THE CORRECTION OF WEIGHTING FACTORS' VALUES IN THE EVALUATING SOLUTION VARIANTS

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Abstract: The Pahl and Beitz methodology is widely used in the product designs. The problem of adding new and/or neglect some objectives in hierarchical structure is described. This paper gives the mathematical equations for weighting factors (w_i) correction when we add or neglect the objectives with the proof. The paper gives a case study as well.
The main motive for the research is the lack of formula for the w_i correction. The goal is to shorten the time to calculate w_i and facilitate this process. Two formulas are the result and they are very simple for implementation. They also allow easier application of computers in this method.
Key words: Evaluation, Method, Weights, Objectives, Tree

1. INTRODUCTION

The Pahl and Beitz method (Pahl & Beitz, 2007) for the evaluating solution variants of some product incorporates the concepts of Cost-Benefit Analysis (Brent, 2006), and of Guideline VDI 2225. The method contains the objectives tree. The objectives are arranged in a hierarchical order with multiple levels (Tiro & Brdarević, 2003). They have different relative contributions on the overall value.

The contributions of objectives are given with the weighting factors w_i (Pahl & Beitz, 2007; Oberšmit, 1991). The factors values are rated from 0 to1. The sum of the factors for all objectives contribute to some objective in hierarchical higher level must be equal to 1.

$$w_1 + w_2 + w_3 + ... + w_n = 1 \qquad \text{or} \qquad \sum_{i=1}^{n} w_i = 1 \qquad (1)$$
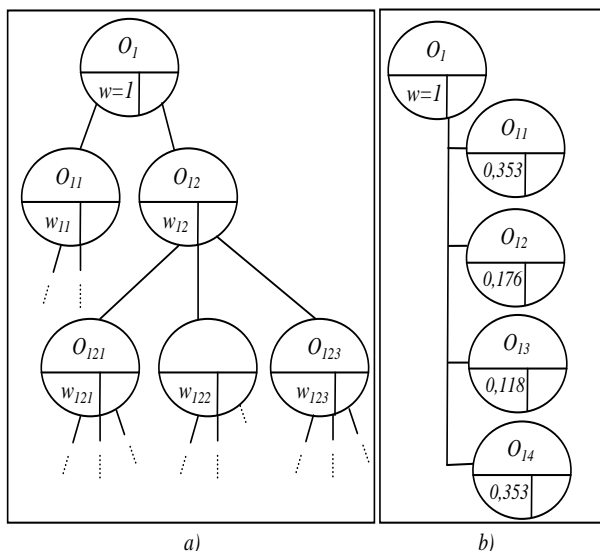

a)   *b)*

Fig. 1. a) Hierarchical structure of the objectives O_i with the weighting factors w_i
b) The objectives O_{11} , O_{12} , O_{13} and O_{14} contribute to O_1 and the sum of the weighting factors must be equal to 1.

For example in figure 1.a) the objectives O_{121}, O_{122} i O_{123} contribute to the O_{12}. The sum of the weighting factors must be equal to 1: $w_{121} + w_{122} + w_{123} = 1$. Also the sum $w_{11} + w_{12} = 1$.

2. THE PROBLEM OF ADDING NEW AND/OR NEGLECTING SOME OBJECTIVES IN HIERARCHICAL STRUCTURE

It is often necessary during the design process to add some new objectives in the hierarchical structure or to neglect some irrelevant objectives. As the sum of weighting factors must be equal to 1, its values must be corrected.
This means that we must re-implement the analysis of the subobjectives' relative influence at the objective of higher hierarchical level. To avoid this job, the question is whether we can write a mathematical equation for correction of the weighting factors' value. **The aim of this study** is to obtain the equation.

3. THE MATHEMATICAL EQUATION WHILE NEGLECTING THE IRRELEVANT OBJECTIVES

It is necessary to correct (increase) the weighting factors of relevant objectives when we neglect the irrelevant ones. The equation is:

$$w_{ik}^{b} = w_i^{b} + \frac{w_i^{b} \cdot \sum_{j=1}^{m} w_j^{n}}{1 - \sum_{j=1}^{m} w_j^{n}} \qquad (2)$$

where w_{ik}^{b} is the corrected weighting factor of relevant objective "i"; w_i^{b} is the weighting factor of relevant objective "i" before the correction; $\sum_{j=1}^{m} w_j^{n}$ is the sum of m irrelevant weighting factors and m is the number of irrelevant objectives.
To prove (2) it is necessary to use the fact that the sum of all **corrected** relevant weighting factors must be 1:

$$\sum_{i=1}^{n-m} w_{ik}^{b} = 1 \qquad (3)$$

So, we need to obtain 1 at the left side of the equation (3). If we include the equation (2) in the equation (3):

$$w_1^{b} + \frac{w_1^{b} \cdot \sum_{j=1}^{m} w_j^{n}}{1 - \sum_{j=1}^{m} w_j^{n}} + ... + w_{n-m}^{b} + \frac{w_{n-m}^{b} \cdot \sum_{j=1}^{m} w_j^{n}}{1 - \sum_{j=1}^{m} w_j^{n}} = 1$$

