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GANTRY ROBOT VOLUMETRIC ERROR EVALUATION USING ANALYTICAL AND FEM MODELING

MARINESCU, D[an] A[ndrei] & NICOLESCU, A[drian] F[lorin]

Abstract: *There are many types of industrial robotic applications where robot's volumetric accuracy is very important. The main factors influencing robot's volumetric accuracy are joints and structural elements stiffness and elastic displacements. In this paper the authors propose an analytical model for gantry robot's volumetric accuracy evaluation based on elastic displacements of joints and structural elements. For joints elastic displacement calculation specific analytical methods were developed, while for links elastic displacements calculation a FEM model was developed. As reference robot model, it is used the Güdel FP$_4$ gantry robot. In order for structural elements optimization various constructive solutions were studied using FEM.*
Key words: *Gantry robot, volumetric error, elastic displacements, analytical model, FEM*

1. INTRODUCTION

In this paper the authors propose an analytical model for gantry robot's volumetric accuracy evaluation based on elastic displacements of joints and structural elements. To properly define the virtual model for robot's overall FEM analysis was necessary to determine first the stiffness of the robot's translational joints including cam-followers components. For this purpose using the mathematical model, developed in the paper (Nicolescu et al., 2010), the overall loading of the gantry robot has been reduced to corresponding axial and radial direction for each cam-follower, the specific loads on each cam-follower being determined in accordance with specific design for each translational joint. To express the displacements in radial and axial direction of ball bearings, the authors have used the mathematical background presented in (De Tedric et al, 2007) in which the elastic displacements are expressed as δ_a (ball bearing axial displacement) and δ_r (ball bearing radial displacement). By using FEM analysis, the elastic behavior of the gantry robot structure subjected to static load was analyzed and errors induced by structural elements elastic displacements were revealed. By studying several possible design alternatives, the optimum design solution having the highest stiffness for robot's structural elements were identified.

2. ANALITICAL MODEL FOR ROBOT'S VOLUMETRIC ERROR EVALUATION

The mathematical model of a gantry robot, according to Denavit-Hartenberg algorithm, can be written as follows:

$$T_{(4,0)}^{HD} = T_{(1,0)}^{HD} T_{(2,1)}^{HD} T_{(3,2)}^{HD} T_{(4,3)}^{HD} \quad (1)$$

However, the matrix expression written above is valid for an ideal robot. In order to obtain a mathematical model taking account of robot's components elastic behavior, it is necessary to include supplementary terms that allow to model component's elastic displacements (Nicolescu A. & Stanciu M, 1996):

$$T_{IR}^{real} = T_I T_1^{HD} T_{II} \Delta_1^J \Delta_1^S T_{III} T_{IV} T_2^{HD} T_V \Delta_2^J \Delta_2^S T_{VI} \Delta_3^J \Delta_3^S T_{VII} T_3^{HD} \quad (2)$$

where T_I... T_{VII} are depending of each robot type specific design, Δ_i^J are depending of robot's joints elastic displacement and Δ_i^S is depending of robot's links elastic displacement.

Thus, the influence of IR's joints and links elastic behavior on IR's volumetric accuracy can be expressed by a total error matrix $[\varepsilon]$:

$$[\varepsilon] = T_{IR}^{real} - T_{(4,0)}^{HD} \quad (3)$$

The Δ_i^J term can be expressed as follows:

$$[\Delta_i^J] = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Where

$$n_x = \cos a_5 \cos a_2 \quad (5)$$
$$n_y = \sin(a_3)\sin(a_5)\cos(a_2) + \cos(a_3)\sin(a_2) \quad (6)$$
$$n_z = -\cos(a_3)\sin(a_5)\cos(a_2) + \sin(a_3)\sin(a_2) \quad (7)$$
$$o_x = \sin(a_5) \quad (8)$$
$$o_y = -\sin(a_3)\cos(a_5) \quad (9)$$
$$o_z = \cos(a_3)\cos(a_5) \quad (10)$$
$$a_x = \cos(a_5)\sin(a_2) \quad (11)$$
$$a_y = \sin(a_3)\sin(a_5)\sin(a_2) - \cos(a_3)\cos(a_2) \quad (12)$$
$$a_z = -\cos(a_3)\sin(a_5)\sin(a_2) - \sin(a_3)\cos(a_2) \quad (13)$$
$$p_x = 0 \quad (14)$$
$$p_y = a_1 \quad (15)$$
$$p_z = a_4 \quad (16)$$

