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Thermo Elasto Hydrodynamic Lubrication Model of Mixed Friction

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Abstract

The paper provides mathematical model for solution of lubrication problem in case of mixed friction, i.e. when total contact load of two bodies is partially transmitted through the peaks of roughness of a solid contact, and partly through the pressure generated in the oil film. Mathematical model used for solution of lubricated contact with mixed friction is based on the bases of thermo-elasto-hydrodynamic (TEHD) theory of lubrication expanded by equation of solid bodies contact. Based on the presented mathematical TEHD model own computer program was developed, which enables rapid analysis of tribological parameters at the point of contact of two bodies (the thickness of the oil film, the pressure transmitted by solid body, the pressure transmitted by fluids, temperature in the oil film) in the function of working conditions parameters.

Keywords: geometry; lubrication; wearing; contact

1. Introduction

Research of tribological system has first of all been aimed at the increase of loading, prolongation of life cycle, enlargement of efficiency and reduction of price [1-5, 11, 12]. In order to optimally design the tribological system in view of the temperature (the heat) limit it is necessary to know the operation losses. Besides the geometry of gearing, which influences the efficiency by the conditions necessary for the hydrodynamic lubrication, the applied lubricant also plays a significant role. For researches performed in this work, the applied lubricant was the mineral oil, with the assumption of a Newtonian rheology.

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Nomenclature

c_{K1}, c_{K2}	specific heat of contact solids, J/kgK
dl	contact line element, m
E	elastic moduls, Pa
F_N	load per unit length, N/m
$h_{(x)}$	film thickness, m
H_F	hydrodynamic portion of total force, -
n_d	rotational speed, min^{-1}
$p_{(x)}$	hydrodynamic pressure, Pa
p_0	maximum Hertzian contact pressure, Pa
$p_{k(x)}$	asperity pressure, Pa
$p_{uk(x)}$	total pressure, Pa
q	heat fluux, W/m^2
R^*	relative radius of roller, m
R_{a1}, R_{a2}	CLA surface roughness, m
R_{p1}, R_{p2}	maximum surface roughness
s_0	Roeland's thermoviscous coefficient parameter, -
v_{1n}, v_{2n}	velocities of surfaces 1 and 2, m/s
v_k	sliding velocity, m/s
v_{sum}	rolling velocity, m/s
x	horizontal coordinate, m
z	pressure-viscosity index, -
α	pressure-viscosity coefficient, m^2/N
η	viscosity, Pas
η_0	viscosity in ambient conditions, Pas
ϑ, $\vartheta_{K1,2}$	temperature of the lubricant and solids, K (^oC)
λ, $\lambda_{K1,2}$	thermal conductivites of the lubricant and solids, W/mK
μ	frictional coefficient, -
ρ	density, kg/m^3

The following combination of materials for contact pair has been chosen: steel and tin bronze. The paper investigates tribological model with two discs in case of mixed friction, which in most cases presents parts of contact line of different transmitters of power and movement. Approximation of contact line with scope of a pair with elementary cylindrical pair is used by the author [2,4,5] for the analysis of degree of efficiency. Cylindrical pairs which replace parts of contact lines approximately represent lubrication condition at contact place and equivalent kinematic pair for analysis for thermo-elasto-hydrodynamic (TEHD) lubrication model. Fig. 1 shows contact line of equivalent pairs of cylinders with dl length. In the area of mixed friction, oil film interrupts solid bodies contact. This form of friction occurs during unfavorable conditions of lubrication and rough surfaces of solid bodies i.e. when oil film thickness h is smaller than a sum of roughness of both contact surfaces $h < R_{p1}+R_{p2}$ .Wearing process in mixed frictio points to the fact that overall load, in the zone of lubricated contact, is transferred partially across the contact of solids and partially across the pressure in the oil film, and that the total value of coefficient of friction μ_{uk} is the result of friction in the oil film μ_F and friction by means of solids μ_{KT}:

$$\mu_{uk} = \mu_F H_F + \mu_{KT}(1 - H_F) \tag{1}$$

where H_F is the portion of load carried by the oil film. The aim of this work is to mathematical modeling of lubricated contact in case of mixed friction. To achieve this, it was necessary to extend the TEHD equation system for ideal smooth surfaces by an additional equation of contact of solids according to [4, 9] and to model the experimental dependence of coefficient of friction of solids when solving energy equation.

