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A Data Driven Approach to Performance Assessment of PID Controllers for Setpoint Tracking

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Abstract

This paper describes an extension to the detrended fluctuation analysis method for control loop performance where the primary focus is on controller setpoint tracking capabilities. The technique utilizes closed loop control error data and does not rely on prior knowledge of the process dead time or the process transfer function (model independent). The performance benchmark is a specific scaling of the Hurst exponent which is computed via the detrended fluctuation analysis method. It will allow control practitioners to efficiently detect problematic control loops since it is a data driven method which requires no prior information. Experimental simulation case studies are provided to validate the efficacy of the performance metric and show that it may be an industrially relevant benchmark for the detection of sluggish and oscillatory servo control loops.

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Keywords: PID control; performance assessment; minimum variance; Hurst exponent

1. Introduction

Controller performance assessment (CPA) is used to verify the current health of a control system. This is achieved by clarifying whether it is operating optimally within certain constraints such as inherent process characteristics, dead time and disturbances. In today's competitive economic climate it has become crucial for controllers to operate optimally in order to reduce product wastage and provide minimal output variance. A modern manufacturing plant possibly contains one hundred to a thousand process control loops and there is significant incentive for maintaining these processes within desired closed loop specifications.

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CPA has been an active area of research following the seminal work of Harris [1]. The methodology uses known process dead time and routine process output operating records to calculate a performance benchmark based on minimum variance control (MVC) framework. It is worth noting that a practical implementation of the MVC algorithm for process control would lead to excessive wear on the final control element. This is due to its wide bandwidth and noise amplification which leads to the aggressive control action. However these problems are not a deterrent when using the algorithm for CPA [2]. Excellent reviews on the subject can be found in [2, 3]. In these reviews, CPA has mainly been applied to industrial process control loops where the focus has been on the regulatory performance of the controller. In this paper we concentrate primarily on the setpoint tracking abilities of proportional-integral-derivative (PID) controllers. The transparency of the standard PID algorithm and its variations, plus the cost versus benefit ratio that they provide has made PID control a popular choice for most process control loops [3].

Poorly tuned PID controllers may result in aggressive, sluggish or even oscillatory and unstable closed loop behavior. Hence a suitable benchmark to automatically track control system performance is important for industrial practice. The work of [4] proposes an iterative approach for servo based systems to achieve optimal PID control performance, based on the process output data and the open loop process model. Further in [5, 6] the proposed methodologies describe theoretical lower bounds for integral of the absolute error (IAE) and dimensionless settling time based on internal model control principles. The main drawback of these techniques is that they are model dependent.

An approach based on detrended fluctuation analysis (DFA) has been proposed by Srinivasan *et al.* [7] which utilizes routine process output data to calculate a controller performance measurement (CPM). No knowledge of process dead time is required to calculate the CPM [7]. Simulation studies conducted by [7] indicate excellent correlation to the minimum variance based Harris index [1] for regulatory process control loops. In this paper, an extension to their method is provided to show the application of DFA to closed loop PID performance assessment where setpoint tracking performance is of general interest. The paper is organized as follows. Section 2 provides a description of the system used in the study; Section 3 gives the steps used to compute the controller performance index via the method of DFA; Section 4 shows the simulation study used to evaluate the effectiveness of the performance benchmark and also provides a discussion on the results; Section 5 concludes the study.

2. System description

We consider the typical closed loop negative feedback control system shown in Fig 1. The process output variable is given as $y(k)$, with the controller signal denoted by $u(k)$ and the disturbance driving white noise being represented by $a(k)$. k is the sample interval. The control error is given as:

$$e(k) = r(k)w(q^{-1}) - y(k) \tag{1}$$

where $r(k)w(q^{-1})$ represents a unit step change and $a(k)$ denotes the white noise driving signal. $g(q^{-1})$ and $h(q^{-1})$ denote the transfer functions of the process and the white noise disturbance sensor, respectively. q^{-1} indicates the backshift operator. The structure of the discrete model PID controller applied in this study is given in (2).

