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THEORETICAL MODEL OF UNWINDING PROCESS FROM PACKAGES

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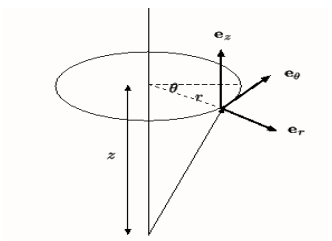
Abstract: *Yarn unwinding from packages plays an essential role in many textile processes. The stability of unwinding has a direct influence on the efficiency of the textile processes in general and on the quality of the final products. Calculations based on theoretical model of unwinding can help us in our pursuit of the optimal design of the packages and the optimal unwinding process. In the article we will derive the system of differential equations that describe unwinding of the yarn.*
Key words: *yarn unwinding, packages, balloon theory, yarn tension*

1. INTRODUCTION

In production of garments thread unwinding exists in a sewing process. In order to achieve low and constant tension of thread or yarn it is necessary to optimize the process of unwinding. Computer simulation are now in use for this purpose, so it is important to obtain a mathematical description of yarn motion (Barr&Catling, 1976, Padfield, 1958).

2. KINEMATICS



We introduce a cylindrical coordinate system that rotates around the z axis with an angular velocity w. The yarn is parametrised with arc length s (s is therefore the length of yarn from the origin of the coordinate system to the given point on the yarn). The coordinates of a point are given by r, the radial distance from the axis, q, the polar angle and z, the vertical distance from the origin (Kong, 1997, Kong et al.,1999).

Fig.1: Cylindrical coordinate system rotates around the z axis with an angular velocity w.

It should be kept in mind that each point has its own triplet of base vectors e_z, e_q, e_r, respectively pointing in vertical, tangential and radial direction. The radius vector pointing to a point on the yarn can be decomposed along radial and vertical directions (the polar angle dependence is hidden in the e_r vector):

$$\mathbf{r}(s,t) = r(s,t)\mathbf{e}_r(\theta(s,t),t) + z(s,t)\mathbf{e}_z \qquad (1)$$

We have emphasized that coordinates of a point depend explicitely on both the time of observation t and on the arc length s, where the point is located at given time t.

The velocity of a point on a yarn that is being withdrawn (with withdrawing speed V) is given by the total time derivative:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{\partial \mathbf{r}}{\partial t} + \frac{\partial \mathbf{r}}{\partial s}\frac{\partial s}{\partial t} \qquad (2)$$

It is important to note that the velocity is not given by the local (partial) time derivative, denoted by µr/µt. This derivative does not take into account that in the infinitesimal time Dt the point moves to a different position along the yarn (i.e. to a d+ifferent arc length s). The contribution to velocity due to this movement is described by the additional term µr/µs µs/µt. The withdrawing speed is equal to V=-µs/µt and we obtain the following expression:

$$\mathbf{v} = \dot{\mathbf{r}} - V\frac{\partial \mathbf{r}}{\partial s} \qquad (3)$$

where the dot denotes the partial derivative with respect to time. Itôs worth noting that t=µr/µs is the unit tangential vector to the yarn. Indeed the direction of the withdrawing velocity at a given point should be in the direction of the yarn.

To calculate the time derivative of the radius vector we make use of a relation between derivatives in an inertial and a rotating frame:

$$\left(\frac{\partial}{\partial t}\right)_K = \left(\frac{\partial}{\partial t}\right)_{K'} + \boldsymbol{\omega} \times . \qquad (4)$$

When applied to a base vector, that is rotating around the Z axis together with the yarn, this equations gives

$$\frac{\partial \mathbf{e}_i(t)}{\partial t} = \boldsymbol{\omega} \times \mathbf{e}_i(t) \qquad (5)$$

The partial time derivative of the radius vector is then found to be:

(6)

The final expression for the velocity of a point is of the form

$$\mathbf{v} = \mathbf{v}_{\mathrm{rel}} + \boldsymbol{\omega} \times \mathbf{r} - V\frac{\partial \mathbf{r}}{\partial s} \qquad (7)$$

The three contributions to the velocity of the point have very simple physical interpretations. The first term is the relative velocity in the non-intertial frame; it describes how the form of the yarn is changing from the point of view of an observer that is rotating together with the yarn, but it is not equal to the velocity of a given point in the non-intertial frame. (This term is dropped in the quasi-stationary approximation that we describe below.) The second term is the circular velocity of the point due to the rotation of the frame; this is the velocity of a point that is fixed in the non-inertial frame. Finally, the last term is the withdrawing velocity that we introduced above.

By analogy, the acceleration of a point is given by the total time derivative of the velocity. By a lengthy but straight-forward calculation we obtain the following expression:

$$\mathbf{a} = \mathbf{a}_{\mathrm{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{\mathrm{rel}} - 2V\boldsymbol{\omega} \times \frac{\partial \mathbf{r}}{\partial s} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r} - 2V\frac{\partial \mathbf{v}_{\mathrm{rel}}}{\partial s} + V^2\frac{\partial^2 \mathbf{r}}{\partial s^2} \qquad (8)$$

This complex expression can be given more compact form if we introduce a differential operator D, which follows the motion of the point in the rotating frame(Fraser &Ghosh&Batra,1992):

$$D = \frac{\partial}{\partial t} - V\frac{\partial}{\partial s}. \qquad (9)$$

The fact that this operator ñfollows the motion of the point in the rotating frameò means, that the partial time derivative operator only operates on the coordinates of the point (r, q, z), but it gives zero when applied on the base vectors e_z, e_q, e_r.