Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
Make Harmony Between Technology and Nature, and Your Mind will Fly Free as a Bird
DAAAM Symposium

CONTROL OF UNSTABLE SYSTEMS A POLYNOMIAL APPROACH

VOLKOVA, N[atalia] & PROKOP, R[oman]

Abstract: *The contribution is focused on control, design and simulation of unstable systems. The feedback is the most important tool how to change the properties of the controlled plant. The appropriate choose of the controller design then can convert an originally unstable system into a stable feedback system. This process is called stabilization. A suitable and effective tool can be found in algebraic methods. The paper adopts the notion of the ring of polynomials and Diophantine equations in this ring. Controllers are obtained via solutions of Diophantine equations for first and second order systems. A Matlab, Simulink program implementation was developed for simulation and verification of the studied approach. Illustrative example supports simplicity and effectivity of the proposed methodology.*
Key words: *Unstable system, feedback, stable and unstable polynomials, controller, equations*

1. INTRODUCTION

Many dynamic systems in industry or transport have an unstable behavior. A classical control theory with traditional PID controllers often hardly overcomes the stabilization of unstable systems; see e.g. *Aström and Hägglund (1995)*. However, the appropriate of the feedback controller can change this unstability. Naturally, also an open-loop stable system can become unstable after feedback control, which is undesirable. Therefore, stability is a fundamental requirement for any controlled feedback system.

An effective and attractive tool how to change the stability properties can be found in algebraic methods, see *Kučera (1993)*, *Prokop and Corriou (1997)*. The general problem for unstable systems was solved in *Prokop et al. (2001)*. Diophantine equations in the polynomial ring are simple and proper tool how to design suitable controllers. The fundamental task is to express a feedback characteristic equation with a stable right hand side. The system design approach utilizing the Diophantine equation is not a state-variable system design technique. It is a “purely algebraic” approach that deals with transfer functions of the controlled plant and controller. The main reason for considering Diophantine equation in this paper is that it allows to address unpleasant but very realistic situations when the controlled plant has right-hand-side zeros.

2. STABILIZATION

The easiest way to check the stability of a linear continuous system is to check the pole locations in the complex plane. If there is a pole with a positive real part, the system is said to be unstable and this pole is referred to as an unstable mode of the system. In other words, if there is one or more poles on the right half plane, the system is unstable. The system is stable if all poles have negative real parts, that is, all poles lie in the left half plane. The imaginary axis then represents a stability border. The systems with poles on imaginary axis are either oscillating or integrating ones, both unstable.

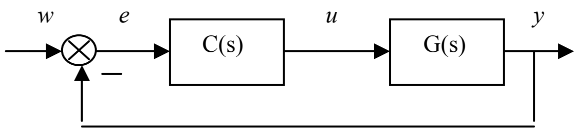

Fig.1. General structure of system

Suppose the feedback system depicted in Fig.1 where G(s) is a controlled (unstable) system and C(s) is a controller. Both are expressed as a ratio of polynomials:

$$C(s) = \frac{q(s)}{p(s)} \tag{1}$$

$$G(s) = \frac{b(s)}{a(s)} \tag{2}$$

and a reference signal is also defined as a ratio:

$$W(s) = \frac{g(s)}{f(s)} \tag{3}$$

The basic Diophantine equation for the stabilization problem in the ring of polynomials can be read as

$$a(s)\cdot f(s)\cdot p(s) + b(s)\cdot q(s) = m(s)\,, \tag{4}$$

Where m is a stable polynomial with the appropriate degree, see *Prokop and Corriou (1997), Prokop et al. (2001)*. The control law is then given by

$$f(s)\cdot p(s)U(s) = q(s)E(s) \tag{5}$$

3. CONTROL DESIGN

According to mentioned methodology, first and second order systems were assumed as unstable ones:

$$G_1(s) = \frac{b_0}{s+a} \tag{6}$$

$$G_2(s) = \frac{b_0 + b_1 s}{s^2 + a_1 s + a_0} \tag{7}$$

Then the controllers take the form

$$C_1(s) = \frac{Q}{P} = \frac{q_1 s + q_0}{s\cdot p_0} \tag{8}$$

$$C_2(s) = \frac{Q}{P} = \frac{q_2 s^2 + q_1 s + q_0}{s(p_1 s + p_0)} \tag{9}$$

for the first and second orders, respectively. The right hand side polynomials are then

$$m_1(s) = (s + m_0)^2 \tag{10}$$

$$m_2(s) = (s + m_0)^4 \qquad (11)$$

Where m_0>0 is a multiple pole (positivity follows from stability) and the value of m_0 can be used as a tuning knob for influencing of response properties.

