Available online at www.sciencedirect.com
ScienceDirect
Procedia Engineering 69 (2014) 952 – 961
Procedia Engineering
www.elsevier.com/locate/procedia

24th DAAAM International Symposium on Intelligent Manufacturing and Automation, 2013

Tolerance Stack Up Analysis for Angularity of Components and their Assembly

A. K. Sahani[a,b,*], Pramod Kumar Jain[b], Satish C. Sharma[b]

[a]*Instruments Research & Development Establishment, DRDO, Ministry of Defence, Dehradun-248008, India*
[b]*Indian Institute of Technology, Roorkee-247667, India*

Abstract

Geometric dimensioning and Tolerancing (GDT) constitutes the dominant approach for design and manufacture of mechanical parts that control inevitable dimensional and geometrical deviations within appropriate limits. The stack up of tolerances and their redistribution without hampering the functionality is very important for cost optimization. This paper presents a methodology that aims towards the systematic solution of tolerance stack up problem involving geometric characteristics.Conventional tolerance stack up analysis is usually difficult as it involves numerous rule and conditions. The methodology presented i.e. generic capsule method is straightforward and easy to use for stack up of geometrical tolerances of components and their assembly using graphical approach. In the work presented in this paper, angularity tolerance has been considered for illustration of the methodology. Two approaches viz. Worst Case (WC) and Root Sum Square (RSS) have been used. An example of dovetail mounting mechanism has been taken for purpose of stack up of angularity. This assembly consists of two parts i.e. dovetail male and dovetail female. Tolerance stack up has been done both for the components and their assembly. Need for computerisation of methodology for geometrical tolerance stack up of large assemblies has emerged out as the limitation of the proposed method.

Selection and peer-review under responsibility of DAAAM International Vienna

Keywords: GDT; Angularity; Stack up Analysis; Graphical Method; WC Approach; RSS Approach

* Corresponding author. Tel.: +91-135-2782503; fax: +91-135-2787161.
E-mail address: sahaniak@yahoo.com

1877-7058
Selection and peer-review under responsibility of DAAAM International Vienna
doi:10.1016/j.proeng.2014.03.075

1. Introduction

Tolerance is an essential part of design and manufacturing. There are two types of tolerancing i.e. dimensional and geometrical tolerancing. The application of GDT for mechanical design has gained widespread acceptance in industry. From a component design perspective, it provides the engineer a communication tool to fully describe the functionality of an item dimensionally. Design and Tolerancing (DT) is used to specify the size, shape, form, orientation, and location of features on a part. Features toleranced with GDT reflect the actual relationship between mating parts. Drawings with properly applied geometrical tolerancing provide the best opportunity for uniform interpretation and cost effective assembly. GDT is used to ensure the proper assembly of mating parts, to improve quality and to reduce cost by proper selection of manufacturing process. Before designers can properly apply geometric tolerancing, they must carefully consider the fit and function of each feature of every part. Properly applied geometrical tolerancing ensures interchangeablity of the parts. Geometrical tolerancing allows the designers to specify the maximum available tolerance and consequently design the most economical parts. A properly toleranced drawing is not only a picture that communicates the size and shape of the part, but it also explains the tolerance relationships between features.

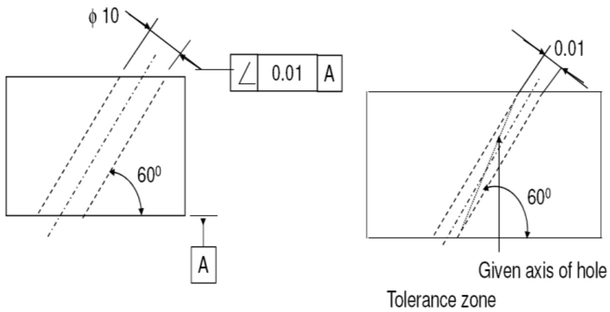

Fig. 1. Angularity.

In this paper, angularity is taken for study. It is defined for a feature (like surface or line) with reference to another feature called reference. It defines the distance between two lines or surfaces which are at an angle to the datum surface and encompass the line or surface as shown in Fig. 1. Tolerance stack ups of individual components and their assembly have been carried out using graphical approach.

