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DEVELOPMENT OF THE IMPROVED METHOD OF GRIDS

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Abstract: *The purpose of this paper is to develop and generalize the improved method of grids (method of finite differences) described in (Kollats, 1969) to calculation a partial derivative. The method of uncertain coefficients is applied to the two dimensional boundary value problems. For obtaining of interpolation polynomials, the matrixes are used. The essential simplification of the calculation formulae is received. The numerical results are presented.*

Key words: *method of grids, matrix equations, interpolation polynomials, method of uncertain coefficients*

1. INTRODUCTION

Approximation by a method of uncertain coefficients and interpolation of derived function (Aryassov et al., 2010) can be extended to calculation a partial derivative. Let's describe one possible variant. Let's accept additional notations and consider the purpose of simplification a two-dimensional problem only. We assume that a function U of two variable is transformed using the dimensionless abscissa ξ and ordinate η. The integer variables ξ and η form a square grid. Nodes of the grid are defined by two indexes. The first index corresponds to integer abscissa ξ, second–to ordinate η. It is expedient to join points laying on the same horizontal or vertical, that means having one and same abscissa or ordinate. Such association of points we shall name a vector–abscissa or vector–ordinate. The generalized matrix is applied function (Aryassov et al., 2010), but two indexes to it are added. They are written to specify coordinates of a point or vector–abscissa or a vector–ordinate, along which the partial derivative are calculated.

The essential simplification of the calculation equation is received with the help of matrix notable symbolic. The used matrix symbolic gives the convenient tool for realization of calculations with the help of computers.

2. TWO DIMENSIONAL PROBLEM

The above-mentioned transformation is carried out by Eq. 1.

$$x = a + \frac{b-a}{n}\xi\ ;\ 0 \le \xi \le n\ ;\ \xi = \frac{n(x-a)}{b-a}\ ; \qquad (1)$$

The integer variables ξ and η form a square grid and are given in Fig.1.

Nodes of the grid are defined by two indexes. The first index corresponds to integer abscissa ξ , second–to ordinate η . It is expedient to join points laying on the same horizontal or vertical, that means having one and same abscissa or ordinate. Such association of points we shall name a vector-abscissa, with ordinate k and to denote $\{U_{jk}\}$ or vector-ordinate with abscissa i and to denote $\{U_{i\eta}\}$, where k = 0, 1, 2, …, r; $r \le n$; i = 0, 1, 2, …, s; $s \le n$.

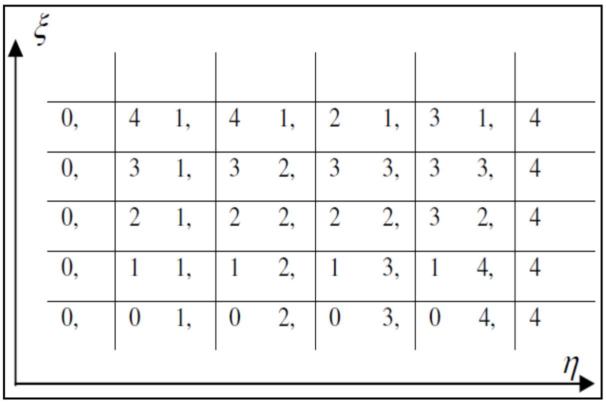

Fig. 1. The form of a square grid

Further in abbreviated form, accordingly "a vector *k* "and "a vector *i*".

The symbol $\left[O_n^{(m)}\right]$ remains, but two indexes to it are added. They are written in brackets at the left and specify coordinates of a point or a vector-abscissa or a vector-ordinate, along which the partial derivative are calculated. The upper index shows, according to the accepted notation, the order of the partial derivative. Interpolation polynomial, appropriate to a vector $\{U_{i\eta}\}$, is differentiated on η .

