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The Investigation of the Stress State near the Crack Tip of Central Cracks Through Numerical Analysis

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Abstract

The paper deals with the fracture problematics which is solved by CAE systems. Main goal of the numerical simulation is to investigate the stress state near the crack tip through determination of stress intensity factors. The investigated component is thick plate with symmetric central crack. Through determination of stress intensity factor for all three fracture modes of cracks can be told whether the crack will be grown by comparison of their values with critical value of the stress intensity factor for used material called as the plane strain fracture toughnessK_{IC}. Numerical simulations were realized by the worldwide used code ANSYS. Through numerical simulation can be solved diverse fracture problems with various shapes of cracks and subsequently we can produce safer and more reliable products.

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Keywords: fracture mechanics; crack; stress intensity factor; ANSYS; CAE

1. Introduction

The numerical methods represent many years leading computational utility. Initially uninteresting finite element method (FEM) has today become one of the main computing resources not only in the engineering industry. The main advantage FEM is graphic interpretation often very abstract phenomena in which classical technique solutions introduces considerable simplification at the expense of accuracy.

The humankind has begun to deal with causes of a components fracture in the second half of the 19th century. However to date comprehensive theory explaining the brittle failure still not exist in its complexities and all

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contexts. With the existence of cracks and defects in the material of the structural members is necessary to calculate, since they can lead to an undesired formation of macroscopic cracks. The danger of macroscopic cracks in terms of thebrittle fracture is the possibility of their instability. Thus may occur a sudden and uncontrollable particle separation of the material through a fracture with the speed nearly propagation of the elastic waves in the material without need to supply an energy to the process from the outside[1, 3, 11].

2. Theoretical base

A crack is the most important case of a failure of the body continuity in terms of strength. A surface violation of continuity with the zero thickness and thus with the zero radius of the curvature on the tip of crack is the most preferable calculation model of a crack.Fig. 1.shows this case. The tip of the crack is in this case singular point in which are obtained infinite mechanical stresses. Therefore the state of stress near the crack tip is very inhomogeneous. Thus the plastic deformation in the crack tip will be occurred also at very small loading [1, 4, 5].

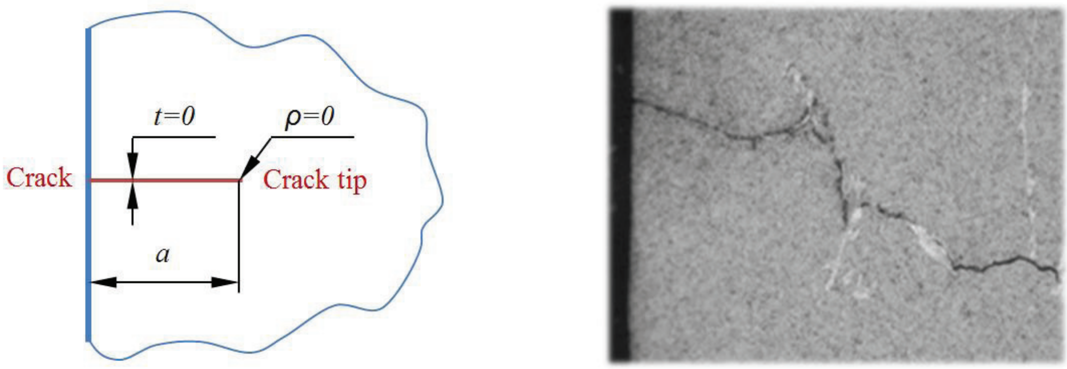

Fig. 1.Ideal computational model of a crack and thereal crack in the material.

Fora classification of a stress state around a crack is consideredan infinite plate witha central crack (Fig. 2.).

