Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
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Annals of DAAAM International

DAMPING OF OSCILLATIONS IN DUTCH ROLL MODE

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Abstract: *Aircraft flying in Dutch roll mode can exhibit certain flight stability deficiencies, prevailing in aircraft with non-assisted controls for which the damping factor is of level 2. This paper deals with the issue of increasing aircraft stability, by damping the lateral-directional motion for a wide envelope, according to flight regulations. In order to meet the required levels of flying qualities, we consider the relation between flying qualities parameters of the Dutch roll mode and gravimetric-inertial and aerodynamic parameters of the aircraft. The proposed method can be used to definine conditions for meeting the required flying qualities levels. These conditions constitute a basis for improving stability, using an automatic stabilization system acting directly in the rudder control circuit.*
Key words: *damping, Dutch roll, controls, aircraft*

1. INTRODUCTION

School and training aircraft are usually equipped with conventional control systems, featuring non-assisted mechanical circuits. These aircraft have a low damping factor at speeds above Mach 0.3 and altitudes of over 5000 m. With this type of aircraft an unsatisfactory level of oscillations damping was found (Hacker et al., 2009), especially in missions requiring Dutch roll mode specific maneuvers, decreasing their stability in these conditions. According to stability calculations (Raport INCREST, 1981), the damping factor of the Dutch roll mode is of level 2 in almost the entire operational envelope.

The paper deals with increasing the damping factor of oscillations for these aircraft, during maneouvers involving the Dutch roll mode. We determine the conditions on the aerodynamic derivatives values which affect the damping factor, so that it is of level 1. To evaluate the flying qualities parameters for Dutch roll, we use a simplified model of the aircrafts lateral-directional motion with three degrees of freedom, which describes this flight mode acurately enough and allows for the analytical expression of the stability parameters based on the aerodynamic derivatives and the inertial parameters. Therefore we can use these conditions in order to improve the stability of aircraft of this type by constructive means, such as an automatic system for increasing stability.

2. EQUATIONS OF THE LATERAL-DIRECTIONAL DISTURBED MOTION

The dimensional equations of the lateral-directional motion, obtained by linearizing the general motion equations, in the hypothesis of small disturbances around the base solution represented by stationary symmetric flight (Stevens & Lewis, 2003), are formulated in system (1):

$$\begin{aligned} m\left[V_0\dot{\beta}_0 + V_0(\cos\alpha_0)r - V_0(\sin\alpha_0)p - g(\cos\alpha_0)\phi\right] &= \mathrm{Y}_\beta\beta + \mathrm{Y}_\mathrm{r}r + \mathrm{Y}_\mathrm{p}p + \mathrm{Y}_{\delta_\mathrm{r}}\delta_r \\ I_x\dot{p} + I_{xz}\dot{r} &= \mathrm{L}_\beta\beta + \mathrm{L}_\mathrm{r}r + \mathrm{L}_\mathrm{p}p + \mathrm{L}_{\delta_\mathrm{r}}\delta_r + \mathrm{L}_{\delta_\mathrm{a}}\delta_a \\ -I_{xz}\dot{p} + I_z\dot{r} &= \mathrm{N}_\beta\beta + \mathrm{N}_\mathrm{r}r + \mathrm{N}_\mathrm{p}p + \mathrm{N}_{\delta_\mathrm{r}}\delta_r + \mathrm{N}_{\delta_\mathrm{a}}\delta_a \\ \dot{\phi} &= (\tan\alpha_0)r + p \end{aligned} \tag{1}$$

By replacing $\dot{\beta}$, $\dot{r}$ and $\dot{p}$ respectively, the equations (1) can be written in the canonical form of a linear dynamic system:

$$\dot{x} = Ax + Bu \tag{2}$$

where:

$$x = [\beta \quad r \quad p \quad \phi]^T \quad ; \quad u = [\delta_r \quad \delta_a]^T$$

$$A = \begin{bmatrix} Y_\beta^{'} - \dfrac{T_0\cos\alpha_0}{mV_0} & Y_r^{'} - \cos\alpha_0 & Y_p^{'} + \sin\alpha_0 & Y_\phi \\ N_\beta' & N_r' & N_p' & 0 \\ L'_\beta & L_r' & L_p' & 0 \\ 0 & \tan\alpha_0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} Y_{\delta_r}^{'} & 0 \\ N_{\delta_r}^{'} & N_{\delta_a}^{'} \\ L_{\delta_r}^{'} & L_{\delta_a}^{'} \\ 0 & 0 \end{bmatrix} \tag{3}$$

To study the stability of an aircraft with fixed controls, we consider the equations system $\dot{x} = Ax$.