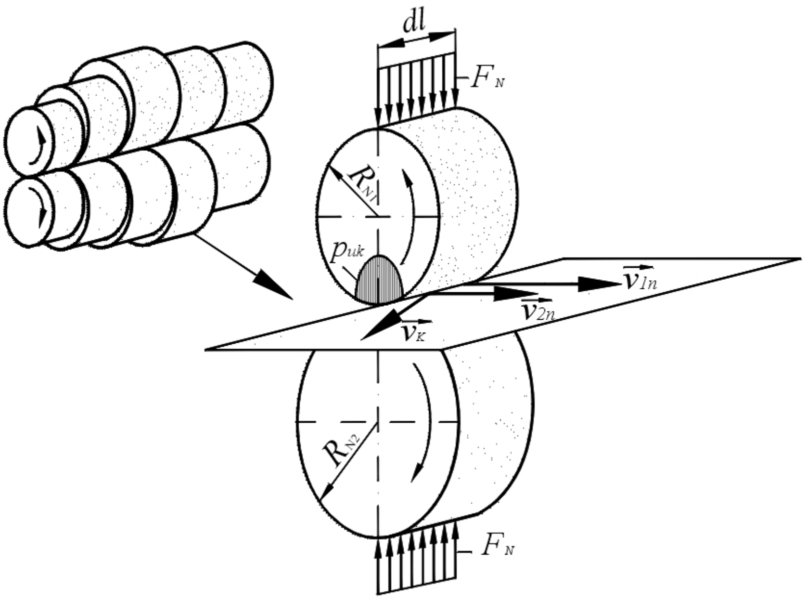

Fig. 1. Contact line with contact roler pairs.

2. Modeling of mixed friction

2.1. The mathematical model of lubricated contact

The extended mathematical model of lubricated contact contains the basic equations of TEHD model for ideally smooth surfaces according to [6 -14] and additional equation of contact of solids for the combination of materials of the observed drive. The equation of elasticity connects elastic deformations of solids in contact, i.e. the oil film thickness with the distribution of total pressure $p_{uk(x)}$, and is given in the form:

$$h(x) = h_{00} + \frac{x^2}{2R^*} - \frac{2}{\pi E^*} \int_{-b}^{a} p_{uk}(s) \ln(x-s)^2 \, ds \quad (2)$$

where are: h_0 the constant of integration, R^* the relative radius of curvature, E^* the equivalent elastic modulus, x the direction of moving, s the integration variable.

Reynolds' equation describes dependences of pressure distribution in the oil film on the oil film thickness at the contact place:

$$\frac{\partial}{\partial x}\left(\frac{\rho}{12\eta} h^3 \frac{dp}{dx}\right) = \frac{(v_{1n} + v_{2n})}{2} \frac{\partial}{\partial x}(\rho h) \quad (3)$$

where are: x the direction of moving, ρ the density of lubricant, η the dynamic viscosity of lubricant, h the oil film geometry, p the pressure in lubricant, v_{1n} and v_{2n} the velocities of surfaces in contact. This equation can be solved by using the Swit-Stieber's border conditions [7].

The contact equation of solids establishes connection between the pressure across the peaks of surface roughness and relative oil film thickness for lubrication $h/(R_{p1}+R_{p2})$, □[6]:

$$p_k = h_m /(R_{p1} + R_{p2}) \tag{4}$$

The equilibrium equation establishes connection between the outer loads and the established distribution of total pressure in the contact place:

$$F_{\mathrm{N}} = \int_{\Omega_c} p_{\mathrm{uk}}(x)dx \tag{5}$$

where F_{N} *i*s outer normal force.