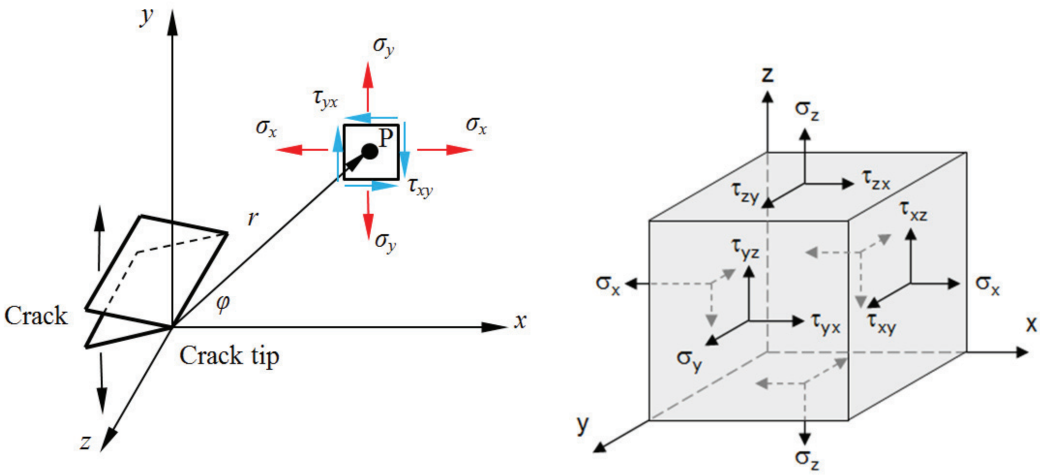

Fig. 2. Infinite plate with a central crack and the Cauchy stress tensor.

In the point P of the object which has coordinates r, φ with regard to the crack tip is the stress state given to the stress tensor $\mathbf{T}_\sigma$. The individual components of the stress tensor can be expressed by the formula

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} \cdot f_{ij}(\varphi) \tag{1}$$

Where $f_{ij}(\varphi)$ is function which depends only on atype of a crack opening mode. The parameter K depends on the geometry of the crack, geometry of the part itself and from the nominal stress in the sufficient distance from the crack tip. Irwin named this parameter as a stress intensity factor. Forthe objects of finite dimensions with variously situated cracks loaded with different modes can be expressed this coefficient in the form

$$K = \sigma \cdot \sqrt{\pi a} \cdot f_k(a,b) \tag{2}$$

where$b$ is the dimension of the part in the crack seam direction and the function $f_k(a,b)$ depends on the boundary conditions of given configuration of the object with a crack.

There are three types of crack opening, termed modes I, II, and III which are shown in Fig. 3. Mode I is a normal opening mode while modes II and III are shear sliding modes [1, 3, 6].

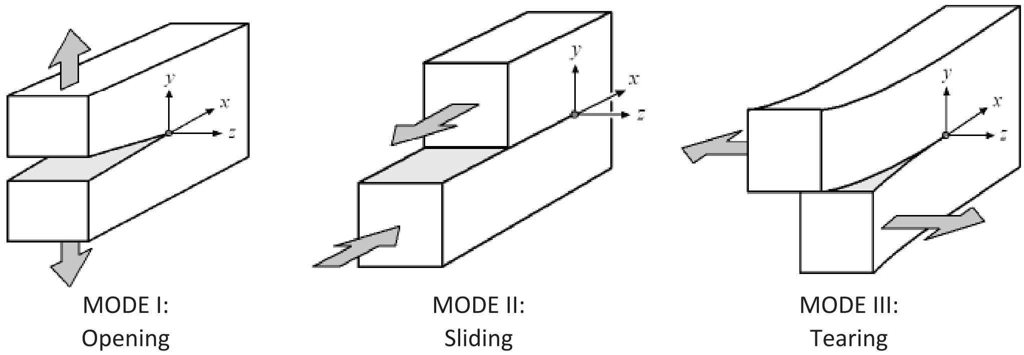

Fig. 3.Three modes of the crack deformation.

If stress intensity factorsK_I, K_{II}, K_{III} are known for all these modes, the stress state near the tip of the crack for linear elastic material is described by the equation (the principle of superposition)

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}} \cdot \sum_{n=I}^{III} K_n \cdot f_{ij,n}(\varphi) \tag{3}$$

The stress intensity near a crack tip characterizes the stress intensity factor by the theory K-conception. If critical value of the stress intensity factor is known for the used material and the calculated stress intensity factors then can be said whether the crack will be grow or not. The critical stress intensity factor for mode I obtained by experimental test of the material K_{IC} is calledas the plane strainfracture toughness [1, 7, 8].

3. Problem definition

A thick plate with the dimensions 0.2 m x 1 m ($2b$ x $2c$) made from the steel is loading with pressure σ = 100 MPa on its ends. The plate containsa central crack which is symmetric. The Crack length $a$can be different. It is needed to determine a state of safety for this type of the crack which can be created on this plate as you can see in Fig. 4.In linear elastic problems, the stress intensity factor is the very important parameter of fracture mechanics from which a singular stress and strain around the crack tip can be obtained [2, 9, 10].