2. Literature Review

A lot of work has been done in the field of conventional tolerancing. Conventional tolerancing methods do a good job for dimensioning and tolerancing of size features and are still used in good capacity. But these methods do not cater precisely for form, profile, runout, location and orientation features as discussed by Cogorno [1], Meadows [2], Drake [3] and ADCATS [4]. GDT is used extensively for location, profile, runout, form and orientation features. In more theoretical terms, there are two types of tolerancing schemes i.e. parametric and geometric. Parametric tolerancing consists of identifying a set of parameters and assigning limits to the parameters that define a range of values which has been discussed by Requicha [5]. Singh *et al.* [6] reviewed different methods of tolerance allocation and found mean shift models and the combination of the basic approaches. Singh *et al.* [7] reviewed tolerance synthesis approaches for tolerance stack up i.e. the worst case and the root sum square approach. Swift *et al.*[8] introduced a knowledge based statistical approach to tolerance allocation. In this approach, a systematic analysis for estimating process capability levels at the design stage is used in conjunction with statistical methods for the optimization of tolerances in assembly stack up. Chase *et al.*[9] demonstrated that the methods for tolerance allocation for minimum production cost can be extended to include process selection from a set of alternate processes. Ngoi *et al.* [10] discussed the stack up of geometrical tolerances using generic capsule method. Ngoi *et al.* [11] presented an elegant approach by using the 'Quickie' technique towards tolerance stack up analysis for geometrical tolerances. Ngoi *et al.* [12] also presented a straightforward graphical approach known as the "Catena" method for tolerance stack up, involving geometric characteristics in form control – flatness, straightness, circularity and cylindricity. He and Gibson [13] developed an extension of computerised trace method to determine the relationship between geometrical tolerances and manufacturing dimensions and tolerances. This method minimizes the cost of scrap as the objective function which is a function of manufacturing tolerances. Requirements of design sizes, geometrical tolerances (both form and position) and machining allowances are expressed mathematically as constraints for the optimization. Shivkumar *et al.* [14] presented a general new methodology using intelligent algorithms for simultaneous optimal selection of design and manufacturing tolerances with alternative manufacturing process selection. Mansuy *et al.* [15] presented an original method that enables to solve problems for the case of serial assembly (stacking) without clearances. This method is based on the use of influence coefficients to obtain the relationship between the functional tolerance and the tolerances associated with the geometry of the mechanism's interface surfaces. Sahani *et al.* [16] presented review of different techniques for stack up for flatness geometrical tolerances.

3. Methodology

The generic capsule method has been used to evaluate stack up of tolerances in this paper. In this method, four steps are to be followed. In labelling step, all the surfaces those are related by dimensions in the drawing are labelled. The labelling is done in one direction, say from top to bottom and in two stages viz. identity and hierarchy. In the former stage, the component is labelled in ascending alphabetical order. It helps in identifying the surfaces in the drawing while referring to the graphical model. The component number is indicated by adding a numeral prefix to the alphabetical label. In next stage of labelling i.e. hierarchical labelling, the surfaces are labelled in ascending numerical order in the same direction as in identity labelling stage. In the next step of modelling, the GDT model is constructed for every component. For constructing the GDT model of assembly, contacts of components are represented by double dashed line. After modelling, formulation is carried out to identify the stack path consisting of the unknown distance. An equation is formulated to calculate the unknown parameter from the stack path. This equation is formed on the basis of principle of summation of vectors. The stack up of tolerances is done by taking the directional arrows as vector and dashed line as scalars. In the last step i.e. evaluation, the desired dimension can be calculated by substituting the known values into the stack path. The stack up of tolerances can be done through worst case and root sum square approaches.

Graphical Approach for Stack up Tolerances

A case is taken up for the stack up of angularity for components and their assembly. This assembly consists of two components i.e. 'Dovetail Female' and 'Dovetail Male' as shown in Fig. 2 &3 and their assembly 'Dovetail Assembly' is shown in Fig. 4.

