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OPTIMISED KINEMATIC MOUNT CONFIGURATION FOR HIGH-PRECISION APPLICATIONS

ZELENIKA, S[asa]; MARKOVIC, K[ristina] & RUBESA, J[osipa]

Abstract: *The generic kinematic mount design configuration, designated as the Maxwell-type kinematic mount, is constituted by three V-grooves on one end and three balls on the other so as to achieve an exact constraint of all six degrees of freedom. The analysis of this coupling configuration comprises force and moment balance equations, as well as expressions for stress-strain and error motion calculations. For a determined external load, the geometry of the mount will thus imply the loads at each groove-ball interface and the respective contact point reactions. The calculation comprises the necessity to deal with the non-linear Hertzian theory of point contacts.*
This work recalls the limits of applicability of the available analytical approaches for the calculation of ball-V groove couplings employed in ultra-high precision positioning. The analytical results are validated experimentally. In the whole range of elastic deformations the correspondence of the theoretical values with the experimental ones is within the intervals of uncertainty of the latter.
The calculation procedure is then used to assess the optimal characteristics of a kinematic mount employed to support a large vacuum chamber of a particle accelerator facility. Stability conditions for different design configurations are established.
Keywords: *kinematic mount, stability, model, optimisation*

1. INTRODUCTION

Kinematic mounts are used in high-precision applications since they are self-locating and free from backlash, allow sub-micrometric re-positioning in static and dynamic applications, can accommodate differential thermal expansions and their behaviour can be represented in a closed form solution [1]. The main drawback of kinematic mounts is constituted by the high contact stresses that can be analysed only by employing the nonlinear Hertz theory [2].

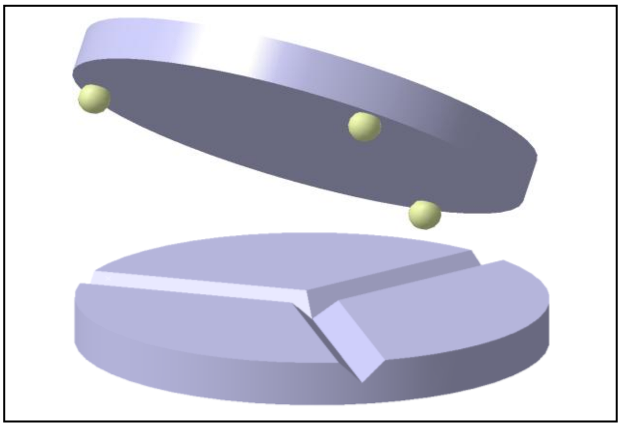

Fig. 1. Maxwell kinematic mount

The most common kinematic mount design configuration is the Maxwell-type mount constituted by three V-grooves on the support and three balls on the supported piece, so as to achieve an exact constraint of all six spatial degrees of freedom (Fig. 1).

The aim of this work is to analyse the influence of mechanical parameters on the behaviour of the considered class of kinematic mounts and especially on their positioning precision and stability. An example of a mount used to support a large structure at a particle accelerator facility is then considered. Stability conditions for different design configurations are established.

2. MODEL OF BEHAVIOUR AND ITS VALIDATION

The analysis of a Maxwell-type kinematic mount comprises force and moment balance equations, expressions for the calculation of stresses and deflections at the contact points and error motion calculation. Knowing the external loads (including the preload acting on the coupling) and the geometry, the loads at each groove-ball interface and the respective contact point reactions can be computed from the overall force and moment balances [1].

Hertz theory describes the nonlinear behaviour of point contacts between elastic isotropic solids loaded perpendicular to the surface, where the contact area is small compared to the radii of curvature and the dimensions of the involved bodies. The respective analytical model entails a lengthy iterative evaluation of transcendental equations involving elliptic integrals [3, 4, 5]. The approximated methods suggested in literature, where the need to calculate the elliptic integrals is obviated by introducing polynomial [1], tabular [6, 7, 8] or graphical [8, 9] approximations of the characteristic parameters, are appropriate for most high-precision applications.

In fact, as depicted in Fig. 2, the introduced errors are always smaller than ± 2%. Given the small entity of the stresses and strains involved in most high-precision applications, these errors can hence be considered negligible in all but those cases where true nanometric accuracies are sought. Only in the case when the curvature of the grooves approaches that of the balls, the errors tend to become appreciable. In this case, however, Hertz theory itself starts to break down [3].