$$c(q^{-1})_{PID} = \frac{c_1 + c_2 q^{-1} + c_3 q^{-2}}{1 - q^{-1}} \tag{2}$$

With regards to (2), $c_1 = K_P + K_I + K_D$, $c_2 = -(K_P + 2K_D)$ and $c_3 = K_D$. K_P, K_I and K_D are the proportional, integral and derivative gains of the PID controller, respectively. When the disturbance signal is ignored the closed loop transfer function can be obtained as:

$$y(k) = \frac{c(q^{-1})g(q^{-1})}{1 + c(q^{-1})g(q^{-1})} r(k) \tag{3}$$

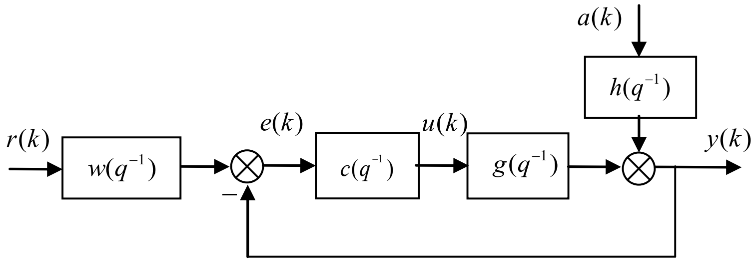

Fig. 1. Typical closed loop system model.

When disturbance has an effect on the system the closed loop can be described as:

$$y(k)=\frac{r(k)w(q^{-1})c(q^{-1})g(q^{-1})+a(k)h(q^{-1})}{1+c(q^{-1})g(q^{-1})} \tag{4}$$

Traditional time domain characteristics used to assess the transient response characteristics of a closed loop system following a setpoint change are the settling time t_s, rise time t_r and percentage maximum peak overshoot Mp, plus a performance index such as the absolute value of the error (IAE) [5] which is given in the discrete form as:

$$IAE=\sum\left|r(k)w(q^{-1})-y(k)\right| \tag{5}$$

The shortcoming of time domain specifications is that no benchmark exists against which these characteristics can be compared. The technique that we propose in this work is an extension to the work of Srinivasan *et al.* [7] and will utilize a benchmark index to quantify the servo tracking capability of the system being considered in the study. This will be shown later in the simulation study where we explore the use of DFA to assess the performance of the controller tuned for setpoint tracking. A brief explanation of the DFA based performance index follows in the next section.

3. Detrended fluctuation analysis based performance benchmark

The DFA technique was first proposed by Peng *et al.* [8] to quantify the correlation property of a non-stationary time series based on human heartbeat signals. It is a scaling analysis method providing a simple quantitative parameter (the Hurst exponent α) to represent the correlation properties of a signal. The advantage of DFA is that it permits the detection of long-range correlation embedded in seemingly non-stationary time series [9]. A time series is considered self-similar if it contains sub-units which resemble the whole structure. The Hurst exponent α is the self-similarity parameter and has the following properties for stationary time series ranging between 0 and 1 [7]:

- $\alpha = 0.5$ represents a white noise sequence.
- $\alpha > 0.5$ indicates the presence of long-term correlation in the time series data.
- $\alpha < 0.5$ shows that the dataset is anticorrelated.

Based on this information the Hurst exponent provides a means of quantifying the predictable component present in the time series data. Srinivasan *et al.* [7] showed that the process output from a minimum variance control loop has a Hurst exponent ≈ 0.5 and therefore can be utilized as a controller performance measure for stochastic systems where the focus is disturbance rejection. Following these developments, we propose that the time series data of the control error $e(k)$ be used in the computation of the Hurst exponent for setpoint tracking systems. The following steps illustrate the DFA algorithm used to compute the exponent followed by the controller performance index.

1) The closed loop control error $e(k)$ time series data of total length N is mapped to a self similar process; $E(k)=\sum_{i=1}^{k}[e(i)-e_{ave}]$ where $e(i)$ is the ith error interval and e_{ave} is the average error signal.

