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## Investigation of Elastic Machine Element Measurement Error

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### Abstract

Today, mechanical engineering is connected with more precise measurements. Especially it is important for elastic machine elements and nanocoatings, which are used for friction surfaces, contacting parts, corrosion surfaces, conducting films etc. In this paper measurement error is described taking into consideration parameters of new ISO standard (EN ISO 25178) for 3D surface texture.

The given paper studies contact of rough surfaces, probability distribution of peak height and influence of material characteristics on measurement error.

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### 1. Introduction

Measurement of machine elements is connected with contact between ferrule of measurement device and machine element (Fig. 1).

The roughness deformation is shown on Fig. 2., where  $P$  is load,  $a_i$  is deformation of free-choice surface peak measured from the top of irregularity.

Surface roughness is described by the normal random field the correlation function of which is monotonically decreasing. To determine the deformation of roughness peaks in this case it is possible to take relatively simple output data  $Sa$  and spacing in two perpendicular directions [1]. To determine the elastic deformation of the details' surface roughness, we admit that contact occurs between the rough surface (measured detail) and the perfectly smooth surface (measuring equipment) (see Fig. 1).

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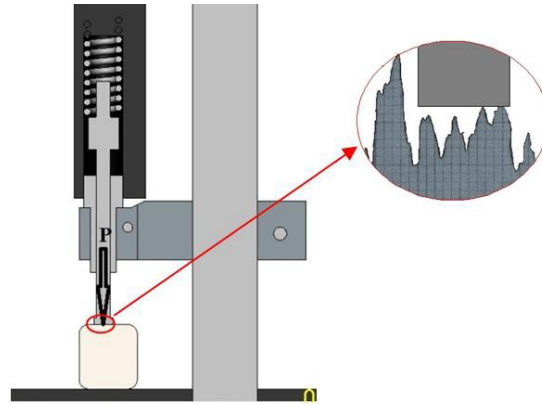


Fig. 1. Scheme of measuring.

Movement of ideal plane is not perfectly parallel to rough surface, but these deviations are insignificant, so in the theoretical calculations they are not taken into account. Let's consider the following ideal plane and rough surface contact diagram: under the influence of the applied force  $P$  the ideal plane moves from the position  $I-I$  to position  $II-II$  where balance between external forces and micro-roughness deformation resistance sets in. In this position, the distance between the ideal plane and the average plane of roughness is equal to  $u$  (Fig. 2).

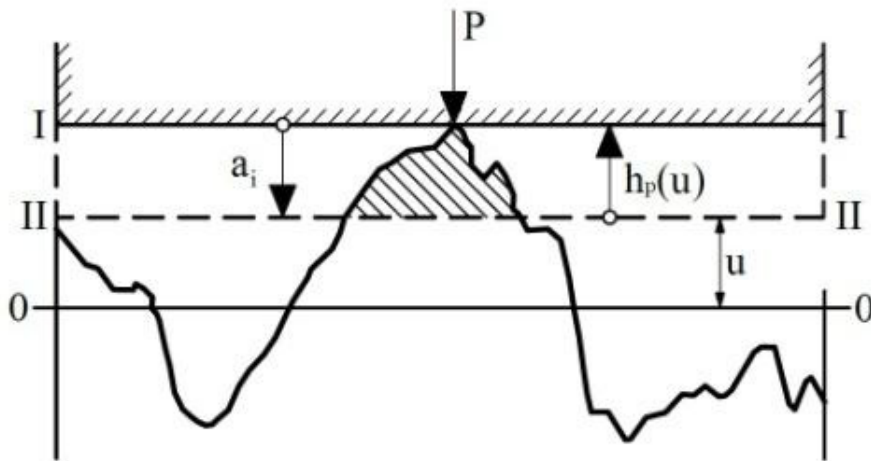


Fig. 2. Diagram of contact of perfect plane and rough surface.

## 2. Deformation of Surface Roughness

Analytical researches show that the shape of peak is close to an elliptic parabola. After elastic deformation of such peaks, deformation of single irregularity peak  $a_i$  is connected with applied load  $P_i$  and is determined by the following equation [1]:

$$P_i = \frac{2}{3 \cdot \theta} \cdot \frac{E(e)^{1/2}}{K(e)^{3/2} \cdot (1-e^2)^{1/2}} \cdot \frac{a_i^{3/2}}{H_i^{1/2}} \quad (1)$$

where

$K(e)$ ,  $E(e)$ - first and second order elliptic integrals;

$P_i$  - load, applied to free – choice surface peak;

$H_i = (k_1 + k_2)/2$  – average roughness curve;  $k_1, k_2$  - roughness peak curves;

$e$  - eccentricity of contact area:  $e = \sqrt{1 - \frac{a_i^2}{b_i^2}}$ ; where  $a_i, b_i$  – small and large semi-axis of the contact area;

$\theta$  - determined by equation:  $\theta = \frac{1 - \mu^2}{\pi E}$ , where  $\mu$  - Poisson’s ratio, bet  $E$  – modulus of elasticity.

Regarding the applied force on one irregularity we can determine the total applied force  $P$  on the surface as follows:

$$P = P_i \cdot N_\gamma \cdot A_a \tag{2}$$

where

$A_a$  – nominal area of contact;

$$N_\gamma \text{ – mean number of deformed surface irregularities per unit of area: } N_\gamma = \frac{\pi \cdot c \cdot n_1^2(0)}{2\sqrt{2\pi}} \cdot \gamma \cdot e^{-\frac{\gamma^2}{2}} \tag{2};$$

$n_1(0)$  – the number of zeroes per unit of length in parallel lay;

$\gamma = u/\sigma$  – relative level of deformation, where  $\sigma$  is the random field standard deviation.