Energy equation describes stability state between the heat produced by friction in the contact place and the heat carried away across contact bodies and is used to determine the coefficient of friction value on the border surfaces solid-oil film:

$$\frac{\partial}{\partial y}\left[\lambda\frac{\partial \vartheta}{\partial y}\right] + \frac{\eta_s^2}{\eta h^2}\left(v_{\mathrm{kx}}^2 + v_{\mathrm{kz}}^2\right) = 0 \tag{6}$$

by using the approach for the calculation of temperature on border surfaces which is based on the solution by Carslaw and Jeager, used in [3,4].

The expression describing the viscosity dependence of pressure and temperature, suggested by Reynolds equation [11], is used in this analysis:

$$\eta = \eta_0 exp\left\{(ln\eta_0 + 9.67)\times\left[-1 + (1 + 5.1\times10^{-9}\,p)^{z_0}\left(\frac{t - 138}{t_0 - 138}\right)^{-s_0}\right]\right\} \tag{7}$$

The density dependence of pressure and temperature is represented by Dowson-Higginson's expression by the following equation [11]:

$$\rho = \rho_0\left[1 + \frac{C_1 p}{(1 + C_2 p) - C_3(t - t_0)}\right] \tag{8}$$

Where C_1=0.6·10-9 Pa^{-1}, C_2=1.7·10-9 Pa^{-1}, C_3=0.00065 K^{-1}.

2.2. Numerical solution of extended TEHD model of equations

Calculation of oil film thickness, pressure transmitted by oil film and solids, temperature distribution is obtained by simultaneous solving of the system of equations for extended TEHD model, (2) to (6). Simultaneous solution of the equation set for extended TEHD model is obtained numerically by using own computer program. The algorithm for solving this problem is shown in Fig. 2.

System of nonlinear integral-differential equations (2) to (5) is discredited by finite difference method, and linearly adopted by Newton's approach of direct influences and solved by iterative Gauss-Seidel's method. By iterative solving of unknown values of the extended TEHD model of equations, the oil film thickness of executed iterations is used to calculate pressure transmitted by the oil film by solving a set of linearly adopted Reynolds equations and also the pressure transmitted by solids using the extended contact equation of solids (4), [4]. The newly established value of total pressure is used to determine the temperature in the oil film by solving the energy equation according to the approach used in [4], and in the new iteration for the calculation of oil film thickness using the equation of deformation (2) and integration constant using the equilibrium of contact equation (5).

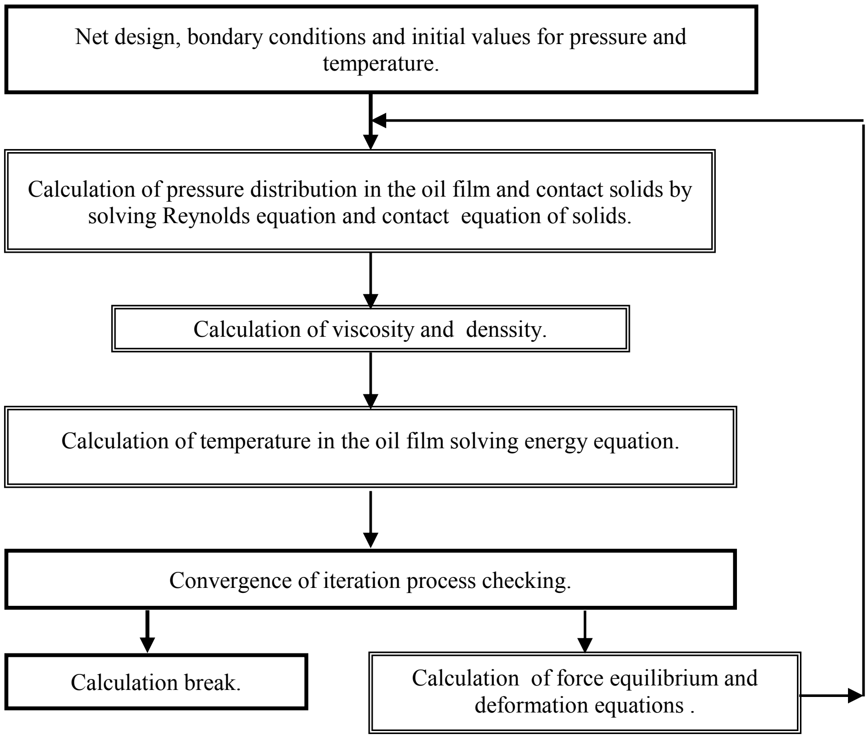

Fig. 2. Solution scheme of expanded TEHD model.