Thus for determining a state of operation safety of the plate have to be known the stress intensity factor at crack tip. Then the calculated stress intensity factor can be compared with the material parameter K_{IC}. For the steel alloy (4340) is this material properties K_{IC} = 50 MPa.m$^{1/2}$. If the calculated stress intensity factor is smaller as the fracture toughness of used material then the crack will not be grown (stabile state). This loading state will be therefore safe for plate with that crack. The numerical approach will be used for the investigating of stress intensity factorsvalues.

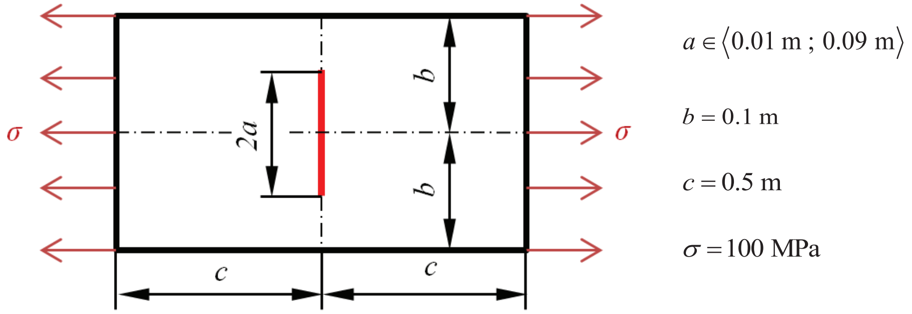

Fig. 4.Plate with central crack together with defined parameters.

4. Analytical approach

The analytical method of fracture mechanics is conservative approach to solution of cracks in the material. Stress intensity factors can be calculated analytically for a central crackby equation

$$K_I = \sigma\sqrt{\pi a}\left(\frac{1-0.5p^2+0.37p^2-0.044p^3}{\sqrt{1-p}}\right), \qquad K_{II} = K_{III} = 0 \tag{4}$$

whereparameter $p = a / b$ and the condition $c / b \geq 3$have to be satisfied for obtaining precise results.The calculated results through the analytical approach will serve only for the investigation whether the results obtained by the numeric method will be calculated enough exactly [1].

5. Numerical approach

The numerical calculations was realized with CAE software ANSYS. The goal of the numerical simulation is to calculate the stress intensity factors. The geometry of the problem is shown in Fig. 4. The geometry represents a thick cracked plate. The plate is made from the steel alloy (4340) which has Young’s modulus E = 210 GPa and Poisson’s ratio v = 0.3. The task was idealized into the simple symmetrical 2D model as illustrated in Fig. 5.

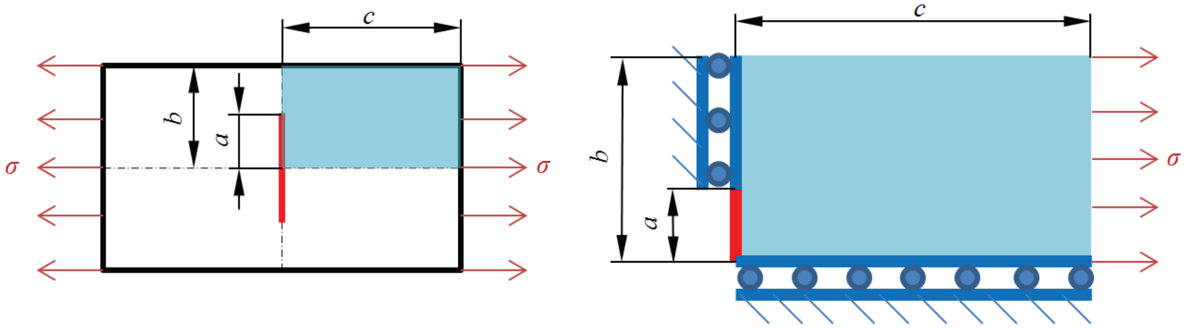

Fig. 5. 2D idealization of the geometrical model along symmetry planes with schematic boundary conditions and the loading.

The crack seam in 2D idealized model is localized bottom left. The top left edge of the model is not be able to move in the X direction and bottom edge in the Y direction. Other degrees of freedom for other nodes were defined default as free. The part was loaded by the pressure $\sigma = 100$ MPa on the right edge of the model in the negative orientation.

In the study of fracture mechanics interest is often focused on the singularity point where quantities such as stress become (mathematically, but not physically) infinite. Near such singularities normal, polynomial-based finite element approximations perform badly.

