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## Some Mathematical Considerations on the Natural Protection of the Danube Delta Coastal Zone in Romania

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### Abstract

Today, more and more scientists agree with the hypothesis concerning the raising of the planetary ocean level due to global climate warming and consequently due to glaciers melting. Therefore, as a consequence of climate change phenomenon, the Black Sea level is raising as well, which led to the loss of significant areas of land due to marine erosion, but also the production of some cracks and penetration of salt water into the Razelm - Sinoe freshwater ecosystem in Romania.

This paper uses a mathematical tool, which addresses the maintenance of the ecological balance of freshwater ecosystem, the transit of freshwater through the Razelm - Sinoe hydrotechnical complex, and from here into the Black Sea, required to ensure the hydraulic gradient from freshwater to saltwater. This would help maintaining vegetation as natural means for protection against the sea erosion, as well as to attenuate the coastal energy of waves, so as to ensure the shielding role together with the other coastal engineering construction. Besides, the strong reed vegetation represents a natural element of protection of levees already made or those in process of development, and also a retention element of fine particles brought by waves at times when sea level is high. The mathematical algorithm used in this paper take into account the principle of fluid flows in porous media. The vegetation of littoral from Razelm - Sinoe ecosystem was considered a porous media. The research results show the capacity of the vegetation to attenuate the waves' energy in the study area. The mathematical model was especially conceived for a unique littoral area, which separates the Black Sea from Razelm - Sinoe ecosystem. The solution of protection exploits the natural environment; it is not expensive and falls into the natural-ecological aspect of the coastal zone.

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## 1. Introduction

The coastal engineering for the defense of littoral zones encounters physical phenomena of the environment, such as wind, waves, sea currents, density currents in the contact zones between freshwater and sea saltwater, storms, level oscillations, the flow of sea alluvium under coastal streams etc. In addition, the coastal engineering influences the chemical phenomena due to the presence of some aggressive substances from the seawater and atmosphere, and interferes with some species from the littoral fauna and flora. The Danube Delta coastal zone that implicitly limits the Razelm-Sinoe freshwater ecosystem from the Black Sea has a recent genesis, because of the alluvial settlements discharged into the sea by the Danube and coastal currents.

The level fluctuations of the Black Sea, the waves actions, especially in the storm periods, the reducing of the alluvium in the upstream of the Danube due to hydrological drought, but also the high level of water uses for the population, industry and agriculture, as well as the silting of hydrotechnical accumulation from upstream have lately resulted a submersion process that means an important loss of land.

It is known that in the process of accumulation and development of the sand banks, hydrophilic vegetation has a special role with its areas, associated with reed, between riverbed and hallow as follows: the grasses, sedge and bulrush area, the rush and reed area [1].

If the first zone serves as attenuation for erosion process, the other three and especially the last one has an important role in the development process of accumulation, due to their behavior as a filtering zone of water and silt retention.

An important category of problems that appears in the design, manufacture, maintenance and operating of coastal constructions refers to the interaction between these works and the waves incidence.

Another problem area is the interaction between the regime of the specific morphological processes to the seaside and coastal constructions [2].

The execution of works in the border zone between sea and land may lead to disturbances in the mechanism of alluvial transport. Accumulation phenomena appear, which determine the development of reed vegetation that act as a protection of the coastal cordon, or as marine erosion when pre-existing stability of the sediment balance flow is affected [3].

The purpose of this paper is to demonstrate the role that the reed vegetation has upon the protection of the coastal area related to the Danube Delta in absence of some protective hydrotechnical works. The mathematical algorithm used in this paper take into account the principle of fluid flows in porous media. The vegetation of littoral from Razelm - Sinoe ecosystem was considered a porous media.

The importance of research results is determined by economic efficiency of protection measures that use such a method of coastal protection and the maintaining of a unique natural ecosystem with specific flora and fauna.

## 2. Methodology

Study of wave generation, propagation, speed and pressure made by the mass of water in terms of a certain regime of waves, reflection and diffraction waves in the coastal zone, stresses made by waves on constructions are some issues of coastal hydraulic engineering. One of the important issues related to coastal works behavior is given by the wave action on permeable constructions, as well as the waves discharging energy on constructions.

So far, the previous research shows that two aspects of the infiltration leaks are more frequently in the coastal constructions:

- a) the wave transmission through permeable levees;
- b) the movement of water pressure in the removal of some waterproof works placed on a foundation layer of porous material.

Both cases can be considered as plane – vertical, impermanent movements, through homogeneous media as a lower contour an impermeable horizontal layer [4].