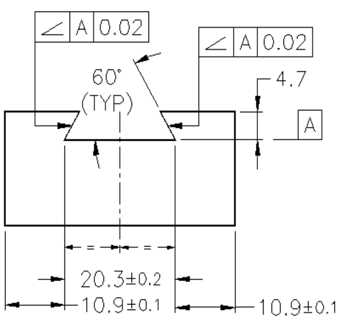

Fig. 2. Dovetail female.

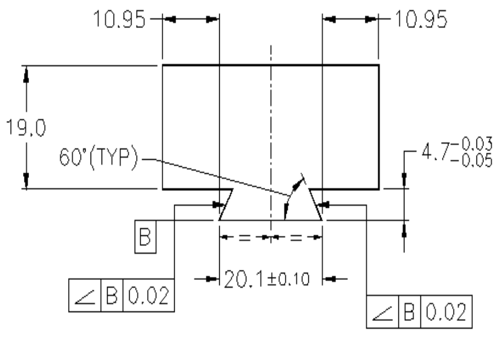

Fig. 3. Dovetail male.

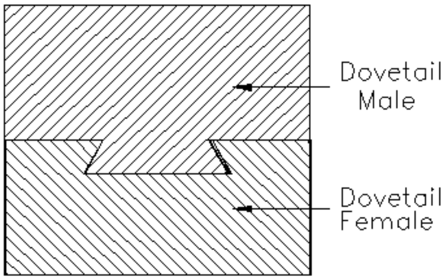

Fig. 4. Dovetail assembly.

The part number assigned for the 'Dovetail Female' component is 1 while the part number for the 'Dovetail Male' component is 2. The labelling of surfaces and vertices of 'Dovetail Female' and 'Dovetail Male' is shown in Fig. 5.

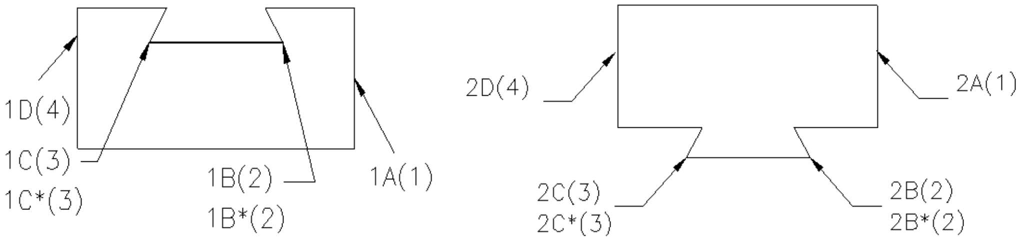

Fig. 5. Labelled dovetail.

The angularity tolerance is the distance between two lines or surfaces that are at an angle to the datum surface (AN) and encompass the line or surface is given at an angle (<ANB), which is transferred to the horizontal surface (AN). The angular tolerance transformation sketch is shown in Fig. 6.

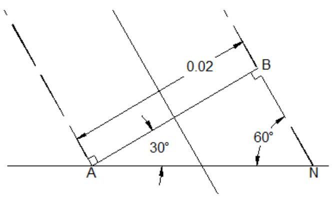

Fig. 6. Angle transformation.

Now from the Fig. 6,

$$\begin{aligned} \text{Sin } 60^{\circ} &= AB / AN \\ \text{So, } AN &= AB / \text{Sin } 60^{\circ} \\ &= 0.02 / \text{Sin } 60^{\circ} \\ &= 0.0231 \end{aligned}$$

Having completed the labelling phase, the graphical model is then constructed for 'Dovetail Female', 'Dovetail Male' and 'Dovetail Assembly' as shown in Fig. 7, 8 &9.

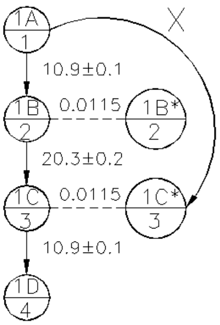

Fig. 7. GDT model for dovetail female.