4. EXAMPLES

1) Let transfer function will be $G_1(s) = \dfrac{1}{s - 0{,}5}$

Diophantine equation then takes the following form

$$(s - 0{,}5) \cdot s \cdot p_0 + 1 \cdot (q_0 + q_1 s) = (s + m_0)^2$$

and the comparison of appropriate coefficients gives a set of linear algebraic equations:

$$s^2 : \quad p_0 = 1$$

$$s^1 : \quad -0{,}5 p_0 + q_1 = 2m_0 \Rightarrow q_1 = 2m_0 + 0{,}5$$

$$s^0 : \quad q_0 = m_0^2$$

The final controller for $m_0=2$ is

$$C_1(s) = \frac{q(s)}{p(s)f(s)} = \frac{4 + 4{,}5s}{s}$$

2) Let transfer function will be $G_2(s) = \dfrac{1}{s^2 - s - 2}$

Diophantine equation then takes the following form

$$(s^2 - s - 2)s(p_0 + p_1 s) + q_0 + q_1 s + q_2 s^2 =$$
$$s^4 + 4s^3 m_0 + 6s^2 m_0^2 + 4sm_0^3 + m_0^4$$

and the comparison of appropriate coefficients gives a set of linear algebraic equations:

$$s^4 : \quad p_1 = 1$$

$$s^3 : \quad p_0 - p_1 = 4m_0$$

$$s^2 : \quad -p_0 - 2p_1 + q_2 = 6m_0^2$$

$$s^1 : \quad -2p_0 + q_1 = 4m_0^3$$

$$s^0 : \quad q_0 = m_0^4$$

The final controller for $m_0=0.5$ is

$$C_2(s) = \frac{q(s)}{p(s)f(s)} = \frac{0{,}06 + 6{,}5s + 6{,}5s^2}{(3 + s)s}$$

The final controller for $m_0=2$ is

$$C_2(s) = \frac{q(s)}{p(s)f(s)} = \frac{16 + 50s + 35s^2}{(9 + s)s}$$

5. MATLAB SIMULINK IMPLEMENTATION

A Matlab Simulink was created for design and simulation of controllers, see *Dingyü Xue, YangQuanChen, Derek P. Atherton* (2007). First and second order systems are considered as nominal plants. The Simulink scheme of feedback system is shown in Fig. 2

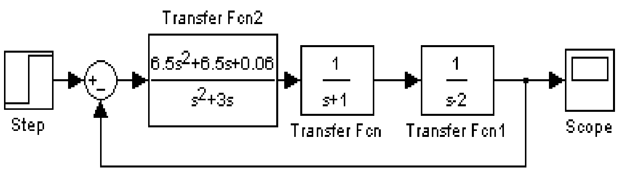

Fig. 2. Simulink Scheme for 2nd order feedback system

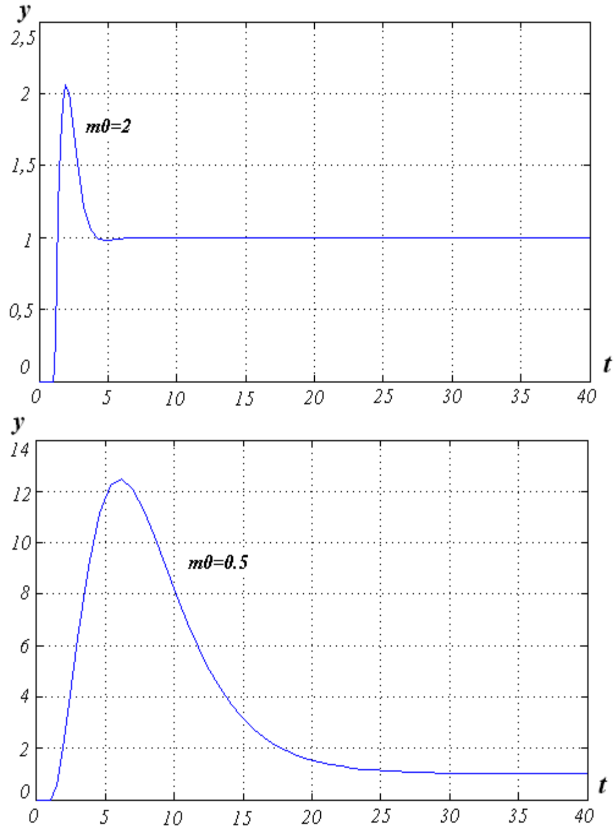

Fig. 3. Control response for 2nd order system

6. CONCLUSION

A design method based on converting an originally unstable system into a stable feedback system was developed for unstable systems generally. Resulting control laws are of PID types. The proposed method enables to tune and influence of response properties and the control behavior by a single parameter m_0 (tuning knob). A Matlab Simulink was developed for controllers design and simulation.

7. REFERENCES

Aström, K.J. and T. Hägglund (1995). *PID Controllers: Theory, Design and Tuning*. Instrument Society of America, USA

Dingyü Xue, YangQuanChen, Derek P. Atherton (2007). *Linear feedback control : analysis and design with MATLAB*. ISBN 978-0-898716-38-2 (alk. paper)

Kučera, V. (1993). Diophantine equations in control - A survey, *Automatica*, **Vol. 29**, **No.6**, pp. 1361-1375

Prokop, R. and J.P. Corriou (1997). Design and analysis of simple robust controllers, *Int. J. Control* , **Vol. 66**, **No. 6**, pp. 905-921

Prokop, R., P. Husták and Z. Prokopová (2001). Simultaneous tracking and disturbance rekjection for unstable systems. . In: *Prepr. of 13th Conf.on Process Control 01, Strbske Pleso*