2) Next the integrated time series $E(k)$ is divided into boxes of equal length n for ease of analysis.
3) For each box a linear least squares fit is performed where the y coordinate of the straight line segments is denoted by $E_n(k)$
4) The integrated time series $E(k)$ is now detrended by subtracting the local trend $E_n(k)$ in each box.
5) The root-mean-square fluctuation of this integrate and detrended time series is computed by:

$$F(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}[E(k) - E_n(k)]^2} \tag{6}$$

6) Steps 2-5 are repeated over different box sizes between a minimum length of 10 samples and a maximum length of *N/4* samples.
7) The log-log plot of the RMS function $F(n)$ versus the box sizes n should yield a straight line. The Hurst exponent α is the gradient of this line.
8) Using the Hurst exponent the controller performance index (CPI) is defined as:

$$CPI = \begin{cases} 2\alpha & if\, \alpha \le 0.5 \\ 1.5 - \alpha & if\, \alpha > 0.5 \end{cases} \tag{7}$$

where the process closed loop control error under MVC should have a CPI → 1 while significant deviations from 1 indicate unsatisfactory closed loop behavior when setpoint tracking is of concern.

4. Simulation studies

4.1. Preliminaries for the case studies

This section provides experimental examples to illustrate the validity of assessing the setpoint tracking performance of PID control loops using the DFA based index. All experiments were conducted in MATLAB Simulink® software package. Process $g(q^{-1})$ and disturbance $h(q^{-1})$ transfer models were taken from literature [10-12] to test the methodology and is given in table 1. Corresponding PID controllers are shown in table 2 with the PID tuned for minimum variance, sluggish and oscillatory responses. The reference input is a unit step signal and the driving white noise has a variance of 0.01. For each case study, the simulation is executed with the disturbance model and then without. This is done to illustrate the effects of disturbances on the performance index.

Table 1. Simulation models used in the experiments.

Case	Process $g(q^{-1})$	Disturbance $h(q^{-1})$
1	$\frac{0.2q^{-5}}{1-0.8q^{-1}}$	$\frac{1}{(1-q^{-1})(1+0.4q^{-1})}$
2	$\frac{q^{-6}}{1-0.8q^{-1}}$	$\frac{1-0.2q^{-1}}{(1-q^{-1})(1-0.3q^{-1})(1+0.4q^{-1})(1-0.5q^{-1})}$
3	$\frac{0.1q^{-3}}{1-0.8q^{-1}}$	$\frac{\sqrt{0.001}}{(1-q^{-1})(1+0.2q^{-2})}$

Table 2. Discrete PID controllers used in the case studies.

Case	Minimum variance	Sluggish	Oscillatory
1	$\frac{2.8327-4.395q^{-1}+1.748q^{-2}}{1-q^{-1}}$	$\frac{1.72-3.3q^{-1}+1.6q^{-2}}{1-q^{-1}}$	$\frac{1.9-2.2q^{-1}+0.75q^{-2}}{1-q^{-1}}$
2	$\frac{0.7251-1.2082q^{-1}+0.52q^{-2}}{1-q^{-1}}$	$\frac{0.01672-0.00403q^{-1}+0.1q^{-2}}{1-q^{-1}}$	$\frac{0.3594-0.409q^{-1}+0.1q^{-2}}{1-q^{-1}}$
3	$\frac{5.1681-6.6294q^{-1}+2.013q^{-2}}{1-q^{-1}}$	$\frac{2.31551-4.26034q^{-1}+2q^{-2}}{1-q^{-1}}$	$\frac{8.3034-9.2q^{-1}+2q^{-2}}{1-q^{-1}}$

4.2. Results of the simulation study

Table 3 and Table 4 provide performance assessment results of the different controllers used in each case study. Time domain characteristics (IAE- integral of the absolute error, POS - percentage overshoot, ts - settling time, tp - time to peak) of the closed loop responses are used to compare against the DFA based performance measure. Settling time occurs when the closed loop process response reaches 10% of the final value and the symbol "-" indicates that the time domain characteristic was not reached within the specified simulation time. For case 1 with disturbance transfer function affecting the system, Fig. 2 shows the log-log plot of the RMS function $F(n)$ versus the box sizes n and Fig. 3 shows the closed loop control error and its corresponding autocorrelation function. Illustrations for the other case studies were similar and therefore not shown.

Table 3. Performance assessment results of the case studies with disturbance in effect.

