CONTROL OF UNSTABLE SYSTEMS A POLYNOMIAL APPROACH

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Abstract: The contribution is focused on control, design and simulation of unstable systems. The feedback is the most important tool how to change the properties of the controlled plant. The appropriate choice of the controller design can convert an originally unstable system into a stable feedback system. This process is called stabilization. A suitable and effective tool can be found in algebraic methods. The paper adopts the notion of the ring of polynomials and Diophantine equations in this ring. Controllers are obtained via solutions of Diophantine equations for first and second order systems. A Matlab, Simulink program implementation was developed for simulation and verification of the studied approach. Illustrative example supports simplicity and effectiveness of the proposed methodology.

Key words: Unstable system, feedback, stable and unstable polynomials, controller, equations

1. INTRODUCTION

Many dynamic systems in industry or transport have an unstable behavior. A classical control theory with traditional PID controllers often hardly overcomes the stabilization of unstable systems; see e.g. Aström and Hägglund (1995). However, the appropriate of the feedback controller can change this instability. Naturally, also an open-loop stable system can become unstable after feedback control, which is undesirable. Therefore, stability is a fundamental requirement for any controlled feedback system.

An effective and attractive tool how to change the stability properties can be found in algebraic methods, see Kucera (1993), Prokop and Corriou (1997). The general problem for unstable systems was solved in Prokop et al. (2001). Diophantine equations in the polynomial ring are simple and proper tool how to design suitable controllers. The fundamental task is to express a feedback characteristic equation with a stable right hand side. The system design approach utilizing the Diophantine equation is not a state-variable system design technique. It is a “purely algebraic” approach that deals with transfer functions of the controlled plant and controller. The main reason for considering Diophantine equation in this paper is that it allows to address unpleasant but very realistic situations when the controlled plant has right-hand-side zeros.

2. STABILIZATION

The easiest way to check the stability of a linear continuous system is to check the pole locations in the complex plane. If there is a pole with a positive real part, the system is said to be unstable and this pole is referred to as an unstable mode of the system. In other words, if there is one or more poles on the right half plane, the system is unstable. The system is stable if all poles have negative real parts, that is, all poles lie in the left half plane. The imaginary axis then represents a stability border. The systems with poles on imaginary axis are either oscillating or integrating ones, both unstable.

![Fig.1. General structure of system](image)

Suppose the feedback system depicted in Fig.1 where \( G(s) \) is a controlled (unstable) system and \( C(s) \) is a controller. Both are expressed as a ratio of polynomials:

\[
C(s) = \frac{q(s)}{p(s)} \quad (1)
\]

\[
G(s) = \frac{b(s)}{a(s)} \quad (2)
\]

and a reference signal is also defined as a ratio:

\[
W(s) = \frac{g(s)}{f(s)} \quad (3)
\]

The basic Diophantine equation for the stabilization problem in the ring of polynomials can be read as

\[
a(s) \cdot f(s) \cdot p(s) + b(s) \cdot q(s) = m(s), \quad (4)
\]

Where \( m \) is a stable polynomial with the appropriate degree, see Prokop and Corriou (1997), Prokop et al. (2001). The control law is then given by

\[
f(s) \cdot p(s)k(s) = g(s)E(s) \quad (5)
\]

3. CONTROL DESIGN

According to mentioned methodology, first and second order systems were assumed as unstable ones:

\[
G_1(s) = \frac{b_0}{s + a} \quad (6)
\]

\[
G_2(s) = \frac{b_0 + b_1 s}{s^2 + q_3 s + q_6} \quad (7)
\]

Then the controllers take the form

\[
C_1(s) = \frac{Q}{P} = \frac{q_4 s + q_2}{s \cdot p_0} \quad (8)
\]

\[
C_2(s) = \frac{Q}{P} = \frac{q_4 s^2 + q_2 s + q_6}{s(p_0 s + p_0)} \quad (9)
\]

for the first and second orders, respectively. The right hand side polynomials are then

\[
m_1(s) = (s + m_0)^2 \quad (10)
\]
Where $m_0 > 0$ is a multiple pole (positivity follows from stability) and the value of $m_0$ can be used as a tuning knob for influencing of response properties.

4. EXAMPLES

1) Let transfer function will be $G_1(s) = \frac{1}{s-0.5}$

Diophantine equation then takes the following form

$$(s - 0.5) \cdot s \cdot p_0 + 1 \cdot (q_0 + q_1, s) = (s + m_0)^2$$

and the comparison of appropriate coefficients gives a set of linear algebraic equations:

$$p_0 = 1$$
$$-0.5p_0 + q_0 = 2m_0 \Rightarrow q_1 = 2m_0 + 0.5$$
$$q_0 = m_0^2$$

The final controller for $m_0 = 2$ is

$$C_1(s) = \frac{q(s)}{p(s)f(s)} = \frac{4 + 4.5s}{s}$$

2) Let transfer function will be $G_2(s) = \frac{1}{s^2 - s - 2}$

Diophantine equation then takes the following form

$$(s^2 - s - 2) \cdot s \cdot p_0 + 1 \cdot (q_0 + q_1, s) = s^4 + 4s^3m_0 + 6s^2m_0^2 + 4sm_0^3 + m_0^4$$

and the comparison of appropriate coefficients gives a set of linear algebraic equations:

$$p_1 = 1$$
$$p_0 - p_1 = 4m_0$$
$$-p_0 - 2p_1 + q_0 = 6m_0^2$$
$$-2p_1 + q_1 = 4m_0^3$$
$$q_0 = m_0^4$$

The final controller for $m_0 = 0.5$ is

$$C_2(s) = \frac{q(s)}{p(s)f(s)} = \frac{0.06 + 6.5s + 6.5s^2}{(3 + 3s)s}$$

The final controller for $m_0 = 2$ is

$$C_2(s) = \frac{q(s)}{p(s)f(s)} = \frac{16 + 50s + 35s^2}{(9 + 3s)s}$$

5. MATLAB SIMULINK IMPLEMENTATION

A Matlab Simulink was created for design and simulation of controllers, see DingYu Xue, YangQuanChen, Derek P. Atherton (2007). First and second order systems are considered as nominal plants. The Simulink scheme of feedback system is shown in Fig. 2

Fig. 2. Simulink Scheme for 2nd order feedback system

6. CONCLUSION

A design method based on converting an originally unstable system into a stable feedback system was developed for unstable systems generally. Resulting control laws are of PID types. The proposed method enables to tune and influence of response properties and the control behavior by a single parameter $m_0$ (tuning knob). A Matlab Simulink was developed for controllers design and simulation.

7. REFERENCES