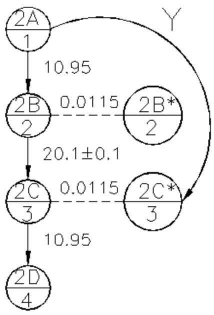

Fig. 8. GDT model for dovetail male.

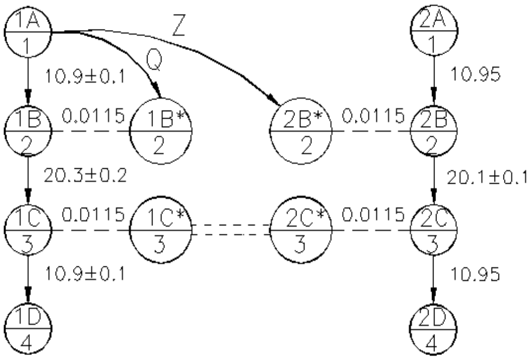

Fig. 9. GDT model for dovetail assembly.

To determine the distance between surface A to edge C for 'Dovetail Female' component, the stack up path is identified for calculation. It will follow the loop 1A - 1C* - 1C - 1B - 1A. The expression derived from the stack path is then

1A1C* - 1C*1C -1C1B-1B1A = 0

Substituting the values,

X - (± 0.0115) - (20.3 ± 0.2) – (10.9 ± 0.1) = 0

Worst Case (WC) Approach:
The total tolerance stack up can be written as

$$\Delta Y = \sum_{i=1}^{n} \delta i$$

Where,

n = Number of constituent dimensions
δi = Tolerance associated with dimension

X - (± 0.0115) - (20.3 ± 0.2) – (10.9 ± 0.1) = 0
X – (31.2 ±0.3115) = 0
X = 31.2±0.3115

Maximum and minimum values of X are

X_{max} = 31.5115
X_{min} = 30.8885

Root Sum Square (RSS) Approach:

Total tolerance of assembly can be written as

$$\Delta Y = \sqrt{\sum_{i=1}^{n} \delta_i^2}$$

Where,

n = Number of constituent dimensions
δi = Tolerance associated with dimension

$X=31.2 \pm \sqrt{(0.0115^2 + 0.2^2 + 0.1^2)}$
X = 31.2 ± 0.2239

Maximum and minimum values of X are

X_{max} = 31.4239
X_{min} = 30.9761

To determine the distance between surface A to edge C for ‘Dovetail Male’ component, the stack up path is identified for calculation. It will follow the loop 2A - 2C* - 2C - 2B - 2A. The expression derived from the stack path is then

2A2C* - 2C*2C -2C2B-2B2A = 0

Putting the values,

Y - (± 0.0115) - (20.1 ± 0.1) – (10.95) = 0

Worst Case (WC) Approach:

Y - (± 0.0115) - (20.1 ± 0.1) – (10.95) = 0
Y = 31.05 ±0.1115

Maximum and minimum values of Y are

Y_{max} = 31.1615
Y_{min} = 30.9385

Root Sum Square (RSS) Approach:

$$Y=31.05 \pm \sqrt{(0.0115^2+0.1^2)}$$
$$Y = 31.05 \pm 0.1007$$

Maximum and minimum values of Y are

Y_{max} = 31.1507
Y_{min} = 30.9493

In the case of an assembly, the graphical model is constructed part by part i.e. one model for the ‘Dovetail Female’ and another model for the ‘Dovetail Male’. The two part models are then linked together by double dashed line that represents contact. The labelling of surfaces and vertices of Dovetail Assembly is shown in Fig.10.

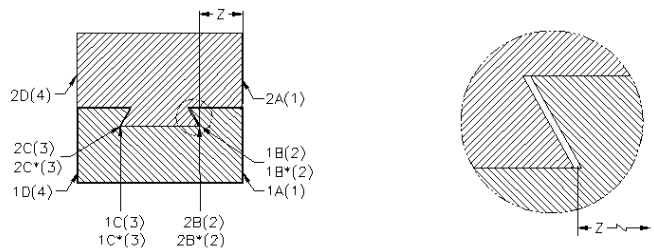

Fig. 10. Labelled dovetail assembly.