The solutions for this system can be found based on the characteristic polynomial. Typically the matrix A has the roots of the polynomial $P(\lambda) = \det(\lambda I - A)$ as eigenvalues, which can be one pair of complex conjugate eigenvalues: $\lambda_{1,2D} = -\zeta_D\omega_{nD} \pm j\omega_{nD}\sqrt{1-\zeta^2}$ and two real eigenvalues, with the time constants $\tau_R = \frac{1}{\lambda_R}$ and $\tau_S = \frac{1}{\lambda_S}$.

Parameters assessing the flying qualities concerning dynamic stability with fixed controls (Hacker et al., 2009; Raport INCREST, 1981) refer mainly to the characteristics of flight modes: the damping factor, natural frequency, number of halving cycles, doubling time, halving time, etc. A flying qualities analysis requires the calculation of the eigenvalues of matrix A (3), which is straightforward if matrix A is indicated numerically. In this case we look for the analytical expressions of the eigenvalues of matrix A. For this purpose the analytical solution of the equation of the fourth degree: $P(\lambda) = \det(\lambda I - A) = 0$ can be difficult to solve and does not express the periodic or aperiodic character of the modes. Consequently we need simplified models of the lateral-directional motion, with fewer degrees of freedom, which approximate only one mode or two modes at most. The adoption of such models, i.e. the choice of degrees of freedom (Etkin, 1959), will be made depending on the weight with which the modes to be approximated occur in the variation of the variables β, r, p, ϕ.

3. THE ROLL AND SPIRAL MODES MODEL WITH THREE DEGREES OF FREEDOM

To obtain a simplified model of the motion, which would characterize only the roll and spiral modes, the following assumptions are considered. In the variations of β, the roll mode is weakly represented and the spiral mode is almost non-existent. Also, the $\dot{\beta} - Y_\beta^{'}\beta$ quantity related to the spiral mode is negligible compared to the sum of the other components of the lateral force. Considering the above observations, the first of the equations $\dot{x} = Ax$ is transformed into an algebraic equation, so that we obtain the following simplified model ($Y_P = 0$), with three degrees of freedom:

$$Y_\phi \phi + \left(Y_r^{'} - \cos\alpha_0\right)r + (\sin\alpha_0)p = 0 \tag{4}$$

$$\dot{r} = N_\beta^{\prime}\beta + N_r^{\prime}r + N_p^{\prime}p \tag{5}$$

$$\dot{p} = L_\beta^{\prime}\beta + L_r^{\prime}r + L_p^{\prime}p \tag{6}$$

$$\phi = (\tan\alpha_0)r + p \tag{7}$$

By solving this system in terms of the main variables, we can obtain a second degree differential equation:

$$\ddot{\phi} + D_1\dot{\phi} + D_0\phi = 0 \tag{8}$$

where D_1 and D_2 are expressions of the aerodynamic parameters $Y_r^{'}$, Y_ϕ , $L_\beta^{\prime}$, $L_p^{\prime}$, $L_r^{\prime}$, $N_\beta^{\prime}$, $N_p^{\prime}$, $N_r^{\prime}$ and α_0.

The characteristic polynomial of this differential equation is $P(\lambda) = \lambda^2 + D_1\lambda + D_0$ and the roots of the polynomial are $\lambda_R = \frac{-D_1 - \sqrt{D_1^2 - 4D_0}}{2}$ and $\lambda_S = \frac{-D_1 + \sqrt{D_1^2 - 4D_0}}{2}$. These expressions of the roots approximate well enough the eigenvalues of the Dutch roll and spiral modes.