Case	Closed loop response characteristics	α	*CPI*	IAE	POS	ts(s)	tp(s)
1	minimum variance	0.5045	0.9955	132.6	66.9	-	280
	sluggish	1.21	0.2894	257.29	191.1	-	335
	oscillatory	0.1569	0.3138	310.95	509.3	-	280
2	minimum variance	0.488	0.976	265.63	71.2	-	281
	sluggish	1.1568	0.3432	539.9	948.8	-	336
	oscillatory	0.2549	0.5098	426.4	1469.6	-	282
3	minimum variance	0.6171	0.8829	7.83	6.1	104	106
	sluggish	1.3823	0.1177	42.1	3.2	176	335
	oscillatory	0.2287	0.4574	33	96.4	188	105

Table 4. Performance assessment results of the case studies without disturbance

Case	Closed loop response characteristics	α	*CPI*	IAE	POS	ts(s)	tp(s)
1	minimum variance	0.6605	0.8395	6.7	17.7	109	109
	sluggish	1.47	0.03	50	0	209	-
	oscillatory	0.2181	0.4362	54.39	97	275	113
2	minimum variance	0.6291	0.8709	9	36.5	123	111
	sluggish	1.4525	0.0475	49.6	0	206	-
	oscillatory	0.2782	0.5564	26.6	78.7	180	114
3	minimum variance	0.6198	0.8802	4.04	5.6	104	106
	sluggish	1.386	0.2	36.3	0	180	-
	oscillatory	0.2289	0.4578	28.32	105	188	105

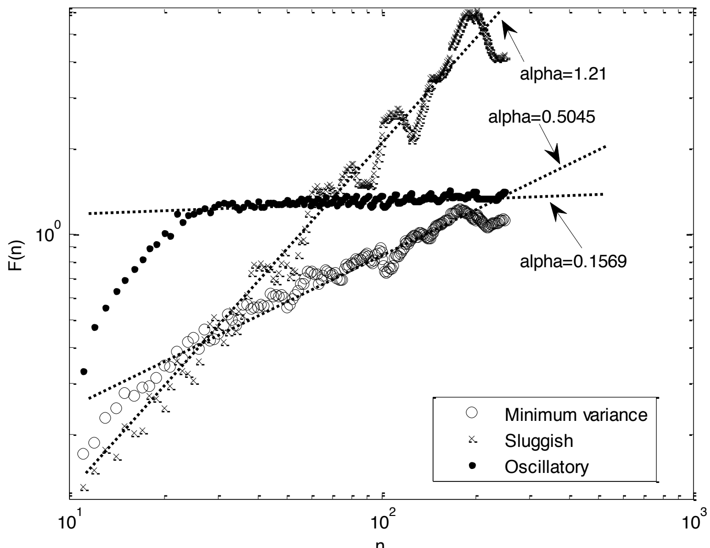

Fig. 2. Log-log plot of the RMS function $F(n)$ versus the box sizes *n* for case 1 with disturbance in effect.