To determine the singularity in strain, the crack faces should be coincident, and the elements around the crack tip should be quadratic, with the midside nodes placed at the quarter points. Such elements are called singular elements and should be used around the singularity point (crack tips) [2].Fig. 6.shows 2D singular elements PLANE 183 (plane strain) placed around the crack tip (node 2).

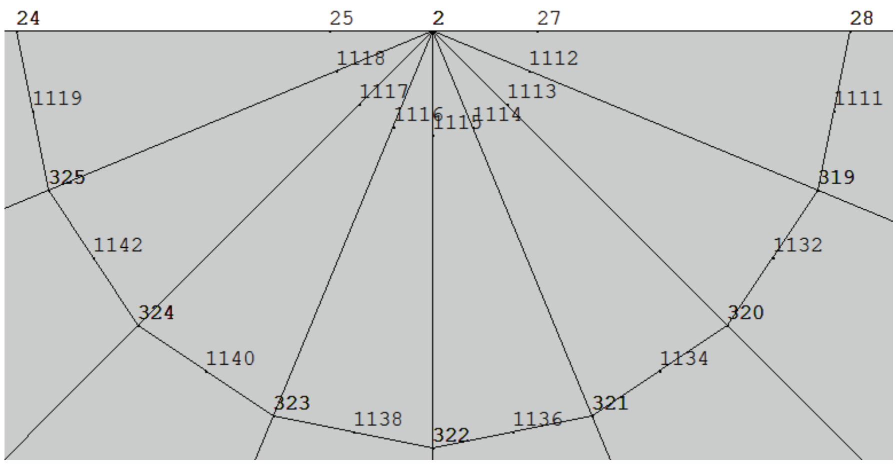

Fig. 6. Mesh around the crack tip with 2D singular elements PLANE 183.