2.3. Friction coefficient of solids

For the calculation of limit temperatures, when solving energy equation it is necessary to know the heat fluxes on border surfaces (the solid/the oil film).

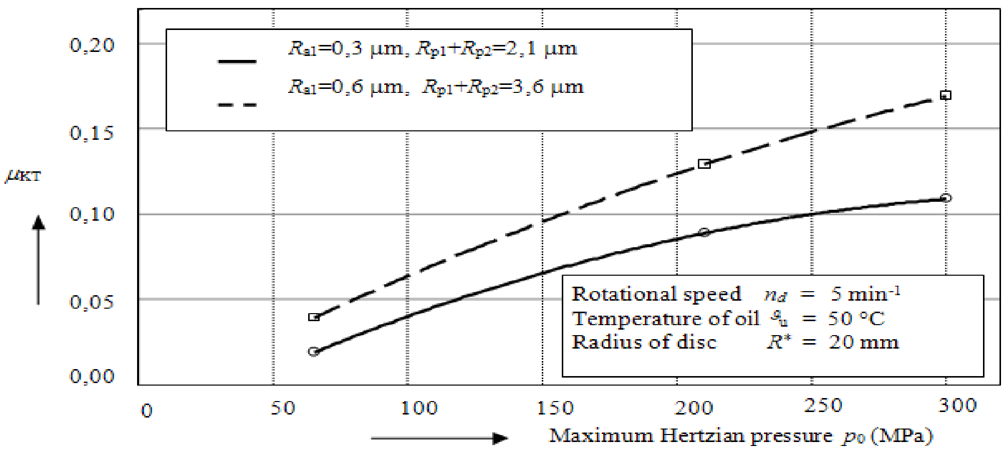

Fig. 3. The coefficient of friction of a single asperity for material combination bronze/ steel and mineral oil.

In order to take into consideration the heat produced by the friction of solids, the expression for heat flux calculation for ideal smooth surface had to be extended by a part of the heat flux caused by the friction of solids:

$$q_{\mathrm{KT}}(j) = \frac{v_{\mathrm{kT}}\ \mu_{\mathrm{KT}}(j)\, p_{\mathrm{k}}(j)}{2} \tag{9}$$

Functional dependence of the coefficient of friction of solids μ_{KT} on the condition of lubricated contact can only be obtained by experiments. The experimental research has been conducted on a model appliance by Timken [1-2]. Measurements of coefficient of friction of solids have been performed for low disc velocities i.e. when it is impossible to establish a more significant hydrodynamic effect and other conditions of experiment which will match the testing conditions of real worm gear drives. Experimental dependence of coefficient of friction of solids and loads is given in Fig. 3.

3. Calculation results of the mixed lubrication

Characteristics of the used lubricant, characteristics of the contact bodies and other parameters used for numerical analysis of line contact according to the described mathematical TEHD model are given in Tab. 1-2.

Table. 1. Parameters of kinematic motion, loads, geometry of contact solid and characteristics of oil for lubrication.