Knowing the loads corresponding to radial and axial direction for each cam-follower and internal load distribution inside each cam-follower, the resulting linear and angular displacements of each robot's mobile element representing the general deformed/un-deformed model for a P-joint, assumed to be vertical (a_1), horizontal (a_4), pitching angle (a_2), rolling angle (a_3) and yawing angle (a_5), may be expressed as following figure shows too by equation (17)… (24):

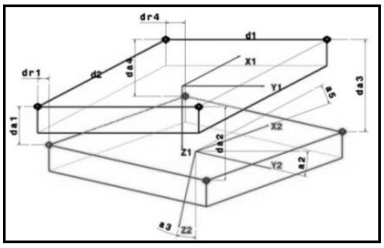

Fig. 1. General deformed/un-deformed model for a P-joint
where a_i factors have particular values for each P-joint, the general calculation formulas were determined as:

$$a_1 = \frac{d_{r1} + d_{r2}}{2} \quad (17)$$

$$a_4 = \frac{\sum d_{ai}}{4} \tag{18}$$

$$\sin a_3 = \frac{d_{a2}+d_{a3}-d_{a1}-d_{a4}}{2\sqrt{d_1^2+\left(\frac{d_{a2}+d_{a3}-d_{a1}-d_{a4}}{2}\right)^2}} \tag{19}$$

$$\cos a_3 = \frac{d_1}{\sqrt{d_1^2+\left(\frac{d_{a2}+d_{a3}-d_{a1}-d_{a4}}{2}\right)^2}} \tag{20}$$

$$\sin a_2 = \frac{d_{r4}-d_{r1}}{\sqrt{d_2^2+(d_{r4}-d_{r1})^2}} \tag{21}$$

$$\cos a_2 = \frac{d_2}{\sqrt{d_2^2+(d_{r4}-d_{r1})^2}} \tag{22}$$

$$\sin a_5 = \frac{d_{a3}+d_{a4}-d_{a1}-d_{a2}}{2\sqrt{d_2^2+\left(\frac{d_{a3}+d_{a4}-d_{a1}-d_{a2}}{2}\right)^2}} \tag{23}$$

$$\sin a_5 = \frac{d_2}{\sqrt{d_2^2+\left(\frac{d_{a3}+d_{a4}-d_{a1}-d_{a2}}{2}\right)^2}} \tag{24}$$

The overall horizontal and vertical elastic displacements of translational joint can be now determined as function of horizontal and vertical elastic displacements of cam-followers by expressing the difference between two reference systems position and orientation ($O_1X_1Y_1Z_1$; $O_1X_2Y_2Z_2$) corresponding to non-deformed and respectively deformed joint by matrix .

3. ROBOT'S STRUCTURAL ELEMENTS FEM ANALYSIS

In order to use FEM for robot's overall elastic behavior analysis, first the authors developed, using Catia V5, the robot virtual model, a simplified model being generated by excluding robot's motors and gearboxes. For robot's most stressed structural elements (transversal beam) tree different types of beams models have been used. Then, using the cam-followers stiffness determined as above mentioned, the robot joints FEM elastic model was determined by introducing appropriate elastic elements in each joint.

