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# Helmert Transformation of Reference Coordinating Systems for Geodesic Purposes in Local Frames

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**Abstract**

The real-time conditions may develop a need for data originating from the concrete reference system. Sometimes this can refer to the Gauss-Krüger coordinates and UTM projections with a specific datum. This paper analyses a connection between UTM projection with datum WGS-1984 and Gauss-Krüger coordinates for a wider area of the City of Rijeka, without precise need for making the reference to the specific reference coordinate system. Paper also includes the observations of mathematical frames which define Helmert transformations as tool that enables switching from one reference system to another. Additionally, it discusses the needs and possibilities of described tool within GIS and GPS navigation and possibilities of computer program which can modularly be implemented into the specialized programs for Geodesic purposes.



*Keywords*: reference systems; transformation of coordinates; Helmert transformation; Gauss-Krüger projection; UTM projection

## 1. Introduction

Geodetic surveys primarily have a pragmatic character. This is a process which basic purpose is to determine spatial coordinates of points specific to a certain object or terrain. Terrain is a synonym that corresponds to the relief or a segregation of the individual objects in space. Geodetic surveys determine location of point's characteristic to the individual objects when they are presented in cartographic-graphic or digital spatial overview. Each point in space is defined by three coordinates.

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Geodetic point is spatially defined by two coordinates, latitude -φ, longitude –λ, while the third coordinate is altitude-h – point above the sea level or a proportional measure opposed to the referent sea surface – geoids.

The concept of datum, in surveys and geodesy, refers to the referent point or a surface which location serves as basis for the measurements. The position of that point originates in the accepted model of Earth's shape as the starting location for measurements. Horizontal datum describes points of Earth's surface, according to the latitude and longitude, or other coordinates when applying some other coordinate systems. Vertical datum is used for measuring heights or underwater depths. In geodetic coordinates, the Earth's surface is an approximate ellipsoid and each point is defined by its' latitude -φ, longitude –λ and altitude -h. When measuring, the spatial ellipsoid should be projected as a flat surface. The process of mapping depends on the cartographic projection. Projections are executed through geometric maps – conforming projections. The Republic of Croatia has accepted a conforming, transversal, cylindrically projection - Gauss-Krüger projection with the original angles.

Pragmatic result of geodetic survey is determination of measures and depicted data on land for different purposes in the continuing processes of making and updating maps and plans in urban planning or documenting of construction sites. Momentarily, the Republic of Croatia should hastily update the immovable property cadastre and the general spatial documentation used for other purposes as well.

The process of legalizing the illegally constructed objects during the last fifty years within the Republic of Croatia's territory requires further measurements and positioning of the concrete objects. This paper tries to describe processes of transformation from different reference systems alongside the possibilities of modern approach and the GPS – satellite positioning of objects. For this purpose, Gauss-Krüger and UTM coordinate referent systems as well Helmert transformations for switching between systems have been used. The practical example refers to the area of North-South Croatia. The second part of paper describes computer programmes used for described purposes.

## 2. Problem formulation

Describing and measuring object within space should begin with its locating in the specific coordinate system that is positioning the centre of coordinate system within a specific point. Since coordinating system is a relative concept, the centre can be located randomly. This enables observing and measuring objects in comparison to the several different originate points. Switch or transformation from one to another system is possible through the creation of specific mathematic connection. Connected systems are referent and such transformations are being realized through transformation of coordinates characteristic to the points of objects measured. [1]

Referent systems applicable in geodesy are developed according to the geodetic datum. Datum is a parameter or a group of parameters which define a location of centre, measures and orientation of the coordinating system. Geodetic datum defines a size of ellipsoid and its position in comparison to the centre of gravity and the central position of the Earth's rotational axis. Local datum describes relationship between coordinating system and local references that is the local points.