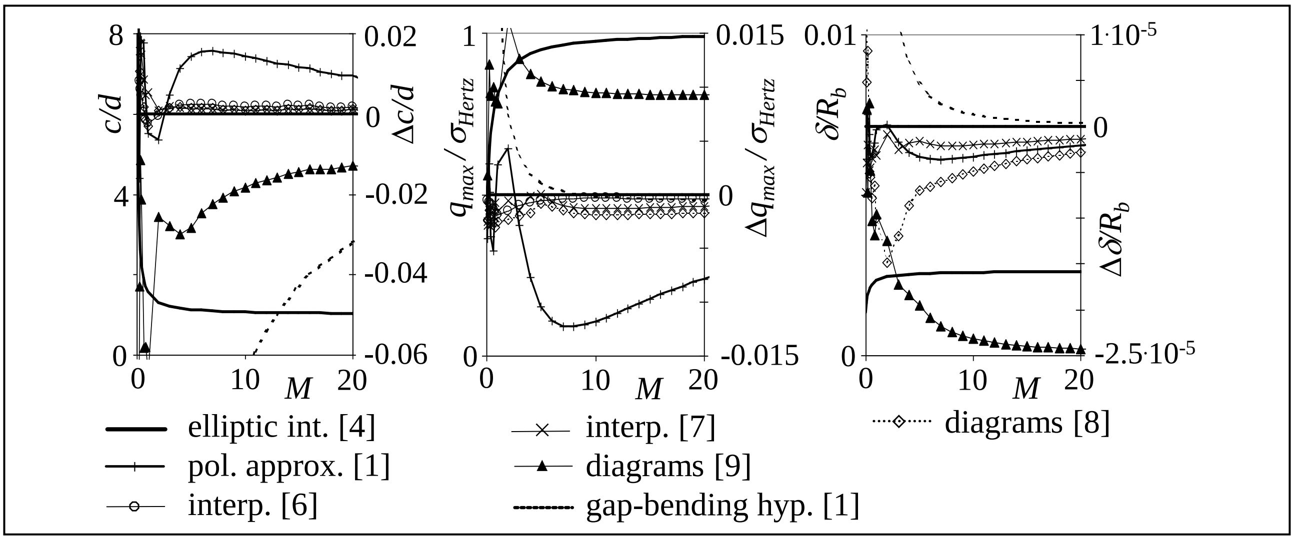

Fig. 2 Ratio of lengths of elliptical contact area, normalized contact stresses and normalized interpenetration distances versus the ratio of the groove and ball radii of curvature *M* calculated with the available analytical models [3]

To validate these results, an experimental set-up was built (Fig. 3). Stainless steel and ceramic (tungsten carbide (WC) and silicon nitride (Si_3N_4)) polished balls and gothic-arch grooves were employed. It was thus proven that in the whole elastic deformations range the correspondence of the theoretical values of the interpenetration distances δ with the experimental ones is within the intervals of uncertainty of the latter, regardless of the used materials and lubrication conditions. The precision of the kinematic mount is thus shown to be comparable to the surface finish of the coupling interface (100 nm range) – Fig. 4 [3].

The calculated stresses and strains allow next, under the assumptions that the change of the distances between the KM supports is small, to calculate couplings' error motions about its centroid [1] – Fig. 5.

3. STABILITY CONDITIONS OF A LARGE STRUCTURE MOUNT

The calculation procedure is hence implemented in structured software algorithms and used to assess the characteristics of a Maxwell kinematic mount used to support a large vacuum chamber of a particle accelerator facility. In fact, as extensively elaborated in [10], these structures are characterized by extremely stringent design requirements in terms of their positioning and repositioning accuracies and precisions.

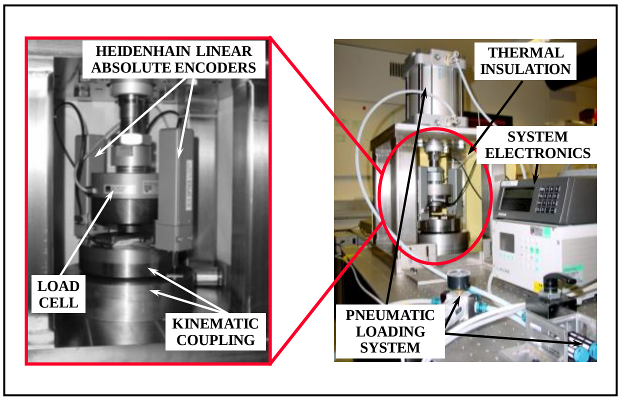

Fig. 3 Experimental set up [3]

In the considered case, the input design data define the maximal support area as a 1 m diameter while the load to be supported is 5 kN with an additional 300 N lateral load at a 1.5 m height from the contact points of the mount. In accordance with the performed experiments, the elements of the mount itself are chosen having Si_3N_4 balls and WC V-shaped grooves.