The general equations characterizing the vertical movement through permeable media can be expressed by considering:

- equilibrium of hydrodynamic forces exerted on the liquid mass within an area covered by a scale parallelepiped  $\Delta x$ ,  $\Delta z$  and 1, the volume of fluid being  $n \Delta x \Delta z$ , porosity due to the environment;
- mass continuity in movement associated parallelepiped where “ $n$ ” is the volume of voids in a unit total volume, respectively.

The infiltration velocities taken into account are those corresponding to the specific flow rates and are lower than the actual ones, for which the cross area available is reduced proportionally to “ $n$ ”. Forces considered in the equilibrium equations are: inertia, pressure, weight and resistance as a result of the interaction between the mass of the fluid in motion and the solid skeleton of permeable environment [5].

One of the protection methods of the coastal area is also the vegetation, especially the reed.

The main issue of the present research is to establish the way to determine the waves’ action on the coastline in the presence of the reed vegetation. The mathematical model has unique application, being dedicated to compute the effect of wave action on the specific vegetation of the coastal zone formed by the natural lake Razelm – Sinoe and Black Sea.

### 3. Mathematical model

It is supposed that in present, the wave propagation through the extensive reed vegetation in the coastline of the Danube Delta can be considered as a movement through permeable media (Fig.1) [6]. In this area, the waves are strongly attenuated due to very high roughness of the vegetation - reed, bulrush, sedge, rush etc.). In order to develop the mathematical model, it were taken into consideration very small waves (small amplitudes in relation to wavelength).

It is considered the equation of hydrodynamic equilibrium (Eq.1):

$$\frac{\partial \varphi}{\partial t} + \frac{v^2}{2g} + y + \frac{p}{\gamma} = 0 \quad (1)$$

In eq.(1), the term  $\frac{v^2}{2g}$  can be neglected in relation to other terms, having in view that it is very small compared with the potential energy, and it is neglected also the vertically component of fluid velocity.

It is symbolized by  $M$ , the ratio between surface of free spaces  $\omega l$  and total surface  $\omega t$  in a vertical plan, perpendicular on the direction of the wave, in the marsh vegetation (Eq.2).

$$M = \frac{\omega l}{\omega t} \quad (2)$$

The bed of the riverbed, on which the waves are propagated, is considered horizontally, and the dike that limits the strip of the coastal area is perpendicularly on the wave direction and parallel with the shore line [7].

It can be assumed that horizontal flow velocity component, symbolized by  $u$ , derives from a potential function  $\varphi$ , which has the expression (Eq. 3):

$$\varphi = -kH + c \quad (3)$$

where:

H - represents hydraulic load;

c - represents a constant.

According with hypothesis assumed, the horizontal flow velocity component  $u$ , is (Eq.4):

$$u = \frac{\partial \varphi}{\partial x}; \quad (4)$$

Hydraulic load,  $H$ , can be expressed as follows (Eq. 5):

$$H = y + \frac{p}{\gamma} \quad (5)$$

where:

$y$  – height of a current point of water;

$\gamma$  – specific weight of water;

$k$  – filtration coefficient through porous media; on a particular case, coefficient of filtering through reed, is symbolized by "b";

$p$  – pressure.

#### *Movement equations*

Continuity equation is a Boussinesq-type equation (Eq.6)

$$m \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( kh \frac{\partial H}{\partial x} \right) \quad (6)$$

where  $h$  is the mean water depth and  $m$  is particle mass.

According to the assumed hypothesis expressed by Eqs. 5, 6,  $h$  is a function on time and space (Eq.7):

$$h = H(x, t) \quad (7)$$

In this case, eq.6 become (Eq.8)

$$m \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left( kH \frac{\partial H}{\partial x} \right); \quad \frac{\partial H}{\partial t} = \frac{1}{m} k \overline{H} \frac{\partial H}{\partial x} \quad (8)$$

where:

$t$  - is time.

$\overline{H}$  - is an average value of hydraulic load  $H$

By linearization and using the notation (Eq.9)

$$A^2 = \frac{k \overline{H}}{m} \quad (9)$$

(Eq.8) becomes as follows:

$$\frac{\partial H}{\partial t} = A^2 \frac{\partial H}{\partial x} \quad (10)$$

4. Results and discussions

The study of the wind waves regime has been done using a model for uniform waves that takes into consideration a system of consecutive identical waves, having constant values for height  $h_0$ , period  $T$  and wavelength  $\lambda$  (Eq. 11).

It is considered the wave spreading in the dam-shore area, taking into account the presence of reed vegetation, as it is illustrated in Fig.1.