6. Obtained results and discussion

In Fig. 7.is illustrated the output format of the results of the stress intensity factors for given crack seam obtained by numerical approach (ANSYS). Here is described that solved problem was defined as the plane strain condition, simplified by the symmetry boundary condition, made from the material with described material properties and assigned node of the crack tip. The last row of this output shows the stress intensity factors for all three modes of the crack deformation. This type of the loading generates only first mode (opening) thus the stress intensity factors K_{II} and K_{III} had zero value.

```
**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****

 ASSUME PLANE STRAIN CONDITIONS

 ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

 EXTRAPOLATION PATH IS DEFINED BY NODES:       2      44      42
  WITH NODE       2 AS THE CRACK-TIP NODE

 USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
  EX =    0.21000E+12   NUXY =    0.30000      AT TEMP =  0.0000

**** KI =  0.17857E+08,   KII =   0.0000    ,   KIII =   0.0000      ****
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Fig. 7. Sample output of stress intensity factors obtained from ANSYS.

From the obtained results using numerical method were made Fig. 8 and Fig. 9. Extreme high stress-level was expected near the crack tip as it is described in linear fracture mechanic (see Fig. 8.). These extreme local stresses are caused by the singularity on the crack tip. The breaking points on the curves in the locations –0.1 mm and +0.1 mm from the crack tip have been caused by the numerical approach. In this region are changed the elements from normal conception to the singular conception of the elements.

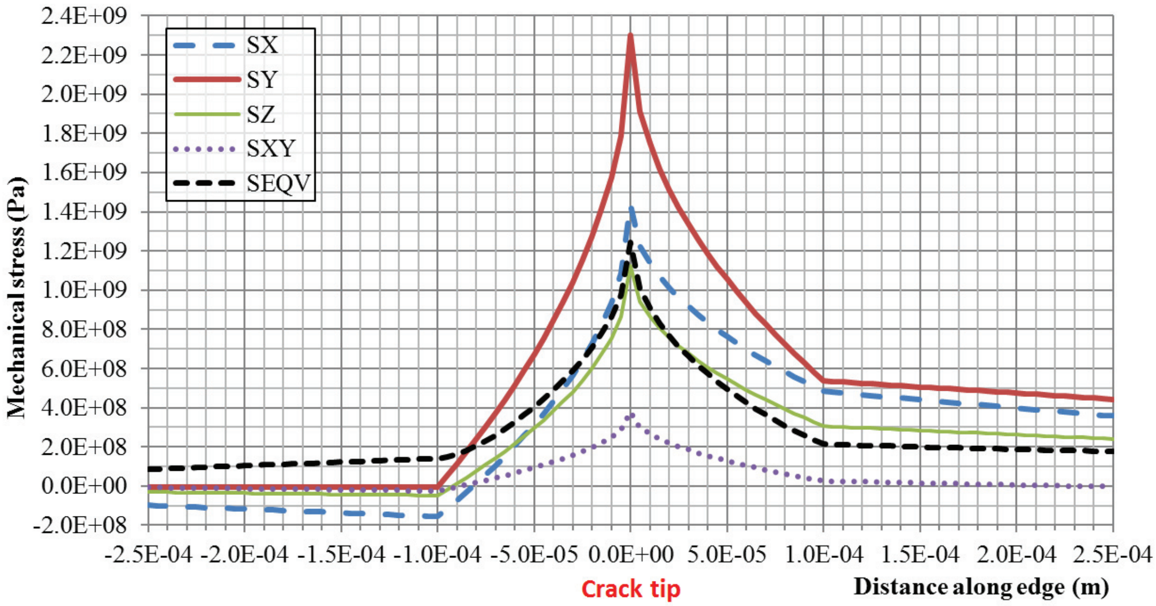

Fig. 8.Mechanical stresses obtained near the crack tip along the edge of the model.

Therefore stress intensity factors have to be found in this location for investigating the stress state. Fig. 9.showsthe curve of the stress intensity factor K_I which was obtained by the numerical analysis of the solved problem in the range of the crack length from 0.01 m to 0.09 m.

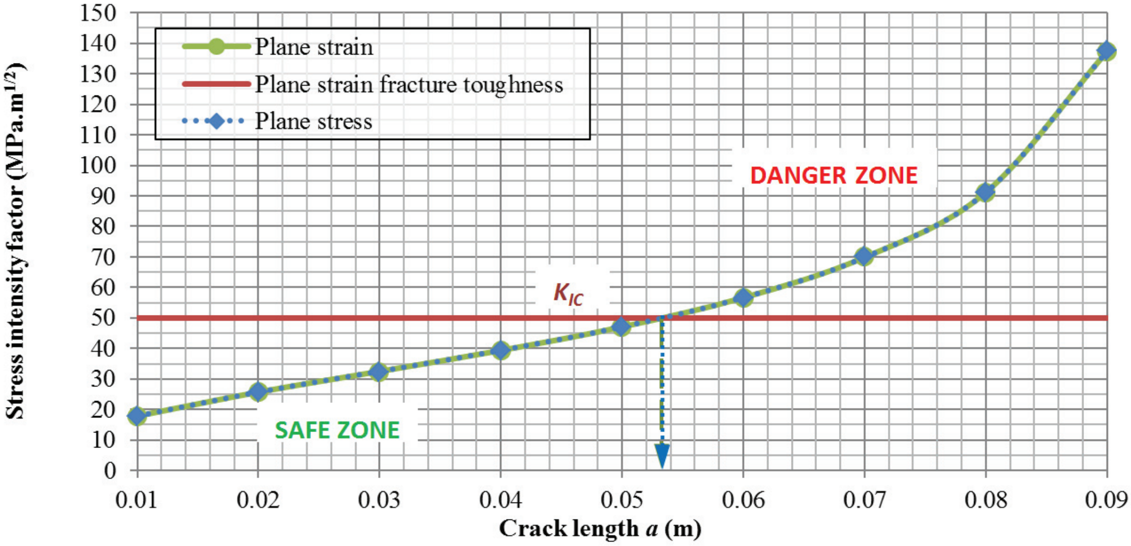

Fig. 9. Stress intensity factorsK_I obtained by numerical method in wide range of crack length together with critical value of fracture toughness.

The material properties called as the plane strain fracture toughness divides the graph on a safe zone and a danger zone. All values of the stress intensity factors, which are above the value of material toughness, are in the danger zone because the crack will be grown if is exceeded this value of the material properties at the plane strain conditions. The values below are in the safe zone which represents a region where is the crack in stabile state thus the crack will not be grown. The fracture toughness K_{IC} is determined in terms the plane strain conditions (thick parts) at the crack tip thus the results obtained from the numerical analysis for the plain strain conditions can be compared directly with the fracture toughness of the material.

For the plane strain conditions has been obtained minimal value of the stress intensity factor 17.86 MPa.m$^{1/2}$ when the length of the crack had the value 10 mm and the maximum value 137.36 MPa.m$^{1/2}$ was calculated when the length of the crack had the value 90 mm. The value of the crack length a = 53.5 mm is critical from the safety point of view. For this crack length the stress intensity factor obtained from the numerical analysis has value 50 MPa.m$^{1/2}$ which is exactly the value of the material fracture toughness.

Table 1. Stress intensity factors obtained by analytical and numerical approach.

Stress intensity factor (MPa.m$^{1/2}$)	Crack length a (m)								
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Analytic K_I	17.82	25.63	32.37	39.19	46.91	56.54	69.88	91.28	137.48
Numeric K_I (Plane strain)	17.86	25.72	32.52	39.38	47.09	56.66	69.88	91.17	137.36