We can find the mean pressure on nominal area by simple equation:

$$q = \frac{P}{A} = P_i \cdot N_\gamma \tag{3}$$

The equation (1) includes deformation  $a_i$  which is equal to deformed peak height  $h_p(u)$ (Fig.2). The peak height is one of the statistical parameters of surface roughness, the value of which must be found.

### 3. Statistical characteristics of deformed peak height

The surface roughness height (SRH) distribution law for an irregular 3D surface (mathematically – for a normal random field) was studied by P.R. Nayak [3], where according to the probability distribution density function  $f_1(\xi_p)$  is:

$$f_{1\gamma}(\xi_p) = \kappa_1 \cdot \frac{\sqrt{3}}{2\pi} \left\{ \frac{\lambda}{4} \sqrt{3(8 - 3\lambda^2)} \cdot \gamma \cdot e^{-\frac{4}{8-3\lambda^2}\gamma^2} + \frac{3\sqrt{2\pi}}{4} \lambda^2 \cdot (\gamma^2 - 1) e^{-\frac{\gamma^2}{2}} \cdot \phi \left( \sqrt{\frac{3\lambda^2}{8 - 3\lambda^2}} \gamma \right) + 4 \sqrt{\frac{2\pi}{3(4 - \lambda^2)}} \cdot e^{-\frac{2}{4-\lambda^2}\gamma^2} \phi \left( \sqrt{\frac{4\lambda^2}{(4 - \lambda^2)(8 - 3\lambda^2)}} \gamma \right) \right\}, \tag{4}$$

where

- $\xi_p = \frac{h_p}{\sigma}$  – standardized value of peak height;
- $\lambda$  – non-dimensional parameter:  $\lambda = \frac{n_1(0)}{m_1}$ ;  $m_1$  – the number of summits per unit of length in parallel lay;
- $c$  – coefficient of anisotropy:  $c = \frac{n_1(0)}{n_2(0)}$ ;
- $\phi(\dots)$  – Laplacian function:  $\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$ , (the numerical values of which can be found in [4]).

Since the formula proposed by P.R. Nayak is complicated for engineering calculations it is essential to find a simpler distribution law, which could substitute the precise formula. Let’s consider two best-known probability distribution laws: normal (Gaussian) distribution law and Rayleigh law. According to these laws the density of probability distribution of surface peak height  $f(\xi_p)$  can be determined as follows:

Truncated normal distribution law: 
$$f_{2\gamma} = \frac{1}{\operatorname{erfc}\left(\frac{\gamma}{\sqrt{2}}\right)} \cdot \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\xi_p^2}, \quad \xi_p \geq 0 \tag{5}$$

Rayleigh law: 
$$f_{3\gamma}(\xi_p) = e^{\frac{1}{2}\gamma^2} \cdot \xi_p \cdot e^{-\frac{1}{2}\xi_p^2} = \xi_p \cdot e^{\frac{1}{2}(\gamma^2 - \xi_p^2)}, \quad \xi_p \geq 0 \tag{6}$$

Diagrams of the distribution density are given on Fig. 3. It shows that, starting from value  $\xi > 1$ , the relevant expressions are approaching and become closer to the precise expression in Rayleigh’s distribution, therefore, the corresponding distribution density for the range  $\xi > 1$  can be used for solving engineering tasks.

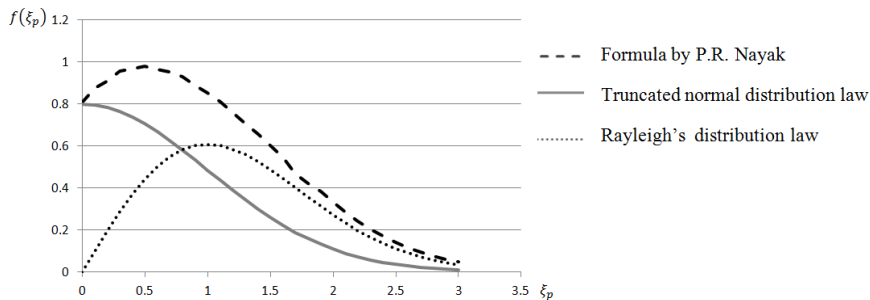


Fig. 3. Density of probability distribution of surface peak heights.

Mathematical expectation of the surface peak height for Rayleigh’s distribution law is [5]:

$$E\{\xi_{p\gamma}\} = e^{\frac{1}{2}\gamma^2} \cdot \sqrt{\frac{\pi}{2}} \cdot \operatorname{erfc}\left(\frac{\gamma}{\sqrt{2}}\right) + \gamma. \tag{7}$$

where:

- Error integral [6].

$\operatorname{erfc}(x)$

Using equations (1), (3) and (7) we can determine the deformation level of irregularities, but because of the limited amount of paper the measurement error is not shown here.

#### 4. Summary

Influence of surface roughness on the measurement error should also be taken into consideration in accurate measurements of elastic elements.

To determine this error the present paper studies the rough surface contact theory, assuming that surface roughness is of irregular character and can be modelled by a normal random field.

Statistical characteristic values of the height of deformed roughnesses are stated for the given roughness model and basic coherences are given for the calculation of mean stress per a unit of contact surfaces' area.

The obtained formulas show that measurement error under the influence of roughness is associated with the deformed surface material characteristics  $E$ ,  $\mu$  and 3D surface texture parameters  $Sa$ ,  $RSm$ .

Experiments will be carried out in the following work to specify theoretical coherences.

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