Geodetic datum is passed upon ellipsoid and its parameters, while local datum is defined according to the globally accepted one. [2]

As mentioned above, survey of the concrete objects in local frames depends upon the specific reference system and is determined by recognizing the specific parameters. Surveying is a process which can be accompanied by eventual mistakes. Elimination of mistakes and correction of the surveys' results are necessary. That process is directly connected to the applied reference system so the possibility of switch between coordinates is necessary. Presently technical possibilities provide a high preciseness and reliable measurements, while the computers enable easy switch – transformation from one system to another. [2]

Coordinating system – plain surface cartographic projection is locally prepared by the Government, while it is being processed by its body or office. Plain surface cartographic projections of the Republic of Croatia, according to the Act made by the Croatian Government are [3]:

- Coordinating system of transversal Mercator (Gauss-Krüger) projection - HTRS96/TM, with the mean meridian 16°30' and the linear measure of the mean meridian of 0,9999 which is defined by Croatian coordinating system for the area of cadastre and state topographic cartography.
- Coordinating system of vertically Lambert conform conical projection - HTRS96/LCC, with the standard parallels 43°05' and 45°55' determined by projection coordinating system of the Republic of Croatia within the official state cartography.

- Coordinating systems of cartographic projections are based on the Croatian terrestrial reference system defined in point a.
- The Republic of Croatia's Army uses the projection coordinating system of the Universal Transverse Mercator – UTM, according to the Standardization Agreement "STANAG 2211", signed by the membership countries of NATO.

As can be concluded from the above mentioned, it is necessary to provide an elegant transformation from one coordinating system to another, which is this paper's hypothesis.

## 3. Backgrounds

Geodetic datum Bessel 1841 is Bessel spheroid originating in 1841 that is the referent ellipsoid, while WGS84 is the referent ellipsoid momentarily used for the GPS satellites.

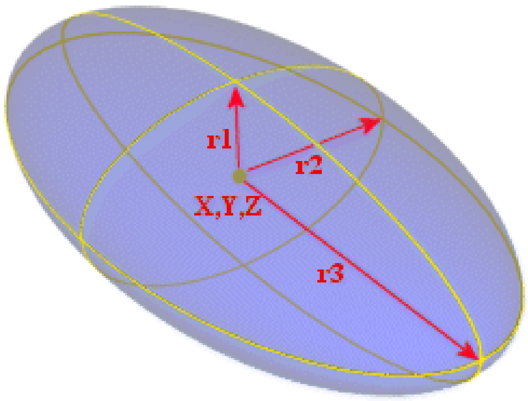


Fig. 1. Ellipsoid [16] (Source: http://www.builder.cz/rubriky/c/c--/jak-vyzrat-na-kolize-2-dil-kolize-v-prostoru-156031cz, loaded 20.IX.2013).

Cartographic projections are mathematical procedures which enable reflections of distorted surface (sphere or rotational ellipsoid, Fig. 1.) of Earth and other celestial bodies. The purpose of projection is to solve theoretical and practical assignments in cartography, geodesy and similar sciences. Within ellipsoid points are determined by transaction of meridian and parallels. The picture of meridian and parallels network is being projected into the plain area projection which is basically the procedure for developing a cartographic network.

Cartographic reflection defines dependence between coordinating points of Earth's ellipsoid and coordinates of their projection points. This dependence can be described by the equations:

$$x = f1(\lambda, \varphi);\ y = f2(\lambda, \varphi) \tag{1}$$

Where: φ and λ – are geographic width and length, while x and y represent rectangular coordinates in plain surface projection. [4]

More info on other datum can be found on the following website (http://www.colorado.edu /geography/gcraft/notes/ datum/edlist.html, downloaded on 15$^{th}$ July 2013).

Table 1. Some of Datum – Ellipsoid [14].

| Ellipsoid | Data |
|---|---|
| **Bessel 1841** | Semi-major axis = 6377397.155, Semi-minor axis = 6356079.0000, 1/flattening = 299.15281 |
| **WGS 84** | Semi-major axis = 6378137, Semi-minor axis = 6356752.3142, 1/flattening = 298.257224 |

Each projection can result in specific deformations, so they can be divided into the following groups:

- Conform – rectangular – preserve angles
- Equivalent – of the same surface – providing the identical surfaces
- Equidistant – of the same length – providing the same length and direction.

Other categorizations use the shape of plain area projection or the location of cartographic network's pole as the main criterion.