Here the mating edges are 1C of first part and 2C of second part. Unknown parameter is the distance between surface A of part 1 to edge B of part 2. Upon completion of the model, the stack path is identified. Since the requirement is to find the minimum value of Z, the correct stack path should pass through the double dashed line that connects between 1C* and 2C*. The expression derived from the stack path is then

$$1A2B^* + 2B^*2B + 2B2C + 2C2C^* + 2C^*1C^* + 1C^*1C - 1C1B - 1B1A = 0$$

Upon substitution,

$$Z \pm 0.0115 + (20.1 \pm 0.1) \pm 0.0115 \pm 0.0 \pm 0.0115 - (20.3 \pm 0.2) - (10.9 \pm 0.1) = 0$$

Worst Case (WC) Approach:

$$Z - (11.1 \pm 0.4) \pm 0.0345 = 0$$
$$Z - (11.1 \pm 0.4345) = 0$$
$$Z = 11.1 \pm 0.4345$$

Maximum and minimum values of Z are

Z_{max} = 11.5345
Z_{min} = 10.6655

Root Sum Square (RSS) Approach:

$$Z=11.1\pm\sqrt{(0.1^2+0.2^2+0.1^2+0.0115^2+0.0115^2+0.0115^2)}$$
$$Z=11.1\pm0.2458$$

Maximum and minimum values of Z are

$$Z_{max} = 11.3458$$
$$Z_{min} = 10.8542$$

Following the same procedure, the expression for the distance 1A1B* is obtained as

$$1A1B^* + 1B^*1B - 1B1A = 0$$

Upon substitution,

$$Q \pm 0.0115 - (10.9 \pm 0.1) = 0$$

Worst Case (WC) Approach:

$$Q \pm 0.0115 - (10.9 \pm 0.1) = 0$$
$$Q - (10.9\pm0.1115) = 0$$
$$Q = 10.9\pm0.1115$$

Maximum and minimum values of Q are

$$Q_{max} = 11.0115$$
$$Q_{min} = 10.7885$$

Root Sum Square (RSS) Approach:

$$Q=10.9\pm\sqrt{(0.0115^2+0.1^2)}$$
$$Q=10.9\pm0.1007$$

Maximum and minimum values of Q are

$$Q_{max} = 11.0007$$
$$Q_{min} = 10.7993$$

Calculation of Clearance (P):

Worst Case approach gives

Maximum Clearance, $P_{max} = Z_{max}-Q_{min}$
$= 11.5345-10.7885$
$= 0.746$

Minimum Clearance, $P_{min} = Z_{min}-Q_{max}$
$= 10.6655-11.0115$
$= -0.346$

Root Sum Square approach gives

Maximum Clearance, $P_{max} = Z_{max} - Q_{min}$
$= 11.3458 - 10.7993$
$= 0.5465$

Minimum Clearance, $P_{min} = Z_{min} - Q_{max}$
$= 10.8542 - 11.0007$
$= -0.1465$

4. Results

Results obtained by Worst Case (WC) and Root Sum Square (RSS) approach for individual parts and their assembly are shown in Table 1. The results of stack up analysis show that the maximum clearance in assembly comes out to be positive value whereas minimum clearance takes negative value. This indicates a situation where the assembly of parts can't be done. Hence, reallocation of tolerances by the designer is required to remove the possibility of negative clearance.

Table 1. Results.

Approach / Part Name		WC		RSS	
		Max	Min	Max	Min
Dovetail Female *X*		31.5115	30.8885	31.4239	30.9761
Dovetail Male *Y*		31.1615	30.9385	31.1507	30.9493
Dovetail Assembly	*Z*	11.5345	10.6655	11.3458	10.8542
	Q	11.0115	10.7885	11.0007	10.7993
	P	0.746	-0.346	0.5465	-0.1465

5. Conclusion

The present paper explains an efficient and effective graphical method that aims towards the systematic solution of tolerance stack up problems. The method is straightforward and easy to use for stack up of tolerances of components and their assembly using graphical approach. The usefulness of the method over the conventional tolerance stack up is demonstrated by considering an example of dovetail mounting mechanism for purpose of stack up of angularity. Based on the results of analysis, reallocation of tolerances can be done to fulfil the functionality of the system. If number of components in an assembly is more, huge amount of mathematical calculations is required. However, it can also be found out by using suitable algorithm. Based on this algorithm, a code can be developed which will solve for the desired dimensions of components and their assembly with proper mating relationship. Further the results of stack up analysis can be validated from measurement of manufactured hardware.