The stress calculations allow establishing that, for a certain coupling radius R_C and a maximal allowable stress q_{all} [11, 12], suitable ball radii are R_B = 18 mm (Fig. 6), V-groove arch radii are R_G = - 21.6 mm and the overall coupling radius is R_C = 300 mm.

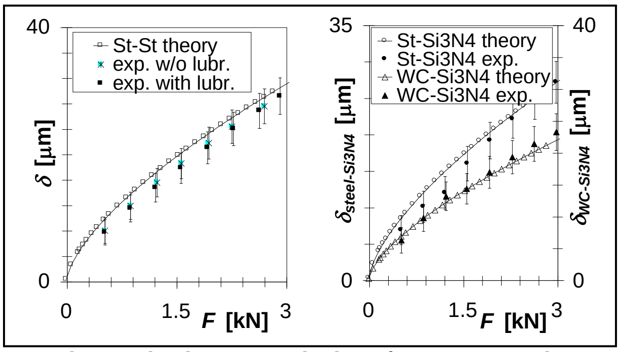

Fig. 4 Theoretical and experimental values of interpenetration distances δ [3]

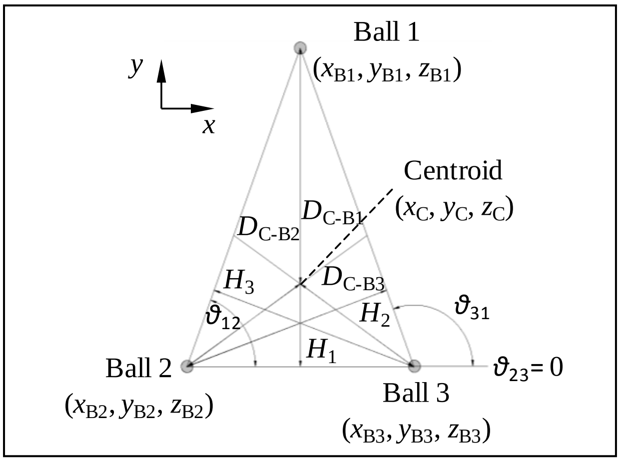

Fig. 5. Top view of the geometry of the mount

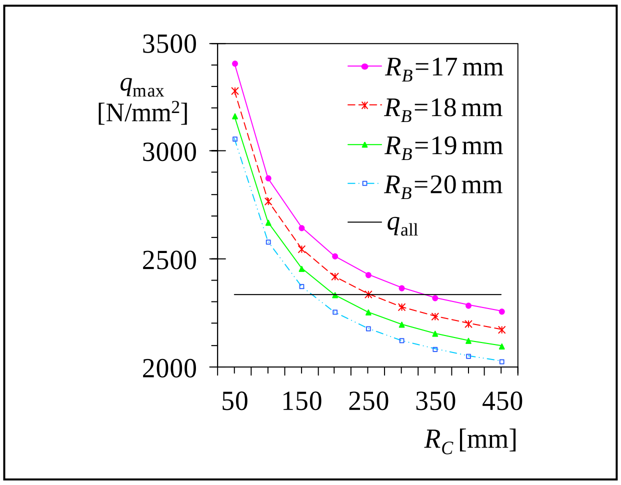

Fig. 6. Dimensioning of the elements of the considered kinematic mount configuration

In terms of the stability of the considered mount, two design configurations suggested in [1] were considered:

- configuration A: the normals to the planes containing the contact forces are directed towards the centroid of the coupling – Fig. 7;
- configuration B: the normals bisect the angles between the balls – Fig. 8.

On these figures is to be noted the definition of an auxiliary angle δ linked to the geometry of the mount and its length-to-width ratio. This angle corresponds to the angle between the segment connecting the centroid of the mount with, respectively, the centre of ball number 1 and that of ball number 2.

Obviously, the support will lose its stability when one of the contact forces F_{Ki} becomes negative. The stability conditions could thus be determined for different geometries of the considered kinematic mount and for various lateral load orientations and their points of application.

As typical examples, on the figures below are depicted the stability regions for both considered design configurations in the case when the lateral load passes through the centroid of the kinematic mount but:

- the lateral load is directed as the positive *x*-axis of the mount (Fig. 5 and Fig. 9),
- the lateral load is directed as the negative *x*-axis (Fig. 10);
- the lateral load is directed as the positive *y*-axis (Fig. 11).

Obviously, the contact force(s) significant for the loss of the stability of the mount will, as shown in the figures, be different for the different considered loading cases.