Especially interesting are conform reflections which preserve angles. Analytical functions describe complex variables and represent conform reflection or transformation of one part of the complex plain surface into another. [4]

### *3.1. Recent developments and research of interest for this paper*

Recent researches exploring a precise implementation of the Earth's geodetic network have been proved as significant efforts made by Wegener group. Wegener group has been originally conceived as a working group comprised of geo-scientists conducting scientific researches for network's implementation. Total geodetic monitoring on a global level has been additionally improved through InSAR (Synthetic Aperture Radar) which tracks the localities. [5] Since the early 1970s, VLBI (Very Long Baseline Interferometry) proved to be a primal spatial-geodetic technique for evaluating precise Earth's coordinates with a simultaneous tracking of changes in its rotation. System is highly précised in measurements and evaluation of other parameters of the Earth's system.[6] Influence of global climate changes, such as increase of a sea level, melting of icebergs and similar, though scientifically important, haven't been analysed in this paper. Due to ensuring a high resolution and precise information, some spatial-geodetic techniques such as GNSS, VLBI, SLR, DORIS and InSAR are being mentioned.[7] For instance, when analysing mathematical models, [xxx] suggest a numerical/topological, grid-search based technique in the $R^m$ space, a generalization and refinement of techniques used in some cases of low-accuracy 2-D positioning. In contrast to conventional solutions which tend to minimize a certain function and directly obtain a point solution of a system of equations, our algorithm has a different strategy.[8]
In their research, [8] consider the movements of geocenter as important factors of changes made in geonetwork. Consequently, geocenter motion is intimately related to the realization of the International Terrestrial Reference Frame (ITRF) origin, and, in various ways, affects many of our measurement objectives for global change monitoring. Due to the importance geodetic network's 3D frames, the authors recommend papers [9], [10] and especially [11], which analyse different techniques of transferring height datum across the sea [9] or modified Stokes integration for calculation of gravimetrical geoid [10]. When precisely determining a local geoid, it is possible to apply different algorithms, especially those available via computer, such as learning based computing algorithms: artificial neural networks (ANNs), adaptive network-based fuzzy inference system (ANFIS) and especially the wavelet neural networks (WNNs) approach in geoid surface approximation. These algorithms were developed parallel to advances in computer technologies and recently have been used for solving complex nonlinear problems of many applications [11].

### *3.2. Gauss projection of ellipsoid to the plain surface*

This is the most important geodetic projection which belongs to the group of so-called geodetic projections for the needs of state surveys. This is a mathematical basis of all computing in the process of developing plans and maps of the highest resolutions. For the needs of state surveys, the majority of European countries momentarily employ Gauss-Krüger projection. Gauss projection of the rotational ellipsoid on the plain surface is defined as conform projection which fulfils two additional conditions:

- central meridian is projected as direction,
- along central meridian there are no lengths' deformations.

Except for the cartography, geodetic projections are used for all calculations in plain surface. They are also applied in the following cases:

- calculation of linear measures from geographic and rectangular coordinates,
- length and azimuth of geodetic line (the first and second geodetic task),
- calculation of reductions for directions and lengths,
- calculation of geographic coordinates from rectangular coordinates within plain surface projection,
- calculation of meridian's convergence from the geographic and rectangular coordinates,
- calculation of rectangular coordinates of points when rectangular coordinates of single point are determined,
- calculation of length and azimuth for geodetic lines from the rectangular coordinates of two points,
- transformation of coordinates between neighbouring coordinating systems.

In Gauss system, ellipsoid is being reflected on the plain surface under the following conditions:

- projection has to be conform and it has to be realised through analytical functions of the complex numbers,
- mid meridian is reflected as direction and its projection represents X-axis of the plain coordinating system to which the projection is symmetric: for $\lambda_0=\lambda$ y=0,
- X-axis of the rectangular coordinating system is compatible to the mid meridian alongside which there are no linear deformations, for $\lambda_0=\lambda$

$$x = \int_0^{\varphi} M d\varphi \tag{2}$$

Where $\lambda_0$ is geodetic length of the central meridian.

### 3.3. Gauss-Krüger's projection

In this projection meridians and parallels are reflected as curves. Meridians are symmetric in comparison to the central meridian which is reflected in direction, while the parallels are also being reflected as directions. The centre can be randomly determined in any point of the central meridian, but is usually defined according to the section between the central meridian and equator.