It is possible to write it in the form:

$$w_1^b + \ldots + w_{n-m}^b + \frac{w_1^b \cdot \sum\limits_{j=1}^{m} w_j^n}{1 - \sum\limits_{j=1}^{m} w_j^n} + \ldots + \frac{w_{n-m}^b \cdot \sum\limits_{j=1}^{m} w_j^n}{1 - \sum\limits_{j=1}^{m} w_j^n} = 1$$

or the form:

$$\sum_{i=1}^{n-m} w_i^b + \frac{w_1^b \cdot \sum\limits_{j=1}^{m} w_j^n}{1 - \sum\limits_{j=1}^{m} w_j^n} + \ldots + \frac{w_{n-m}^b \cdot \sum\limits_{j=1}^{m} w_j^n}{1 - \sum\limits_{j=1}^{m} w_j^n} = 1 \,,$$

$$\sum_{i=1}^{n-m} w_i^b + \frac{\sum\limits_{j=1}^{m} w_j^n}{1 - \sum\limits_{j=1}^{m} w_j^n}\left(w_1^b + w_2^b + \ldots + w_{n-m}^b\right) = 1 \,, \text{ i.e:}$$

$$\sum_{i=1}^{n-m} w_i^b + \frac{\sum\limits_{j=1}^{m} w_j^n}{1 - \sum\limits_{j=1}^{m} w_j^n} \cdot \sum_{i=1}^{n-m} w_i^b = 1 \qquad (4)$$

The objectives are composed of the relevant weighting factors and the irrelevant ones. So, the **equation (1)** can be written:

$$\sum_{i=1}^{n-m} w_i^b + \sum_{j=1}^{m} w_j^n = 1, \quad \text{i.e.} \quad \sum_{i=1}^{n-m} w_i^b = 1 - \sum_{j=1}^{m} w_j^n$$

We include the last equation in the (4):

$$1 - \sum_{j=1}^{m} w_j^n + \frac{\sum\limits_{j=1}^{m} w_j^n}{1 - \sum\limits_{j=1}^{m} w_j^n} \cdot \left(1 - \sum_{j=1}^{m} w_j^n\right) = 1$$

After the elimination:

$$1 - \sum_{j=1}^{m} w_j^n + \sum_{j=1}^{m} w_j^n = 1$$

From where we get $1 \equiv 1$. With this the equation (2) is proved.

4. THE MATHEMATICAL EQUATION TO ADD OBJECTIVES

Another case is when we need to add one or more objectives that affect a hierarchically higher-order objective.

The analysis of the subobjectives' relative influence at the objective of higher hierarchical level is performed for determination the weighting factors. For example, in Figure 1.b) the objectives O_{11} and O_{14} have the same influence. But they have two times higher influence then the objective O_{12} and three times than O_{13}. Before the correction of weighting factors, we need to give a weighting factor's value for the added objectives. When we add a new objective, we give it a weighting factor value of the existing objective which has the same influence. For example, if we add one more objective O_{15} in the Figure 1.b), which has the same importance as O_{13}, then we give $w_{15} = w_{13} = 0{,}118$.

The sum of weighting factors is more than 1, and we have to correct them using the equation:

$$w_{ik} = w_i - \frac{w_i \cdot \sum\limits_{i=1}^{r} w_i^d}{1 + \sum\limits_{i=1}^{r} w_i^d} \qquad (5)$$

where w_i^d is the weighting factor of added objectives "i"; r is the number of added objectives. The proof of equation (5) is analogue one as for (2).

5. CASE STUDY

If we neglect the objective O_{13} in Figure 1.b), we calculate the weighting factors using the equation (2). The obtained values are shown in Figure 2.a).
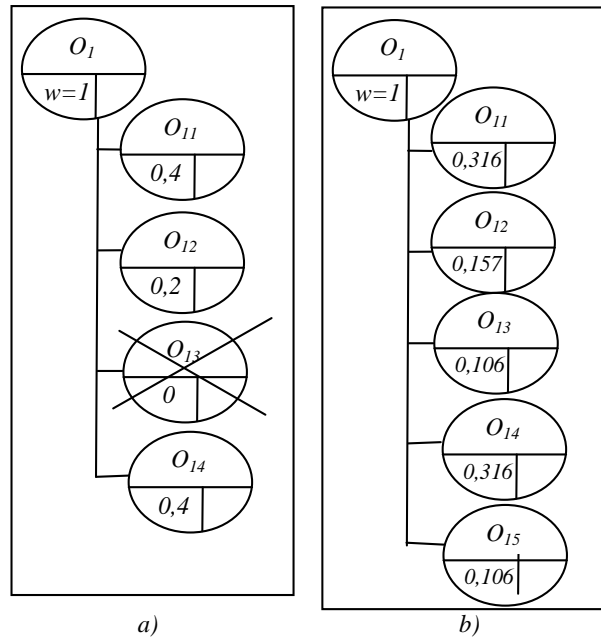

a)   *b)*

Fig. 2. a) The weighting factors calculated using the equation (2) after the neglecting of objective O_{13} in Figure 1.b)
b)The weighting factors calculated using the equation (5) after the adding of objective O_{15} in Figure 1.b)

However, if we add the objective O_{15} in Figure 1.b), which has the same importance as the objective O_{13}, then we obtain the values using the equation (5). The Figure 2.b) shows the calculated values.

This example is simple, because we add (neglect) just one objective. But if we add (neglect) more objectives, the procedure remains the same. We use the equations (2) and (5).

6. CONCLUSION

The equations (2) and (5) are **the results** of the study. It is possible to correct the weighting factors' value with them. The equations are relatively simple. Their application is easy and they significantly speed up the implementation process of evaluation method.

In addition, they also allow easier usage of computers in this method, and a method's software can be developed in further research.

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