References

[1] G.R. Cogorno, Geometric Dimensioning and Tolerancing for Mechanical Design, McGraw Hill, New York, 2006.
[2] J.D. Meadows, Geometric Dimensioning and Tolerancing Handbook: Applications, Analysis & Measurement, ASME Press, New York, 2009.
[3] P.J. Drake Jr., Dimensioning & Tolerancing Handbook, McGraw Hill, New York, 1999.
[4]ADCATS website, http://adcats.et.byu.edu.
[5] A.A.G. Requicha, Mathematical Definitions of Tolerance Specifications, Manufacturing Review, 6(4), 1993, pp. 269-274.
[6] P.K. Singh, P.K. Jain, S.C. Jain, Important Issues in Tolerance Design of Mechanical Assemblies (Part 1: tolerance analysis), Proc. ImechE, J. Engineering Manufacture, Vol. 223 Part B, 2009, pp. 765-778.

[7] P.K. Singh, P.K. Jain, S.C. Jain, Important Issues in Tolerance Design of Mechanical Assemblies (Part 2: tolerance synthesis), Proc. ImechE, J. Engineering Manufacture, Vol. 223 Part B, 2009, pp. 1249-1287.
[8] K.G. Swift, M. Raines, J.D. Booker, Tolerance Optimization in Assembly Stacks Based on Capable Design, Proceedings of the Institution of Mechanical Engineers, Journal of Engineering Manufacture, Part B, 1999, pp. 213: 677.
[9] K.W. Chase, W.H. Greenwood, B.G. Loosli, H.F. Loren, Least Cost Tolerance Allocation for Mechanical Assemblies with Automated Process Selection, Manufacturing Review, Vol. 3, No. 1, 1990, pp. 49-59.
[10] B.K.A. Ngoi, A.M. Agrawal, C.S. Chua, The Generic Capsule Approach to Tolerance Stack Analysis, International Journal of Production Research, 36:12, 2010, pp. 3273-3293.
[11] B.K.A. Ngoi, C.S. Tan, L.C. Goh, Graphical Approach to Assembly Tolerance Stack Analysis—the 'Quickie' GDT method", Proc Institute of Mechanical Engineers Volume 211 Part B, 1997, pp. 463-472.
[12] B.K.A. Ngoi, L.E.N. Lim, P.S. Ang, A.S. Ong, Assembly Tolerance Stack Analysis for Geometric Characteristics in Form Control – the "Catena" Method, Int J Adv Manuf Technol, 15, 1999, pp. 292–298.
[13] J.R. He, P.R. Gibson, Computer-Aided Geometrical Dimensioning and Tolerancing for Process-Operation Planning and Quality Control, lnt J Adv Manuf Technol, 7, 1992, pp. 11-20.
[14] K. Sivakumar, C. Balamurugan, S. Ramabalan, Simultaneous Optimal Selection of Design and Manufacturing Tolerances with Alternative Manufacturing Process Selection, Computer-Aided Design, Volume 43, Issue 2, February 2011, pp. 207-218 .
[15] Mathieu Mansuy, Max Giordano, Pascal Hernandez, A New Calculation Method for The Worst Case Tolerance Analysis and Synthesis in Stack-Type Assemblies, Computer-Aided Design, Volume 43, Issue 9, September 2011, pp. 1118-1125 .
[16] A.K. Sahani, A.K. Sharma, P.K. Jain, Satish C. Sharma, J.K. Bajpai, Review of Assembly Tolerance Stack up Analysis Techniques for Geometrical Tolerances, Int J of Adv Manufacturing Systems, Volume 14, Issue-1, 2012, pp. 1-9.