Let's consider an example, where the interpolation is carried out on five points, that is n=4. We assume that it is necessary to find the second partial derivative from the interpolation polynomial in the point (2, 1) (Korn & Korn, 2000).

$$_{(2,1)}\left[O_4^{\partial^2/\partial\xi^2}\right]U = {}_{(2,1)}\left[O_2''\right]\{U_{\xi 1}\} =$$

$$= \{0\,0\,2\,12\,48\}^T[W]^{-1}\{U_{\xi 1}\} =$$

$$= \ldots\frac{1}{24}\{-2\ 32\ -60\,32\ -2\}^T\{U_{\xi 1}\} =$$

$$= \frac{1}{2}\{-2\ 32\ -60\,32\ -2\}^T\begin{pmatrix}U_{01}\\ U_{11}\\ U_{21}\\ U_{31}\\ U_{41}\end{pmatrix} =$$

$$= \frac{1}{24}\left(-\,2U_{01}\,32U_{11} - 60U_{21}\,32U_{31} - 2U_{41}\right) \qquad (2)$$

To increase the accuracy of calculation of this derivative $_{(4,2)}\left[O_4^{\partial^2/\partial\xi^2}\right]U$ in the point (4, 2) must transfer the

beginning of coordinates of the interval [a, b] to the point (4, 2). It will correspond to differentiation of the interpolation polynomial, which is defined by a vector $\{U_{\xi 2}\}$, x = 2, 3, 4, 5, 6.
The above-mentioned derivative can be calculated as

$$
{}_{(4,2)}\left[O_4^{\partial^2/\partial\xi^2}\right]U = {}_{(4,2)}\left[O_4''\right]\{U_{\xi 2}\} =
$$

$$
= \{0\,0\,2\,12\,48\}^T [W]^{-1} \{U_{\xi 2}\} =
$$

$$
= \ldots \frac{1}{24}\{-2\;32\;-60\,32\;-2\}^T \{U_{\xi 2}\} =
$$

$$
= \frac{1}{2}\{-2\;32\;-60\,32-2\}^T \begin{pmatrix} U_{22} \\ U_{32} \\ U_{42} \\ U_{52} \\ U_{62} \end{pmatrix} =
$$

$$
= \frac{1}{24}\left(-2U_{22}\,32U_{32} - 60U_{42}\,32U_{52} - 2U_{62}\right) \qquad (3)
$$

The calculation of partial derivative with respect of variable η is made in similar way.
Mixed partial derivative are calculated by the same rules, which are applied to interpolation polynomials along axes ξ and η.
In case of numerical differentiation of ξ, it is necessary to use so much vectors abscissa, how many points the used operator demands at numerical differentiation of η. For example, at n = 4 five vectors are required.
The result of calculation mixed derivative does not depend on the order of differentiation of ξ and η.
For increase of accuracy it is necessary to use overlapping of intervals. From general formula proposed in this paper we receive the special case of well-known result (Korn & Korn, 2000).

Such approach for calculation derivative allows rather easily changing a step of a grid and number of nodes taken into account. In the case of change of the step it is necessary to resort to overlapping of intervals so that in any point the nodes of two next, adjacent intervals would coincide.
In case of discrepancy of nodes it is necessary to calculate the interpolation polynomial values on square-law interpolation or more exact.

3. CONCLUSIONS

The received formulae allows to carry out the approximation of functions and their derivatives not resorting to differences as it is made in a classical method of grids (method of finite difference). The use of overlapping of interpolation intervals allows increasing an accuracy of the solution.
The calculation results show that it is possible to adjust the accuracy of the solution either by changing the degree of the interpolation polynomial or with the help of overlapping of intervals. That is the main difference not only from usual, but also from the “improved” method of grids. The essential simplification of the calculation formulae is received; in particular case they are the L. Kollats' formulae. Their derivation is carried out with the help of matrix notable symbolic.

The received results can be applied to the solution of boundary problems of various classes, for example problem of eigenvalues of orthotropic plates (Aryassov & Petritshenko, 2009). The last is especially urgent in vibrodiagnostic of structures, designs, machines, equipment, industrial and civil buildings and so on (Gere & Timoshenko, 1997). In particular, it is supposed to use the given approach for calculation of the stress condition in threaded joints (Aryassov & Petritshenko, 2008) and (Aryassov & Strižak, 2000). The used matrix symbolic gives the convenient tool for realization of calculations with the help of computers.

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