As a further interesting variant, in Fig. 12 is shown the change of the entity of contact force F_{K2} in the design configuration A depending on the angle δ as well as on the direction of the lateral load with respect to the *y*-axis of the mount (angle β_V).

From all these considerations it could hence be clearly shown that, from the stability point of view, when the length of the mount is extended with respect to its width, design configuration B is generally better.

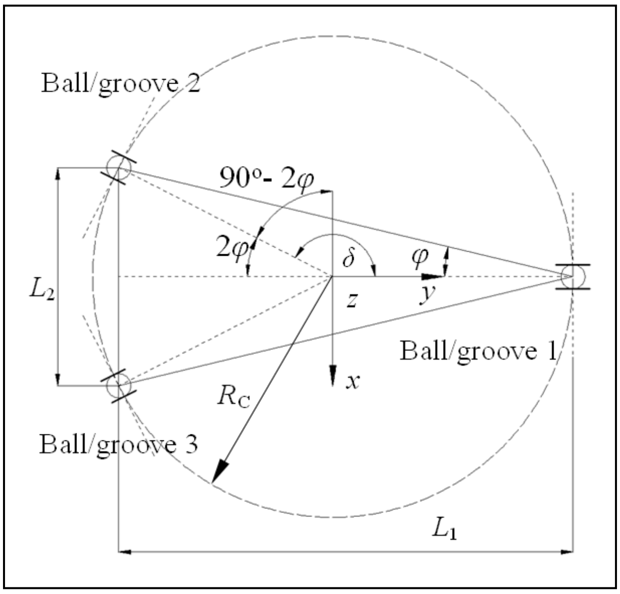

Fig. 7. Design configuration A

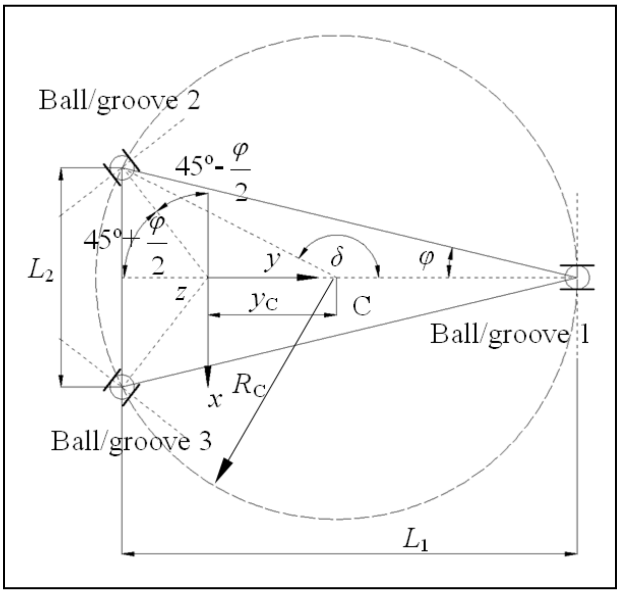

Fig. 8. Design configuration B

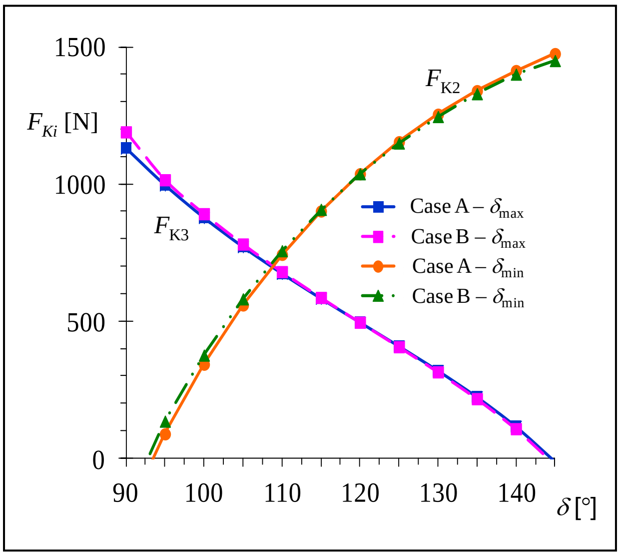

Fig. 9. Stability regions when the lateral load passes through the centroid of the mount and is directed as its positive *x* –axis

