Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
Make Harmony Between Technology and Nature, and Your Mind will Fly Free as a Bird
DAAAM Symposium

A TECHNIQUE OF DETERMINATION AND CONSTRUCTION OF A SINGULAR SURFACES IN THE MANIPULATOR WORK SPACE

POLIAKOV, O[leksandr] M[ykhaylovich]

Abstract: *Determination of a singular configurations is one of the necessary steps during the manipulator synthesis and control. Because in these configurations the instantaneous kinematics is locally undetermined that causes serious problems at the manipulator movement control. A definition technique of singular configurations and corresponding to them singular surfaces in the workspaces of serial manipulators with rotary kinematic pairs is presented. Visualization of singular surfaces allows making at a level of the subjective analysis the estimations of reachability for manipulators with various variants of parameters of kinematic structures.*

Key words: *manipulator, workspace, singularity, manifold*

1. INTRODUCTION

In the literature a questions of primary manipulators description, as a not free multilink's mechanical systems, are presented insufficiently. At the same time they have special importance at formation of the mathematical models in the symbolic form, at the solving of the synthesis problems, at the trajectories planning and at the manipulator control.

One of the most important problems which solve at the stage of primary description is a definition of the manipulator working zone and its singular configurations. In the sixties the last century the first attempts to solve this problem by means of numerical and graphic methods have been undertaken. Thus a main objective of such researches was to obtain the exact swept volume of motion of a rigid body in space (Abdel-Malek et al., 2006).

In number of works published before, for definition of a set of singular configurations the authors used methods based on known theorems about implicit functions reduced to the analysis of the manipulator Jacobian (Haug et al., 1996, Abdel-Malek & Yeh, 1997). However in these cases to receive all possible solutions is difficult enough, as there is a necessity to use a nonformal approach. The manipulators with superfluous mobility are characterized by not square Jacobi matrix owing to what its analysis is reduced to the analysis of submatrixes. Some singular configurations at such approach cannot be defined at all.

The aim of the given work is the substantiation of definition technique of the singular configurations of manipulator, set on the basis of the formal description of its kinematic structure and representation in the form of some variety.

2. INFORMATION

In (Velichenko, 1988) it is shown that some variety S with a dimension m as mathematical mapping of a not free mechanical system can be described in two ways: by definition of n component of the vector x in the form of continuous functions of m - dimensional argument q :

$$q = (q_1 \quad q_2 \quad \dots \quad q_m)^T : x = (x_1(q) \quad x_2(q) \quad \dots \quad x_n(q))^T \qquad (1)$$

and by means of the equations system

$$\eta_i(x) = 0 \,,\ i = 1,2,\dots, n-m \qquad (2)$$

where $\eta_i(x)$ - in special way picked up functions assumed as independent. It is necessary to notice that from set of functions $\eta_k(x)$ which can be received for the set of mechanical systems, including manipulators only $n-m$ will be linearly independent. Any set $n-m$ of such functions represents the system of a base functions.

Let

$$A = \begin{pmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \dots & \frac{\partial x}{\partial q_m} \end{pmatrix} = \frac{\partial x}{\partial q^T} \qquad (3)$$

$$K^T = \begin{pmatrix} \frac{\partial \eta_1(x)}{\partial x} & \frac{\partial \eta_2(x)}{\partial x} & \dots & \frac{\partial \eta_{n-m}(x)}{\partial x} \end{pmatrix}^T = \frac{\partial \eta}{\partial x^T} \qquad (4)$$

accordingly, the basis of a tangential space $T(q)$ and an orthogonal space $N(x)$ of manifold S (Dubrovin et al., 1985). Then $G^q = A^T A$ and $\Gamma^\eta = K^T K$ - a metric tensors of manifold. The manifold is described correctly, if

$$\det G^q \neq 0 \qquad (5)$$

$$\det \Gamma^\eta \neq 0 \qquad (6)$$

But the vectors $a^i = \frac{\partial x}{\partial q_i}, i = 1..m$ and $k^j = \frac{\partial \eta_j}{\partial x}, j = 1..n-m$ is a linearly independent and their association allows to receive a full basis P of manifold S in the form of the following block matrix (Velichenko, 1988)

$$P = \begin{pmatrix} k^1 & \dots & k^{n-m} & a^1 & \dots & a^m \end{pmatrix} = (K|A) \qquad (7)$$

Obviously,

$$rankP = rankK + rankA = n\,. \qquad (8)$$

Under condition of a correct description of manifold *S*, the expression (8) should be carried out in all systems of the generalized co-ordinates and not to depend on their choice.