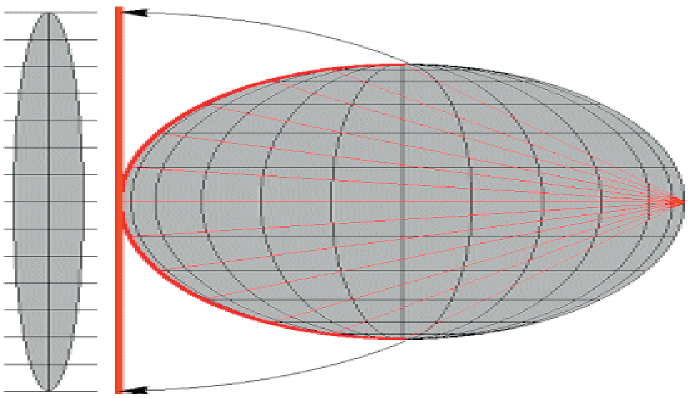

Fig. 2. Gauss-Krüger's projection (source: http://free-ri.htnet.hr/geocaching/_private/gausskruger.htm, loaded 12.VI.2013) [15].

In Gauss-Krüger projection (Fig.2.), conform projection is an ellipsoid oppose to the plain surface since the projection doesn't have deformations in angles. In Gauss- Krüger projection the measure of central meridian equals $m_0$=0.9999. Each coordinate zone has 3° geodetic length. The first zone has a central meridian (Greenwich) 0°, while the second zone equals 3° etc. The central meridians of 15° and 18° of Eastern length are especially interesting for this paper. The number of zone is added as prefix to coordinating movement of ordinate $y_0$. In Gauss-Krüger this movements equals $y_0$=500 000 m. That is the reason why $y_0$ in the sixth coordinating zone equals $y_0$=6 500 000m. Gauss-Krüger's projection uses Bessel ellipsoid from 1841 as datum. [4]

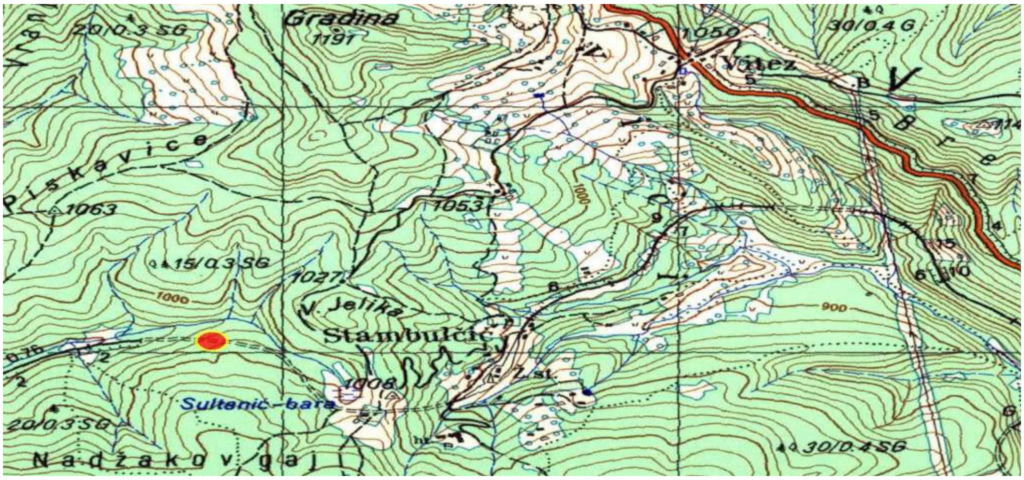


Fig. 3. Example maps of G-K. Projection

### 3.4. UTM projection

Universal Transverse Mercator projection (Gauss-Krüger) represents a projection system for the entire Earth (Fig 3.). This is Gauss-Krüger projection with linear measure of mid meridian $m_0$ = 0.9996. UTM system relies on datum WGS-84 with 6° width of meridian zone. Instead of the central meridian, two parallel sections distanced app 180 km from the central meridian are reflected. The area of section is smaller in projection, with the enlargement in the outer area of 1.00015 for the marginal meridian (φ=50°). UTM-system isn't applied in the poles' zone (app 10°). For this area the Universal Projection of Stereography (UPS) is being applied. The Republic of Croatia is covered in zone 33, with the central meridian 15° and zone 34, with the central meridian 21° (Fig. 4.).