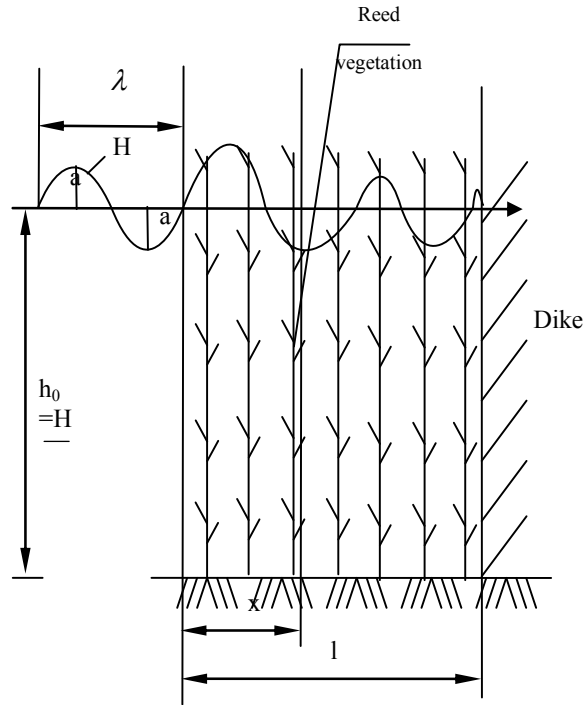


Fig. 1. Waves spreading in dike – shore area.

The calculus model is a statistical one in which the waves field is considered to be a sequence of independent waves, having the features  $h_i$ ,  $T_i$  and  $\lambda_i$ , but with certain laws of distribution around the average values of  $H$  (Eq. 11).

The modeling of wave propagation in the dike-shore area is done "without the initial conditions" (assuming regular waves).

Given  $\omega = \frac{2\pi}{T}$  frequency,  $T$ -wave period and  $r = \frac{2\pi}{\lambda}$  the wave number, it can be written

$$T = \frac{2\pi}{\sqrt{2grthrl}} = \sqrt{\frac{2\pi^2}{gth \frac{2\pi h_0}{\lambda}}}; \bar{H} = h_0 \tag{11}$$

It can be considered the boundary condition (Eqs.12, 13) [8]:

$$x = 0; \quad H = a \cdot e^{i\omega t} \tag{12}$$

$$x = l; \quad u = -k \frac{\partial H}{\partial x} = 0 \quad (13)$$

where:  $H = a \cos \omega t$  or  $H = a \sin \omega t$  and  $a$  represents wave amplitude at shore.

It will be obtained a complex solution as follows (Eq.14):

$$H = X e^{i\omega t} \quad (14)$$

where  $X$  is a unknown function of variable  $x$ .

Setting (eq.14) into (eq.10), results as:

$$X'' - b^2 X = 0 \quad (15)$$

$$b^2 = \frac{i\omega}{A^2}; \quad b = \frac{\sqrt{2\omega}}{2A} (1+i) \quad (16)$$

Solution of equation (eq.15) can be obtained immediately:

$$X = c_1 e^{bx} + c_2 e^{-bx} \quad (17)$$

where:

$c_1$  and  $c_2$  are constants that are obtained from conditions at limit (eq.12) and (eq.13):

$$c_1 + c_2 = a \quad (18)$$

$$c_1 e^{bl} - c_2 e^{-bl} = 0 \quad (19)$$

where:

$$c_1 = \frac{a}{1 + e^{2bl}}; \quad c_2 = \frac{ae^{2bl}}{1 + e^{2bl}} \quad (20)$$

After some transformations, it results the in the complex solution:

$$H = a \frac{chb(1-x)}{chbl} e^{i\omega t} \quad (21)$$

In practice, condition at limit  $H = a \sin \omega t$

$$H = alm \frac{chb(1-x)}{chbl} e^{i\omega t} \quad (22)$$

For the condition at limit  $H = a \cos \omega t$

$$H = a \operatorname{Re} \frac{chb(1-x)}{chbl} e^{i\omega t} \quad (23)$$

*Example calculation*

The example of calculation is performed using a numerical computation assisted by a computer, using hypothetical entry data regarding the wavelength  $\lambda$ ,  $h_0$ ,  $l$  and  $H$ . The presented example of calculation, carried out for  $\lambda = 45$  m, highlights the fact that for such values found by using the mathematical model for calculation, it occurs the attenuation of the energy propagation (Table 1).