Meaning	Parametar	Dimension	Value
Maximum Hertzian pressure	p_0	N/m^2	$600 \cdot 10^6$
Velocities of contact solid 1	v_{1n}	m/s	1
Velocities of contact solid 2	v_{2n}	m/s	0,2
Sliding velocity	v_{kT}	m/s	0.8
Relative radius of roller	R^*	m	0,01
Maximum surface roughness	$R_{p1}+R_{p2}$	μm	0 (Fig. 4) 1,1 (Fig. 5) 2,2 (Fig. 6)
Oil for lubrication	ϑ	°C	50
Temperature of contact solids	ϑ_{k1}, ϑ_{k2}	°C	50
Viscosity in ambient conditions	η_o	Pa s	0,224
Density of oil in ambient conditions	ρ_o	kg/m^3	870
Thermal conductivities	λ	W/mK	0,15
Pressure-viscosity coefficient	α	m^2/N	$1{,}98 \cdot 10^{-8}$
Pressure-viscosity index	z	-	0,476
Roeland's thermo viscous parameter	s_o	-	1,54

Table. 2. Characteristics of materials of contact solids.

Meaning	Parametar	Dimension	Value	
			Steel	Tin bronza
Density	$\rho_{K1,2}$	kg/m^3	7850	800
Specific heat of contact solids	$C_{K1,2}$	J/kgK	461	380
Thermal conductivities	$\lambda_{K1,2}$	W/mK	52	48
Poisson's ratio	$\nu_{1,2}$	(-)	0,3	0,33
Modulus of elasticity	$E_{1,2}$	N/m^2	$206 \cdot 10^9$	$88 \cdot 10^9$

In the first numerical example (Figure 4) the case of contact with smooth surfaces has been simulated i.e. when the sum of roughness $R_{p1}+R_{p2}=0$. Pressure distribution in the contact zone is given in Fig. 4a, while the temperature distribution and the oil film thickness are shown in Fig. 4b. The results of the simulation of contact with smooth surfaces, which are obtained using the developed program, show good compliance with the results of other authors [2, 4, 11] and, in the same time, the program itself is stable concerning the convergence of solutions.

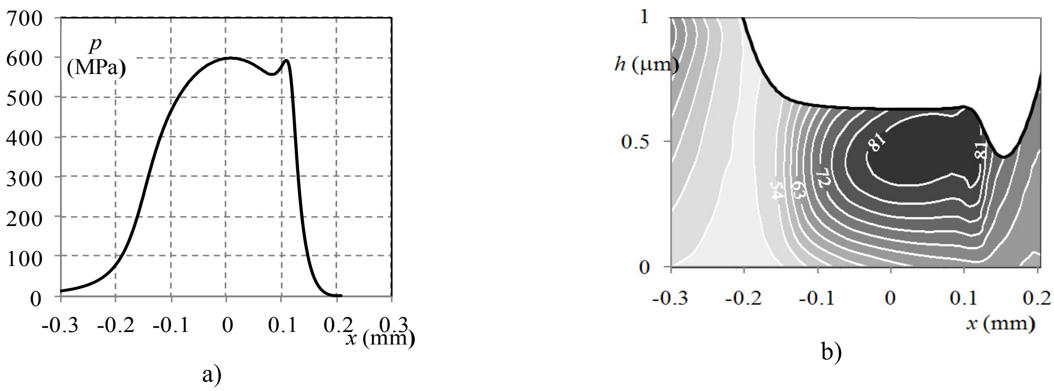

Fig. 4. TEHD case of contact with ideally smooth surfaces: (a) pressure distribution and; (b) film thickness and temperature conturs.