Some configurations in which at least one of coordinates q_k or several $(q_k, q_l \dots, k \neq l \neq \dots)$, such that $q_k = f_r(q_r, q_s \dots), k \neq r \neq s \neq \dots$, $q_l = f_l(q_r, q_s \dots), l \neq r \neq s \neq \dots$ etc., according to the known theorem of implicit functions (Litvin et al., 1986) $\det G^q = 0$, and the condition of correctness of the manifold description (5) is not carried out. From (8) follows, that in these cases the dimension of tangential space $T(q)$ decreases for size $u \geq 1$, and the dimension of orthogonal space $N(x)$ increases by the same size. In other words, in this case the mechanical system loses u degrees of freedom owing to realization of the same number of the additional restrictions caused by its own properties. On the contrary, in cases when $\det \Gamma^\eta = 0$ the mechanical system gets additional degrees of freedom, owing to elimination of one or several restrictions. Thus the dimension of tangential space increases on some size $v \geq 1$ and dimension of orthogonal space decreases for the same size. Such manipulator configurations name as singular. Thus the base point of the manipulator end-effector moves along some surfaces S_{sur} also named as singular.

In cases, when on the mechanical system are imposed an additional nonkeeping restriction kind of $q_k^{\min} \le q_k \le q_k^{\max}$ or similar to them, the set of singular configurations is supplemented with set of boundary singular configurations. So, introduction of m -dimensional vector $\alpha = (\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_m)^T$ allows to present the vector q as

$$q = (f_1(\alpha_1) \quad f_2(\alpha_2) \quad \dots \quad f_m(\alpha_m))^T, \qquad (9)$$

where $f_k(\alpha_k), k = 1,2,..., m$ - some functions set. For example (Abdel-Malek & Yeh, 1997):

$$f_k(\alpha_k) = c_{1,k} + c_{2,k}\sin(\alpha_k);\ \alpha_k \in R;$$

$$c_{1,k} = \frac{q_k^{\max} + q_k^{\min}}{2};\ c_{2,k} = \frac{q_k^{\max} - q_k^{\min}}{2} \qquad (10)$$

and, thus, the generalised co-ordinates are limited.

The theorem. At the description of manifold S in new generalised co-ordinates α and in the presence of their connections with co-ordinates q in the form of (10), $rank G^{\alpha} = 0$ at $\alpha_k = \pm\frac{\pi}{2}$ at least for one k from set $k = 1,2,..., m$.

The proof. As $x = x[q(\alpha)]$ that by a rule of differentiation of the composite function, assuming that $rank\frac{\partial q}{\partial \alpha^T} = m$ and matrix $\frac{\partial q}{\partial \alpha^T}$ is non-singular, we will receive: $\frac{\partial x}{\partial \alpha^T} = \frac{\partial x}{\partial q^T}\frac{\partial q}{\partial \alpha^T}$. Otherwise

$$B = [b^1 \quad b^2 \quad \dots \quad b^m] = \left[\frac{\partial x}{\partial \alpha_1} \quad \frac{\partial x}{\partial \alpha_2} \quad \dots \quad \frac{\partial x}{\partial \alpha_m}\right] = \frac{\partial x}{\partial \alpha^T} \qquad (11)$$

- a matrix of tangential basis of manifold S in a new system of a curvilinear co-ordinates α. Thus the metric tensor of the manifold S it will be transformed under the law

$$G^{\alpha} = \frac{\partial q^T}{\partial \alpha} G^q \frac{\partial q}{\partial \alpha^T} \qquad (12)$$

For the expressions (10) the matrix $\frac{\partial q}{\partial \alpha^T}$ has a diagonal form

$$\frac{\partial q}{\partial \alpha^T} = \begin{pmatrix} c_{2,1}\cos(\alpha_1) & 0 & \dots & 0 \\ 0 & \dots & \dots & \dots \\ \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & c_{2,m}\cos(\alpha_m) \end{pmatrix} \qquad (13)$$

As G^q - a symmetric positively definite $m \times m$ matrix and $G^q = Q^T D Q$, where Q - an orthogonal matrix;