4. THE AUTOMATIC SYSTEM FOR INCREASING STABILITY

For the purpose of correcting the described stability deficiencies, we can consider an automatic control system for increasing aircraft stability by acting on the rudder or aileron control circuits. This auto-stabilization system is described at a simplified level, exclusively in terms of flight dynamics and not of automatic systems dynamics. In this case the control system equations can be written as follows:

$$\dot{x} = A \cdot x(t) + B \cdot u(t) \tag{9}$$

where $u(t)$ is the command and has the form $u(t) = K \cdot x(t)$.

Command $u(t)$ will be "quick" when reading state *x*, without any delays specific to the aircraft subsystems, whose inertia is manifested differently. *K* expresses the global behavior of the automatic system, without regard to the fact that each subsystem separately has its own frequency response.

The states *x* can include the Washout filter and the drive system states. In this case the control law is written as:

$$\delta_r(t) = K_r \cdot r(t) \tag{10}$$

Considering a delay of the states which must be corrected with the amplification factor, the control expression becomes

$$u(t) = K \cdot x(t + \tau_r) \tag{11}$$

By applying the Laplace transform to the resulting system after the replacement of $u(t)$ the characteristic equation is obtained in the variable *s*, written in matrix form:

$$\det(sI - [A + B_{\delta r}K_r e^{-\tau s}]) = 0 \tag{12}$$

where $B_{\delta r}$ is the first column of matrix B (3).

In order to estimate the required accuracy, simulations are made of the states time response and of the signal collected from the feedback loop, without pilot control, at various initial values of the disturbed state *r*. For the disturbances not to be confused with measurement errors, the precision of the measuring instrument must be at least an order of magnitude higher than the measured values.

An auto-stabilization system which acts on the rudder control circuit performs the function of a yaw damper, with feedback depending on yaw rate *r*⟶∂*r*. Fig. 1 shows the block diagram of this system.

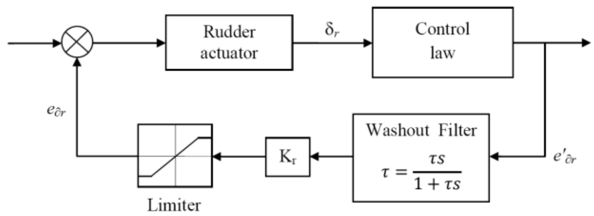

Fig. 1. The block diagram of the auto-stabilization system

Yaw control damping is based on determining the amplification factor K_r and the time constant of the Washout filter τ, so that the Dutch roll mode damping factor of the closed circuit system, composed of the aircraft and the yaw damper, is of level 1 across the entire flight envelope, according to the MIL-F-8785C regulation.

The disturbance of the yaw rate (*r*) relative to a reference value is detected by the gyrometer. It delivers an electrical signal (e_r) to the main computer which will send an output control signal ($e'_{\partial r}$), passed through a Washout filter and limiter. This results in a final control signal ($e_{\partial r}$) applied to a servo-actuator, which acts directly in the rudder control mechanical circuit in order to eliminate or attenuate the disturbance. The feedback depends on the filter time constant τ, so that the damping factor increases with the decreasing of its value. The control law implemented in the main computer is also determined by the amplification factor K_r , which can vary with altitude and airspeed.

The actuator and the control law can form a closed-loop drive subsystem, which allows low actuator response times and therefore rapid displacement corrections of the rudder. These corrections ensure that oscillations in the yaw control circuit are well damped, increasing the stability of the aircraft.

5. CONCLUSIONS

The paper deals with a flight dynamics problem in aircraft with non-assisted controls, which have a low damping factor for maneouvers involving the Dutch roll mode. We propose a solution for increasing stability, by damping the oscillations specific to these maneouvers and keeping them at level 1 as required by flight regulations.

We present a simplified model of the lateral-directional motion, with three degrees of freedom, which can be used to evaluate the flying qualities modal parameters and establish the periodic or aperiodic character of flight modes. This method allows the analysis of the possibilities to correct the stability deficiencies, by implementing an automatic system for increasing aircraft stability functioning as a yaw damper with feedback depending on yaw rate (*r* ⟶ ∂*r*). The system uses a servo actuator, which can act directly in the rudder control mechanical circuit.

This research presents a method for increasing the damping factor for maneouvers involving the Dutch roll mode. Based on this method it is possible to obtain the increase of the damping factor and improve aircraft stability for other flight modes, such as the spiral and roll subsidence modes.

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