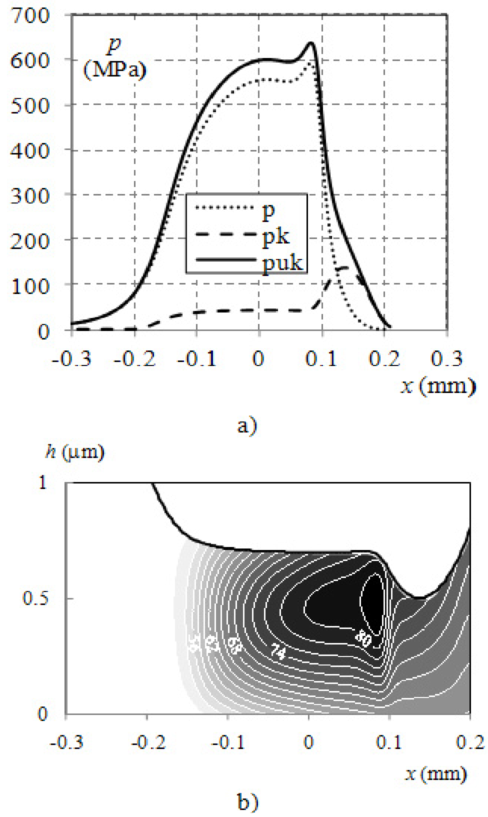

Fig. 5. Mixed lubrication: (a) pressure distribution and; (b) film thickness and temperature conturs.

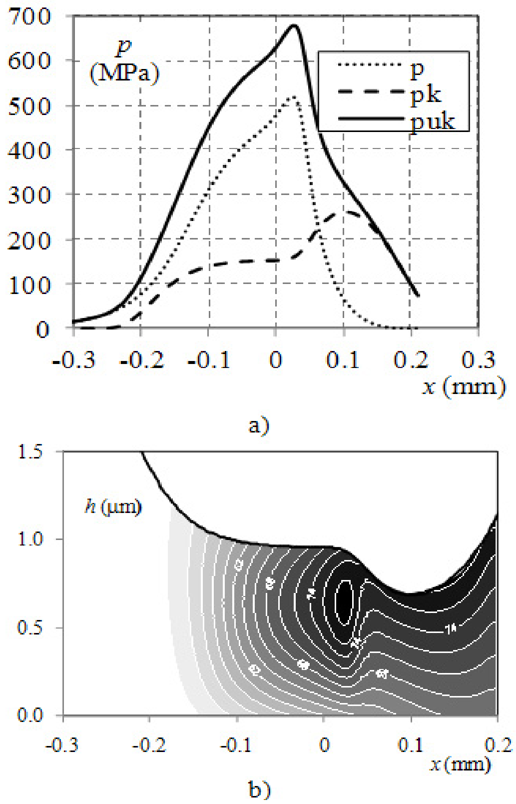

Fig. 6. Mixed lubrication: (a) pressure distribution and; (b) film thickness and temperature conturs.

In the second numerical example (Fig. 5 and Fig. 6) the case of contact with rough surfaces has been simulated, i.e. when the sum of maximal roughness of both bodies in contact $R_{p1}+R_{p2}$=1,1 μm (Fig. 5), $R_{p1}+R_{p2}$=2,2 μm (Fig. 6), and the coefficient of friction by solids μ_{KT}=0,1, at identical input parameters as in the previous example. Fig. 5a(6a) shows pressure distribution in the oil film p, pressure transmitted by solids across peaks of roughness p_k, and the distribution of total pressure p_{uk} in the contact zone, while the temperature distribution and the oil film thickness are shown in Fig. 5b (6b). By comparing the three cases the change of the oil film thickness and the temperature distribution in the oil film becomes evident. Developed a program for contact with rough surfaces is stable concerning the convergence of solutions.

4. Conclusion

The aim of this work is to mathematical modeling of lubricated contact in case of mixed friction. To achieve this, it was necessary to extend the TEHD equation system for ideal smooth surfaces by an additional equation of contact of solids and to model the experimental dependence of coefficient of friction of solids when solving energy equation.

According to the exposed mathematic model of TEHD line contact, a personal computer program was developed for the analyse of pressure distribution in fluid films, and estimating fluid film thickness and temperature distribution in the oil film. The analyses carried out point to the conclusion that the presented mathematical model, which has been numerically solved using the developed computer program, is suitable for :

- Calculation of oil film thickness, pressure distribution in the oil film under mixed friction conditions by extending the TEHD lubrication model of equations for ideally smooth surfaces,
- Calculation of temperature distribution in the oil film, under mixed friction conditions, by including experimentally formed dependences of coefficients of friction of solids into the TEHD model of equations for ideally smooth surfaces,

The mathematical model numerically solved by the developed computer program could, with slight modifications, be applied for solving problems contact with power transmission.

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