frac{l}{4}\xi;\ \xi=\frac{4x}{l}; \tag{9}$$

$$\frac{d^2v}{d\xi^2}+\lambda v=0 \tag{10}$$

Where

$$\lambda=\frac{Fl^2}{16EI} \tag{11}$$

With the boundary conditions $v(0)=v(l)=0$.

With the help of (4) the transition from the given differential equation to the appropriate system of the linear algebraic equations becomes simpler. The matrix operators $\lfloor O''_2 \rfloor$ and $\lfloor O''_4 \rfloor$ will be accordingly

$$\left[O''_2\right]=[-]''_2\left[W_2\right]^{-1}=\begin{bmatrix}002\\002\\002\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\-1{,}5 & 2 & -0{,}5\\0{,}5 & -1 & 0{,}5\end{bmatrix}=\begin{bmatrix}1 & -2 & 1\\1 & -2 & 1\\1 & -2 & 1\end{bmatrix} \tag{12}$$

$$\left[O_4''\right]=\left[-\right]_4''\left[W_4\right]^{-1}=\frac{1}{24}\begin{bmatrix}70 & -208 & 228 & -112 & 22\\ 22 & -40 & 12 & 8 & -2\\ -2 & 32 & -60 & 32 & -2\\ -2 & 8 & 12 & -40 & 22\\ 22 & -112 & 228 & -208 & 70\end{bmatrix} \tag{13}$$

Using the expression (7) and operator $\left\lfloor O_2'' \right\rfloor$ (12) the characteristic system of the algebraic equations is obtained from equation (10), taking into account overlapping of intervals

$$\begin{aligned} 2v_1 - v_2 \qquad &= \lambda v_1 \\ -v_1 + 2v_2 - v_3 &= \lambda v_2 \\ - v_2 + 2v_3 &= \lambda v_3 , \end{aligned} \tag{14}$$

which gives the critical force value for II (see fig.4)

$$F_{kp} = \frac{9.38EI}{l^2} \tag{15}$$

with an error 5.2 %.
Applying the operator $\left\lfloor O_4'' \right\rfloor$ (33), according to the expression (12) in equation (10) without taking into account the overlapping of intervals, we receive the characteristic system of the equations with the same number of unknowns

$$\begin{aligned} 40v_1 - 12v_2 - 8v_3 &= \Theta v_1 \\ -32v_1 + 60v_2 - 32v_3 &= \Theta v_2 \qquad \Theta = 4! \cdot \lambda \\ -8v_1 - 12v_2 + 40v_3 &= \Theta v_3 \end{aligned} \tag{16}$$

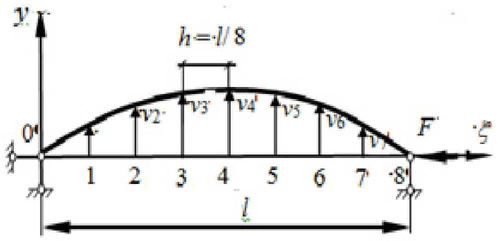
which gives the value

$$F_{kp} = \frac{9.395EI}{l^2} \tag{17}$$

with an error 5.0 %.
Increasing the division numbers or nodes (Fig.5), using operator $\left\lfloor O_2'' \right\rfloor$, we receive

$$F_{kp} = \frac{9.79EI}{l^2} \tag{18}$$

with an error 0.81 %.



$$x = \frac{l}{8}\xi;\ \xi = \frac{8x}{l};$$

Fig. 2. Numerical results

Applying operator $\lfloor O_4'' \rfloor$ in case of double number of nodes (Fig.2) and using the overlapping of intervals so, that a beginning of an interval [a, b] is consistently combined with the nodes of the extended interval [A, B] one after another 0, 0, 1, 2, 3, 4, 4, the characteristic system of the equations is obtained, where

$$\lambda = \frac{F \cdot l^2}{64EI} \tag{19}$$

$$\begin{aligned}
40v_1 - 12v_2 - 8v_3 + 2v_4 &= 4!\lambda v_1 \\
-32v_1 + 60v_2 - 32v_3 + 2v_4 &= 4!\lambda v_2 \\
2v_1 - 32v_2 + 60v_3 - 32v_4 + 2v_5 &= 4!\lambda v_3 \\
2v_2 - 32v_3 + 60v_4 - 32v_5 + 2v_6 &= 4!\lambda v_4 \\
2v_3 - 32v_4 + 60v_5 - 32v_6 + 2v_7 &= 4!\lambda v_5 \\
2v_4 - 32v_5 + 60v_6 - 32v_7 &= 4!\lambda v_6 \\
2v_4 - 8v_5 - 12v_6 + 40v_7 &= 4!\lambda v_7
\end{aligned} \tag{20}$$

the value of critical force will be $F_{kp} = \frac{9.87544EI}{l^2}$ with an error 0.05 %.

3. Conclusion

The above-mentioned technique for receiving ordinary derivative can be extended to calculation a partial derivative. The received formulae allowed carrying out the approximation of functions and their derivatives not resorting to differences as it is made in a classical method of grids. The use of overlapping of interpolation intervals allows increasing an accuracy of the solution. The calculation results show that it is possible to adjust the accuracy of the solution either by changing the degree of the interpolation polynomial or with the help of overlapping of intervals. That is the main difference not only from usual, but also from the “improved” method of grids. The essential simplification of the calculation formulae is received; in particular case they are the L. Kollats' formulae. Their receiving is carried out with the help of matrix notable symbolic.

The received results can be applied to the solution of boundary problems of various classes, problem of eigenvalues etc. In particular, this is supposed to use the given approach for calculating the stress condition in threaded joints [8]. The last is especially urgent in vibrodiagnostic of structures, designs, machines, equipment, industrial and civil buildings and so on [9-13]. The used matrix symbolic gives the convenient tool for realization of calculations with the help of computers.

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