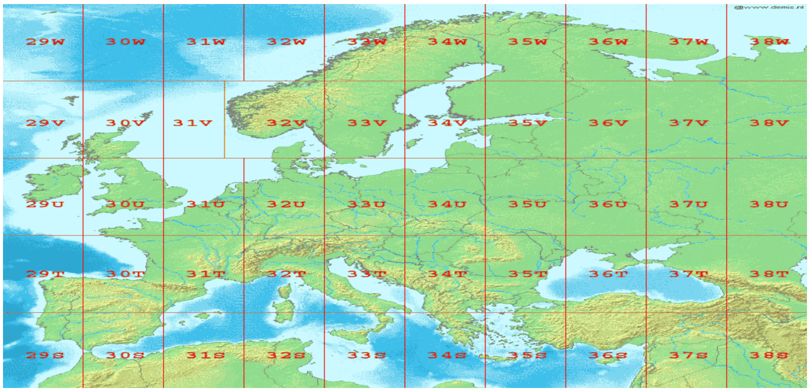


Fig. 4. Distribution Continental Europe in UTM zone
(source : http://commons.wikimedia.org/wiki/File:LA2-Europe-UTM-zones.png, downloaded 4.I.2014.)

Measure alongside the central meridian equals $m_0$=0.9996. In UTM projection, the number of zone isn't put in front of the transformation $y_0$.

UTM system is presently widely used standard among different organisations and in GPS navigation. Today GPS receivers enable easy transfer to the plain surface coordinates. [4]

## 4. Helmert transformation

Transformation of point's position from one to another reference system is possible through numeric operations. [12] This process isn't characteristic solely for geodesy. The process itself depends upon characteristics of coordinating systems. This is usually a mathematic model of linear type that is transformations are described through the linear functions. The general linear transformation is called affine transformation. System of equations depends upon number of dimensions so for two-dimensional coordinating system it is provided as:

$$\begin{aligned} x &= a_1\xi + b_1\eta + x_0 \\ y &= a_2\xi + b_2\eta + y_0 \end{aligned} \tag{3}$$

where coordinates (ξ,η) should be transformed into the coordinates (x,y) of the new system. In all these transformations local coordinates are always transformed into the global coordinates. Sizes $a_1$, $a_2$, $b_1$, $b_2$, $x_o$, $y_o$ are parametric affine transformations. Special example of the affine transformation is conform transformation important for measurements made in geodesy. If the affine transformation is conformal, then these conditions should be fulfilled:

$$a_1 = b_2 = a,;\ b_1 = -a_2 = b \tag{4}$$

so the functions (3) become

$$\begin{aligned} x &= a\xi - b\eta + x_0 \\ y &= b\xi + a\eta + y_0 \end{aligned} \tag{5}$$

Linear confirm transformation is efficient in when the parameters a, b, $x_0$ and $y_0$ are familiar. However, how these parameters are usually not defined, this could be a problem in geodesy. Since the parameters aren't specified, in order to transform local coordinates into the global, they should be calculated indirectly from the group of points which positions are known in both coordinating systems. This way the affine transformations require at least three points in both coordinating systems so to provide the right description. Condition states that points shouldn't be collinear. For conform transformation at least two points should be completely defined in both systems so that values of the parameters a, b, $x_0$ and $y_0$ can be calculated. However, usually for the matter of preciseness one should define several points in comparison to the number of unknown parameters. This enables the usage of statistic apparatus and reliability of parameters assuming they are normally distributed. Each point which is familiar in both systems is defined by two equations (3) with four unknowns a, b, $x_0$, $y_0$. The optimal case refers to more than 10 points which are equally distributed. When the coordinates from both systems (x,y) and (ξ, η) are measured, then the equations (3) represent a general mathematical model which equating demands several iterations and is generally more complex than the parametric model. The simple solution of the problem, equating the parameters of linear orthogonal transformation represents a model provided by the German geodesist Helmert. In this model, the local coordinates (ξ, η) are taken as constants, with only the global coordinates (x,y) measured as the stochastic variables. Therefore * becomes a parametric model. Helmert transformation therefore cannot be transformation of the coordinates but a model for calculating the parameters of transformation through least square method.