Entry data:

$$\lambda = 45 \text{ m}; \quad h_0 = \overline{H_0} = 5 \text{ m}; \quad A = 2H_0$$

$$l = 100 \text{ m};$$

$$T = \sqrt{\frac{2\pi\lambda}{gth \frac{2\pi H_0}{\lambda}}}; \quad T = 6.9125$$

$$\omega = \frac{2\pi}{T}; \quad \omega = 0.909$$

$$b = \frac{\sqrt{2\omega}}{2A}(1+i); \quad b = 0.0674 + 0.0674i$$

$$\frac{K}{m} = 20$$

$$A = \frac{kH_0}{m} = 10$$

$$\left(\frac{H}{a}\right)_{(s,j)} = lm \frac{\cosh[b(1-x_s)]}{\cosh(b-1)} e^{i\omega t_j}$$

$$\left|\frac{H}{a}\right| = \left| \frac{\cosh bl \left(1 - \frac{x_s}{b}\right)}{\cosh bl} e^{i\omega t_j} \right|$$

$$\left|\frac{H}{a}\right| = \left| \frac{\cosh \left[ bl \left(1 - \frac{x_s}{b}\right) \right]}{\cosh bl} e^{i\omega t_j} \right|$$

$$i := \sqrt{-1}$$

$$b := 0.0674 + 0.0674 \times i$$

$$l := 100$$

$$j := 0 \dots 50 \quad s := 0 \dots 10 \quad \omega := 0.909$$

$$x_0 := 0 \quad x_{s+1} := x_s + 0.1 \quad t_0 := 0 \quad t_{j+1} := t_j + 1.9 \frac{\pi}{10}$$

$$z_{j,s} := \left[ \frac{\cosh[b \cdot l \cdot [1 - x_s]]}{\cosh(b \cdot l)} \right] \cdot \exp[i \cdot \Omega \cdot t_j]$$

$$\|Z_{(5,s)}\|, \quad \|Z_{(2,s)}\|$$

Table 1. The variation of the amplitude's values at different distances from the coast level.

S	$ z_{2,s} $
0	1
2	0.260
3	0.132
4	0.067
5	0.034
6	0.018
7	0.009
8	0.004
9	0.003
10	0.002

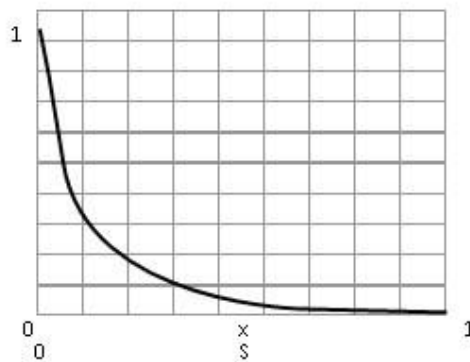


Fig. 2. The variation of the amplitude's values at different distances from the coast level  $lm[z(j.0)]. lm[z(j.1)]. lm[z(j.2)]. lm[z(j.3)]. lm[z(j.6)].$



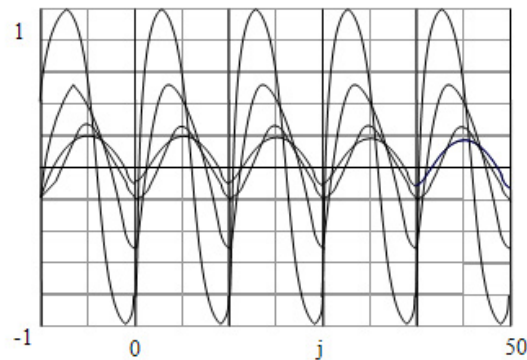


Fig. 3. The variation of the wave energy's values at different distances from the coast level.

The example calculation shows the application of the model for real situations, and consequently might be used as a pre-assessment tool for other coastal zones with similar natural and environmental conditions.

## 5. Conclusion

The digital processing made by a computer program that use Simion Hâncu method and graphic interpretation of results, lead to the following conclusions:

- at distances between 50-100 m from the shoreline, because of intense reed vegetation, it occurs an attenuation of wave propagation energy until at its cancellation and also a values amplitude cancellation;
- in addition, the intense reed vegetation is a natural element of protection of the already built dikes or the ones which are in construction, also represents, because of its roughness, a retention element of fine particles brought by waves at times when the sea level has been raised.
- because of the fact that the hydraulic gradient must always be higher from freshwater to brackish water, after the withdrawal of the sea, the aquifer supplies fresh water to the intense reed vegetation, leading to thickening it and, in time, due to accumulation of fine particles, to raising the share of natural terrain above sea level;
- the presented protection solution is a natural one, is not expensive and is a part of the natural-ecologic aspect of the seaside.

These results are very important because they show the role that the reed vegetation has as a means of protection against coastal erosion.

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