Assuming that rotational elastic displacements are much less than axial displacements (Fig.2), the Δ_i^S mentioned in the precedent chapter have been modeled using following form:

$$\Delta_j^S = \begin{pmatrix} \delta_x \\ \delta_y \\ \delta_z \\ 1 \end{pmatrix} \tag{23}$$

where global displacement vector difference between two reference systems position ($O_1X_1Y_1Z_1$; $O_1X_2Y_2Z_2$) is defined as:

$$\delta_{TOT} = \sqrt{\delta_x^2+\delta_y^2+\delta_z^2} \tag{24}$$

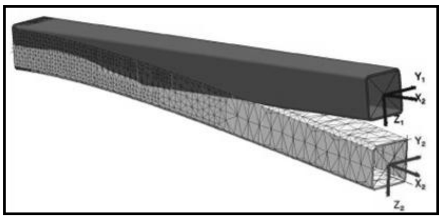

Fig. 2. Elastic displacement on beam

Finally elastic displacements for each structural element, corresponding to various structures design, were calculated using FEM (Tab. 1).

4. FEM MODELING RESULTS

The FEM analysis was performed for two purposes: first for identifying robot's overall elastic behaviour and second for testing which is the most rigid constructive solution for the travelling beam along X axis.

Beam 'A'	Max Von Mises Stress	98.767 MPa
	Max Displacement	0.614 mm
Beam 'B'	Max Von Mises Stress	72.737 MPa
	Max Displacement	0.467 mm
Beam 'C'	Max Von Mises Stress	46.235 MPa
	Max Displacement	0.359 mm

Tab. 1. Von Mises Stresses and Displacements for each beam

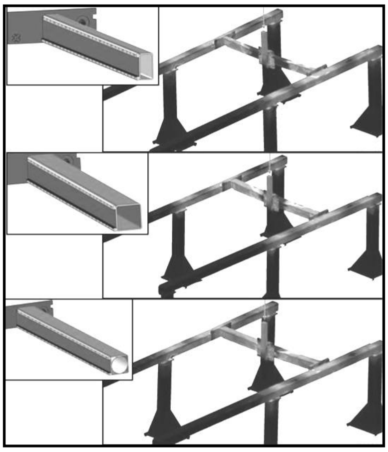

Fig. 3. Comparative view for different construction solution

5. CONCLUSION

An analytical model for gantry robot volumetric error evaluation has been presented. Specific terms of this model allows to analytical model joint's and link's elastic behavior. FEM analysis was performed for presenting overall gantry robot elastic behavior. Various possible constructions variants for robot transversal beam structural element were analyzed, making easier to choose the right design corresponding to precision needs specific for each type of robot application.

For future papers, the authors intend to study cam-follower's internal elastic behavior and guiding ways using FEM analysis as well as compare various design solutions for optimizing robot's guiding system elements.

6. REFERENCES

De Tedric, A. Harris, Michael N. Kotzala (2007). *Rolling Bearing Analysis: Essential concepts of bearing technology*, ISBN: 978-0-8983-7183-7, Boca Raton FL 33487-2742

Nicolescu A., Stanciu M (1996). Elastic Behavior Modeling of Industrial Robot's Base Translation Modules. Part.1. The Mathematical Model, *Proceeding RAAD '96*, Katalinic, B. (Ed.), pp. 239-243, ISBN: 963 420 482 1, Budapest Hungary, June 1996

Nicolescu, A.; Marinescu, D. & Ivan, M. (2010). Elastic displacement influence of translation joints on volumetric accuracy for gantry industrial robots (Part 2), *Proceedings OPTIROB 2010*, Olaru, A.; Ciupitu L. & Olaru S. (Ed.), pp. 28-32, ISBN: 978-981-08-5840-7, Calimanesti, May 2010,

*** (2009) http://www.gudel.com/fileadmin/guedel-com/download/catalogs/brochures/Components/Linear/gudel-01-guideway-medium-screen.pdf - Güdel Components Guideway Systems for medium duty application, *Accessed on: 2010-05-21*

*** (2009) http://www.gudel.com/fileadmin/guedel-com/download/catalogs/brochures/Components/Linear/gudel-02-guideway-heavy-screen.pdf - Güdel Components Guide-way Systems for heavy duty application, *Accessed on: 2010-05-22*