When measurements do not include heights from some particular reason, two-dimensional transformation is applied; when geodic heights are significantly changeable, this form has its lacks, so the local network is applied (y,x), with heights (H) randomly determined.

Global coordinates are considered as measurable sizes assuming the surveys have the same weight. Each point defined in local and global coordinates is described with two equations for every coordinate independently. If there are *n* points, the measurement vector will have 2*m* elements.

$$l^t = [x_1\ y_1\ x_2\ y_2 \ldots\ldots x_m\ y_m] \tag{6}$$

Two-dimensional orthogonal transformations always have four parameters.

$$x^t = [a, b, x_0\ y_0], u = 4 \tag{7}$$

Helmert two-dimensional transformation is a linear parametric model and the first design matrix A is derivation of measurements by parameters, that is equation (5) by vector of parameters (6), which is quite simple in this case

$$A = \begin{bmatrix} \xi_1 & -\eta_1 & 1 & 0 \\ \eta_1 & \xi_1 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots \\ \xi_m & -\eta_m & 1 & 0 \\ \eta_m & \xi_m & 0 & 1 \end{bmatrix} \tag{8}$$

The approximate values of the parameters don't necessarily have to be familiar. Any values can be set. When calculations in equating are highly precise, then for the elements of approximate vector can be chosen these values:

$$x_0^t = [1\ 0\ 0\ 0] \tag{9}$$

The elements of differential vector are differences between local and global coordinates

$$\omega^t = [\xi_1 1 - x_1\ \eta_1 - y_1\ \xi_2 - x_2\ \eta_2 - y_2 \ldots \ldots \xi_m - x_m\ \eta_m - y_m] \tag{10}$$

Any positive number can be chosen as weight for all global coordinates, but due to the simplicity of calculation usually one is chosen, so the matrix of weights is the unit matrix

$$P = I \tag{11}$$

so the creation of normal equations is simplified:

$$N = A^t A, u = A^t \omega \tag{12}$$

When there is a matrix of variance-covariance for the global coordinates, then the weight matrix is calculated as the inverse matrix of variance covariance of the global coordinates:

$$P = C_{GC}^{-1} \tag{13}$$

If the approximate values of the unknown parameters are defined by (8), then differential vectors (9) are numerically high values. This requires high preciseness in the process of equating, that is the greater number of figures and values, which isn't a significant problem when computers are being used. [13]
Finally, when switching from Gauss-Krüger coordinates to UTM coordinates, the local area will be presented by affine transformation (ξ,η)→(X,Y). Naturally, preciseness on the marginal area will be smaller in comparison to the central area where the situation is opposite. Depending on the sort of geodetic works this kind of transformation will be appropriate and its preciseness satisfying.

## 5. Conclusion

Aim of this paper is to imply a simple possibility of switching from one reference system to another. This intrudes an urge for standardisation and (not necessarily) unification of the procedures. Cadastral survey of the concrete object in local frames can be connected to the specific reference system. It can be accompanied by the possible mistakes or preconditioned by recognizing the specific parameters. Since the mistakes in cadastral survey can occur, their corrections are necessary and these are directly connected to the applied reference system. Different conditions and different needs have lead to the situations where the same problems and issues are being interpreted and solved in different ways. However, there always occurs a moment in which a need develops for transformation from one form to another, alongside the necessary interface and shape of transformation. Identical transformations of space, two- and three-dimensional can be qualitatively described by mathematical tool. Usual detailed geodetic calculation can be replaced by the computer processing. Three-dimensional Helmet transformations assume that for the identical points in the global system and local system coordinates (x,y) are well defined and they reflect orthometric heights. At the same time the computer programmes can be more or less sophisticated and adjusted to specialized equipment and modern equipment such as laptops, PDAs and mobile phones.

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