$D = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & .. & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & \lambda_m \end{pmatrix}$; $\lambda_i, i = 1,2,..., m$ - eigenvalues of the matrix G^q, we will receive

$$\det G^{\alpha} = c_{2,1}^2 \dots c_{2,m}^2 \cos^2(\alpha_1) \dots \cos^2(\alpha_m) \lambda_1 \dots \lambda_m = 0. \qquad (14)$$

I.e., the manipulator accepts singular configurations in cases when at least one of the generalised co-ordinates accepts boundary value ($\cos(\alpha_k) = 0 \Rightarrow \alpha_k = \pm\frac{\pi}{2}$) or at least one of eigenvalues of matrix G^q is equal to zero ($\lambda_l = 0$).

First of the resulted conditions allows to define the set of singular boundary configurations, second - the set of internal.

Boundary singular configurations are defined simply enough, and definition of internal, on the basis of equality to zero one of eigenvalues represents a difficult mathematical problem.

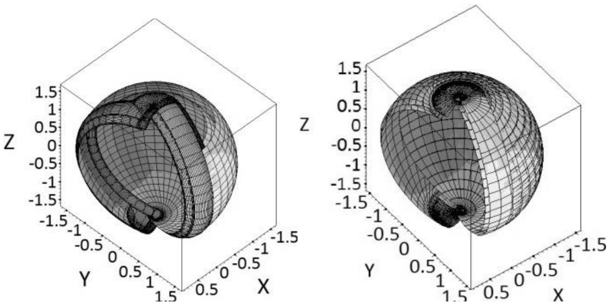
Let's write down $\det G^{q(\alpha)}$ in the next form:

$$\det G^{q(\alpha)} = g_{1,1} G_{1,m-1}^{q(\alpha)} - g_{1,2} G_{2,m-1}^{q(\alpha)} + \dots - (-1)^m g_{1,m} G_{m,m-1}^{q(\alpha)}, \qquad (15)$$

where $g_{1,j}, j = 1,2,..., m$ - the elements of the first line of $G^{q(\alpha)}$; $G_{k,m-1}^{q(\alpha)}, k = 1,2,..., m$ - the minors of $G^{q(\alpha)}$ order of $m-1$. That expression (15) was equal to zero, it is necessary, that all minors of order $m-1$ simultaneously were equal to zero. This is one more condition, allowing to define internal singular configurations. In many cases it is more effective from the computing point of view, than the condition of equality to zero of the tensor $G^{q(\alpha)}$ eigenvalues.

Thus, the expressions (14) and (15) can be used from each other independently for definition of all singular configurations, and the matrix (5), (6) and (7) - for their verification.

As an example, in Fig. 1 are presented the boundary surfaces (a) and the internal surfaces (b) corresponding to set of singular configurations of a serial 3-links manipulator,which are defined by means of the technique stated above.



a) b)

Fig. 1. Singular surfaces of the manipulator: a) the joint boundary surface; b) the joint internal surface

3. CONCLUSION

The offered approach allows effectively to solve the problems connected with definition of all singular configurations of the manipulator. It is necessary to develop reliable algorithm of numerical definition of all own numbers of a matrix for increase of efficiency of use of this approach. It is especially important at the synthesis of the manipulator which work in given working zone and at the planning of end-effector movements.

4. REFERENCES

Abdel-Malek, K.,; Yang, J.; Blackmore, D. & Joy, K. (2006). Swept volumes: Foundations, perspectives, and applications. *International Journal of Shape Modeling*, Vol.12, No.1, 87-127, ISSN: 0218-6543

Abdel-Malek, K. & Yeh, H. (1997). Analytical boundary of the workspace for general 3-DOF mechanisms. *International Journal of Robotics Research*, Vol.16, No.2, 198-213, ISSN: 0278-3649

Dubrovin, B.; Fomenko, A. & Novikov, S. (1985). *Modern geometry – methods and applications. Part II. The geometry and topology of manifolds*, Springer-Verlag, ISBN 0-387-96162-3, New York

Haug, E.; Luh, C.; Adkins, F. & Wang, J. (1996). Numerical algorithms for mapping boundaries of manipulator workspaces. *ASME Journal of Mechanical Design*, Vol. 118, No.2, 228-234, ISSN: 1050-0472

Velichenko, V. (1998). *Matrix geometrical methods in mechanic with applications to robotics problems*, Science, ISBN 5-02-013888-6, Moscow