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MODEL OF PIPELINE WITH PUMP FOR PREDICTIVE CONTROL

KOVAR, J[iri] & BREZINA, T[omas]

Abstract: *The present paper deals with the model of pipeline with pump for large-scale networks based on hydro-electrical analogy. For this purpose, the equation describing one-dimensional flow in the pipeline and the equation describing the water pump were defined. A numerical solution of governing equation is presented at the end of article.*
Key words: *hydro-electrical analogy, pipeline model, large-scale network, numerical model*

1. INTRODUCTION

The model of pipeline with pump is a fundamental part of the pipeline network model for water supply (Zhu et.al., 2001). Such a model then can be used for predictive control of water pumps in the supply network (Anderson, 2009).

For numerical modelling of flow in pipelines it is necessary to have a mathematical model. Usually, such a model is represented by two partial differential equations (Matko et.al., 2000; Habán & Pochylý, 2000; Blažič et.al., 2004). The general closed-form solution of these equations has not been known yet. The solution is then obtained by using of numerical methods. Another approach is to use the hydro-electrical analogy (Tao & Ti, 1998).

The numerical model of pump is usually represented by differential equations (Wozniak, 1990). For purpose of modelling is usually used state space model or expression by transfer functions.

The subject of this paper is a mathematical model of pipeline with pump.

2. MODEL OF PIPELINE

The analogy between electrical and fluid networks has been known for a long time and is successfully used for simulating pipeline networks (Ke & Ti, 1999). The method is based on the fact that the mathematical descriptions of the various systems are qualitatively very similar. This allows us to apply knowledge from different fields of science to others.

Hydro-electrical analogy is based on these relations:

$$p \approx U \tag{1}$$

$$Q \approx i \tag{2}$$

where p is pressure, U is voltage, Q is flow rate and i is current.

Based on the analogy of these physical quantities, hydraulic resistance, hydraulic inductance and hydraulic capacity are defined. These three elements are called a hydraulic resistances.

2.1 Model of pipeline

Modelling of pipe is based on the theory of quadrupole from discipline of electricity. There are multiple ways of how to sort hydraulic resistances into one model. In this paper, L-segment will be used.

For computational purpose the pipeline should be divided into several segments. Number of segments must satisfy the condition of stability:

$$\frac{lf}{c_S} < n < 10\frac{lf}{c_S} \tag{3}$$

where l is length of pipeline, c_S is speed of sound in medium, f is maximum rate of pressure change and n is number of segments.

This segment consists of a hydraulic resistance (R), inductance (L) and capacitance (C). For the purpose of developing the pipeline model we can modify the equation describing of the RLC-segment from the theory of circuits.

Each segment is characterized by two variables – pressure and flow rate.

$$\dot{p}_i = \frac{1}{C_{Hi}}(Q_{i-1} - Q_i) \quad \text{for} \quad i = 1,2,\dots,n+1 \tag{4}$$

$$\dot{Q}_i = \frac{1}{L_{Hi}}(p_i - R_{Hi}Q_i - p_{i+1}) \quad \text{for} \quad i = 1,2,\dots,n \tag{5}$$

where i is a number of the segment. R_H used in equation (4) must be linearized. Equations (4) and (5) are the governing equations for transient pipeline.

2.2 Hydraulic resistances

The resistance in a pipeline depends on several factors (Tao & Ti): roughness of the pipe, viscosity of the fluid, type of the flow and geometry of pipe. The mathematical model of resistance in a pipeline depends on the equations describing this effect. These can generally be defined as (Kozubkova, 2009):

$$R_H = \frac{d\Delta p}{dQ} \tag{6}$$

where R_H is hydraulic resistance and Δp is pressure drop between output and input of pipeline.

The inductance in a pipeline depends on physical properties of the transmitted medium.

Generally can be defined as (Kozubkova, 2009):

$$L_H = \frac{\Delta p}{\frac{dQ}{dt}} \tag{7}$$

where L is hydraulic inductance.

The capacity in a pipeline also depends on physical properties of the transmitted medium.