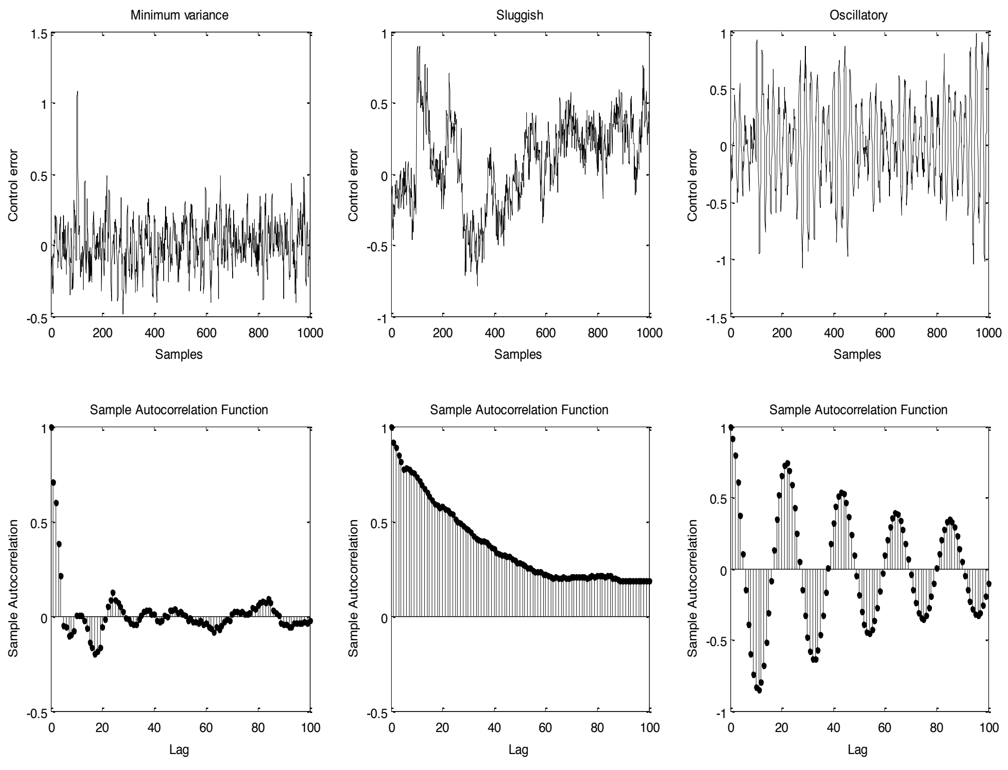

Fig. 3. Control error and corresponding autocorrelation function for case 1 with disturbance in effect.

4.3. Discussion of results

From the results presented in Table 3 and Table 4 we can make the following salient observations:

(1) In all cases the presented CPI gives a value close to 1 for PID tuned for minimum variance thus indicating excellent control performance from a servo tracking point of view. This is verified by the corresponding time domain characteristics that indicate good closed loop performance as opposed to the sluggish and oscillatory systems.

(2) For cases were sluggish closed loop behaviour dominates, a high Hurst exponent indicates the presence of long term correlation in the control error time series. Conversely, lower Hurst exponent values demonstrate closed loop oscillatory behaviour and anti-correlation in the data set. This is also evident from Fig. 3, which clearly shows the autocorrelation of the closed loop control error for each response.

(3) For CPI values approaching the ideal limit, the corresponding IAE is low. Conversely, as CPI moves away from 1 the IAE increases.

(4) From the results presented in the preceding section a simple performance measure to quantify setpoint tracking capabilities of the controller is presented in Table 5.

In Fig. 2 it is observed that a breakaway point occurs at the point of inflection at a window size of $n \approx 30$.This is especially evident from the oscillatory graph. Therefore a window size of $n < 30$ for the given data does not produce reliable estimates of the Hurst exponent (α). MATLAB *polyfit* function was used to fit a straight line graph to the data with an order of 1. The methodology compares favourably to Hägglund's *Idle Index* [13] which describes the relation between times of positive and negative correlation between the control signal and the process output increments, Δu and Δy respectively. Characteristic for the sluggish response is that after a process change has taken place, a very long time period occurs where the correlation between the two signals increments is positive. The *idle index* however is susceptible to noise and requires pre-filtering of raw data before analysis. For oscillatory loops the results are comparable with the oscillation detection procedure of Miao and Seborg [14]. Their method is based on the analysis of the auto-covariance function of the normal operating data of the control error. Using the decay ratio of the of the auto-covariance function, a measure of the oscillation in the trend is provided. A threshold value is required by the methodology to conclude that the loop is oscillatory, which is subjective and application dependant.

Table 5. Proposed performance measure for setpoint tracking .

CPI	α	Setpoint tracking performance assessment
≈ 1	≈ 0.5	Minimum variance : good tuning
≤ 0.3	≥ 1	Sluggish response: poor tuning
≤ 0.5	≤ 0.3	Oscillatory response: poor tuning

5. Conclusion and future work

The simulation study presented in the paper utilizes the DFA based CPI to determine the servo tracking capabilities of a PID controller. CPI is computed directly from closed loop control error signals using only recorded data and is model independent. Case studies reveal that the CPI index successfully identifies poor performing loops when setpoint tracking is of primary concern. Elevated Hurst exponent values indicate sluggish closed loop behavior whilst smaller values are a sign of oscillatory behavior. Future research will focus on analyzing the CPI against ramp and variable setpoint signals. In addition, the methodology will be applied to real world process data.

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