Relative error (%)	99.78	99.65	99.54	99.53	99.62	99.79	100.00	100.12	100.09

To compare the results obtained by numerical method will calculate stress intensity factor by analytical approach too. As reference value of stress intensity factor was defined analytical results. In Table 1.can be seen relative errors which are very close to analytic obtained results.To compare of the results were made the same calculations for the plane stress conditions of this problem. This condition simulates same problem but the plate is in this case the thin plate. Results of the stress intensity factorswereidentical as results obtained for plane strain conditions.

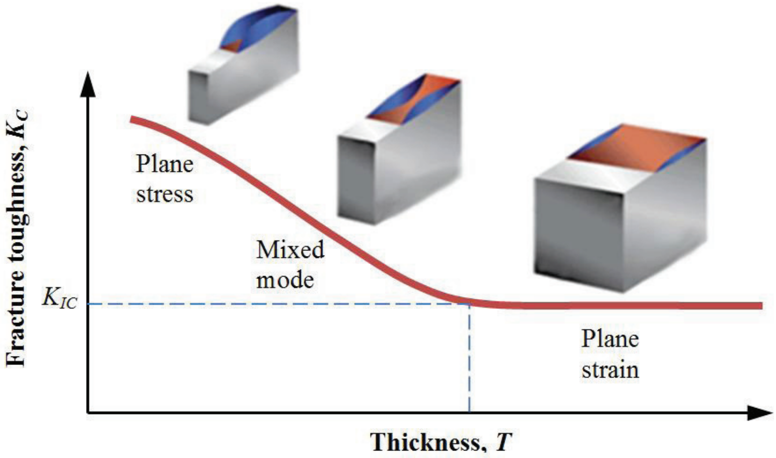

Fig. 10. Critical stress intensity factor of the material K_C called as fracture toughness depending on the material thickness.

But the critical value K_C of the stress intensity factor is for the plane stress conditions greater than the value of the plain strain fracture toughness K_{IC}. Therefore the results obtained from the plane stress model are not able to compare with the plane strain fracture toughness from the view of dimensioning of mechanical components which have to ensure safety aspect but can be established that thin part will have definitely greater limiting value of the crack length as thick part with the same material properties, cross section, boundary conditions and loading state. This fact can be seen in Fig. 10. The fracture toughness at the plane strain conditions is less than the plane stress condition because a plastic zone on the crack tip in the plane strain conditions is very constrained. The material of the part under these conditions will be broken through brittle fracture.As illustratedin Fig. 10.The plane strain fracture toughness above a certain value of the material thickness is not changed thus in other words the parameter K_{IC} is independent of the thickness.

7. Results

When machine parts are dimensioned, the design engineer should always keep in the mind that materials can contain cracks which are very large concentrators of mechanical stresses.The cracks can cause the state of disrepair even though the components will be designed very well in terms of the carrying capacity. Thus the components can be destroyed by brittle fracture even before some signs of plastic deformation will be able to visible.By the analytical method of the calculation of cracks parameters may be solved very exactly but only for simple models with simple crack seams. Therefore main advantage of the numerical approach to the calculation of the stress intensity factors in the crack tip is that a shape and position of cracks in a model are not limited. Thus in other words, solving of complicated cracks in a complex shaped model through the analytical approach is almost impossible or very inefficient in the terms of the time.From the obtained results can be seen that the numerical results give very exactly data if their compare with the analytically obtained results. Thus the numerical approach can be used for the solving of a global crack problem with very well accuracy of results.The numerical results had the deviate from the analytics results in the range only ±1%. To ensure this very good accuracy should be defined around the crack tip the singular elements otherwise obtained results will be incorrect.By solving the defined problem with the thick plate was determined the critical value of crack length.If the value of the crack length a = 53.5 mm or greater will be achieved then part will be broken by brittle fracture because will be occurred the critical value of the stress intensity factor of thematerial toughness for the plane strain conditionsK_{IC} on the crack tip which will not be able to carry the loading without damage. This approach can lead to design very quality, safety and reliable products for end users.

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