Generally can be defined as (Kozubkova, 2009):

$$C_H = \frac{Q}{\frac{d\Delta p}{dt}} \tag{8}$$

where C is hydraulic resistance.

3. MODEL OF PUMP

The model of pump can be described, like Wozniak (Wozniak, 1990), by four differentional equations. First equation describes rotor dynamics:

$$T_a \dot{n} = m - (M_p + cn) \tag{9}$$

where T_a rotations inertia time constant, n is angular velocity, m is torque, c is load constant and M_p is motor torque.

Second equation describe conduit:

$$-\Delta p = T_w \dot{q} \tag{10}$$

where Δp is pressure drop between output and input of hydrogenerator, T_w is a conduit time constant and q is a flow rate.

Third a fourth equations are describing moment and flow of the pump:

$$m = \frac{\partial m}{\partial \Delta p}\frac{\Delta p}{\rho} + \frac{\partial m}{\partial n} n \tag{11}$$

$$q = \frac{\partial q}{\partial \Delta p}\frac{\Delta p}{\rho} + \frac{\partial q}{\partial n} n \tag{12}$$

where ρ is density of transmitted media.

This equations are linear, thereby can be expressed by transfer functions. First transfer function describes transfer from pressure drop to flow rate, second transfer from motor torque to flow rate (s is Laplace variable):

$$F_{q\Delta p}(s) = \frac{q}{\Delta p} = \frac{sT_a \frac{\partial q}{\partial \Delta p} + \left[\frac{\partial m}{\partial \Delta p}\frac{\partial q}{\partial n} + \left(c - \frac{\partial m}{\partial n}\right)\frac{\partial q}{\partial \Delta p}\right]}{sT_a \rho + \left(c - \frac{\partial m}{\partial n}\right)\rho} \tag{13}$$

$$F_{qMp}(s) = \frac{q}{Mp} = \frac{\frac{\partial q}{\partial n}}{sT_a + \left(c - \frac{\partial m}{\partial n}\right)} \tag{14}$$

Finally, open-loop pump transfer function is:

$$q = F_{q\Delta p}\Delta p + F_{qMp} M_p \tag{15}$$

4. RESULTS

This model was numerically solved for one working point on pump and one working point of flow in pipeline.

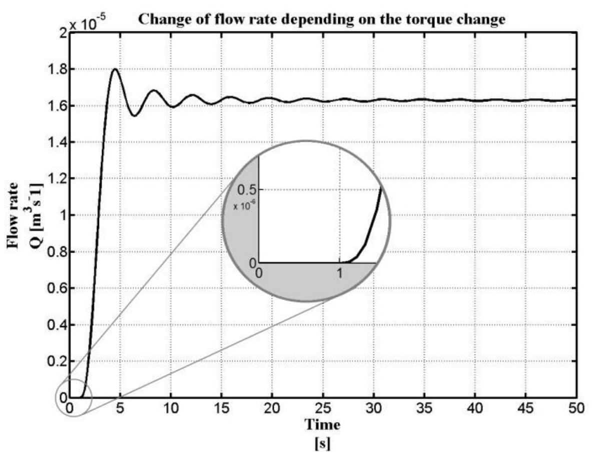

Fig. 1. Change of flow rate at the end of pipe after step change of motor torque. Enlarged picture shows time delay of change of flow rate

The dynamic change of flow rate at the end of pipeline is shown at fig. 1. Lenght of pipeline is 100m and transmitted medium is water. Flow in the pipeline is considered laminar.

5. CONCLUSION

In the present paper a model of pipeline and pump was developed.

A mathematical model of pipeline is based on hydro-electrical analogy. For this purpose, a hydraulical resistances (hydraulical resistance, inductance and capacitance) was derived. Advantage of this method is that it allows for a pipeline network to use knowledge from the circuit theory. Using knowledge of circuit theory implies advantage to solve pipeline networks.

A model of pump was derived from differential equations and expressed by transfer functions. This approach can be used only if governing equations are linear, which is restriction of this method.

Finally a numerical solution was presented.

This approach of modelling can be used for predictive control of pipeline network. The restriction is that it model can be used only around neighbourhood of operation point of pump and pipeline.

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