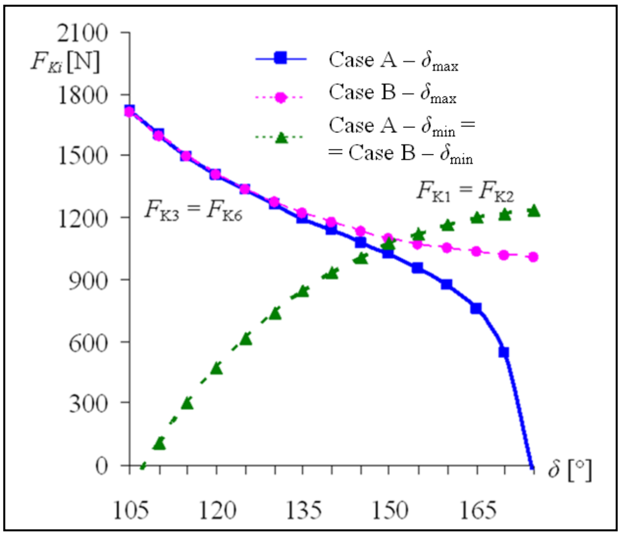

Fig. 10. Stability regions when the lateral load passes through the centroid of the mount and is directed as its negative *x*-axis

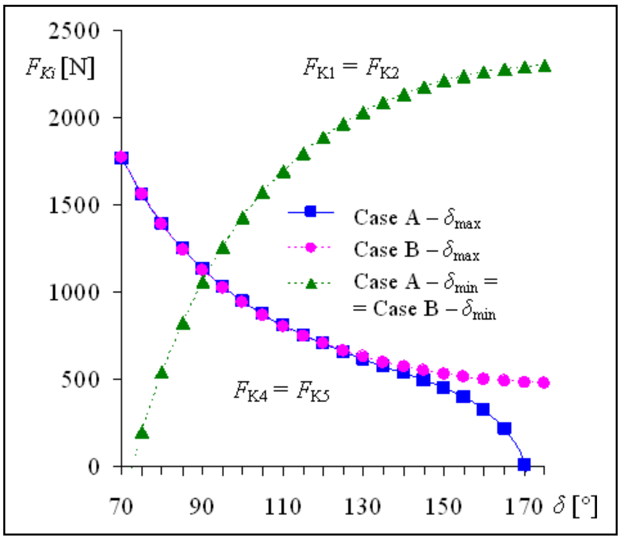

Fig. 11. Stability regions when the lateral load passes through the centroid of the mount and is directed as its positive *y*-axis

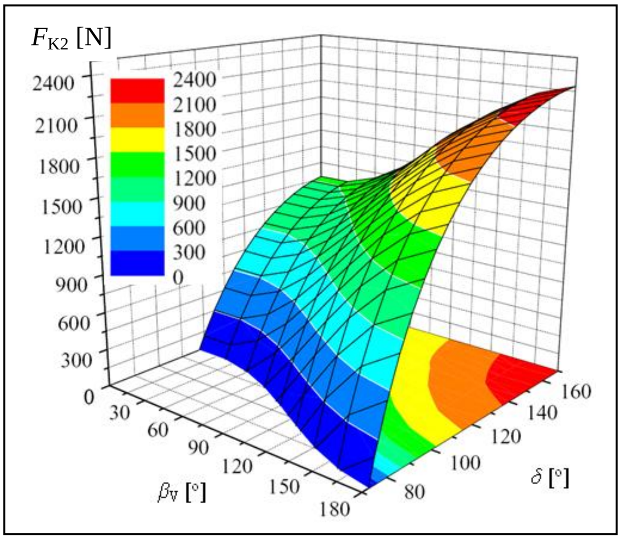

Fig. 12.Variation of the contact force F_{K2} for design configuration A depending on the geometry of the mount (angle δ) and the direction of the lateral load (angle β_v).

4. CONCLUSIONS

Thorough analyses of the precision and the stability of a Maxwell-type kinematic mount were performed.

In terms of precision, it was shown that the approximated analytical approaches available in literature are giving accurate results. Experimental measurements allowed, in turn, establishing that the correspondence of the theoretical values with the experimental ones is within the intervals of uncertainty of the measurements. The repeatability of the kinematic mount was thus shown to be comparable to the surface finish of the used elements, i.e. in the 100 nm range.

In terms of the stability, various design configurations and lateral load conditions of a specific design example were considered. By employing suitable software algorithms it was thus shown that, when the length of the kinematic mount is extended with respect to its width, the configuration where the normals to the planes containing the contact forces bisect the angles between the balls is generally better from the configuration where the normals are directed towards the centroid of the coupling.

In the near future the optimised configuration will be built and installed at a synchrotron radiation particle accelerator facility, where its suitability for the foreseen application will be confirmed.

5. ACKNOWLEDGEMENTS

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