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GENERIC MODELLING OF MATHEMATICAL ROBOT MODELS

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Abstract: *It is necessary to have a good knowledge of robot wheelframe properties in order to be able to design mobile robots and control their motions. These properties are most useful in a form of a mathematical model of a robot wheelframe. The models can be structured into two groups – kinematic and dynamic. Sometimes a dynamic model is necessary which reflects also forces acting on the robot (f.e. centrifugal force created during passing through a curve), possibility of a wheel skid etc. However, for the majority of mobile robots, kinematic models are sufficient.*

The major part of wheels applied in robotics can be structured into several basic categories. Each wheel type has its own definition of basic conditions, which rule out the possibility of wheel skid and slip. These conditions are formulable via mathematical equations. Through the combination of equations for all the wheels of a robot wheelframe it is possible to gain the mathematical model of the whole wheelframe. This is a universal method of creating the mathematical model of any robotic wheelframe

Key words: *mobile robot, model, wheelframe, skid, ICR*

1. INTRODUCTION

The basis of every mobile robot is a wheelframe. The majority of mobile robots uses wheel wheelframes. For a good design of robot control an exact description of robot´s wheelframe is necessary. (Cumpion 1996) Such a model gives us also the information about which motions the robot is able to carry out.

There are two types of robotic wheelframes models kinematic and dynamic. The kinematic model of a robot is able to define the trajectories of motion and the behaviour of the wheelframe at a relatively low speed. This model however does not reflect the motion dynamics. For a more exact description of a wheelframe the dynamic model is used than. This model involves also the acting of different forces upon the wheelframe, the possibility of wheel skidding etc. For the majority of mobile robots the kinematic model of a wheelframe is sufficient.

2. WHEELS OF MOBILE ROBOTS

The major part of wheels applied for mobile robots can be divided in several basic categories.

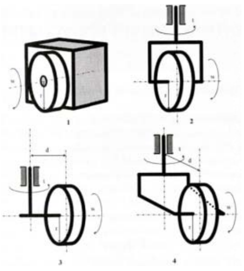

Fig. 1. Categories of mobile robots wheels

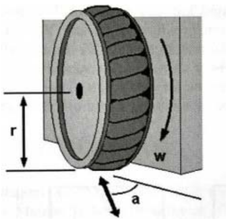

Fig. 2. Categories of mobile robots wheels

We can see the basic categories on the Figure 1. The first category is represented by fixed wheels. These wheels are tightly connected with the robot´s body and they are just rolling away. The second and the third figured wheel is controlled. These wheels are rolling away the same way as the fixed wheel, but in addition they are rotating along the axis which is perpendicular to the axis of the wheel rotation. The fourth figured wheel is a castor one. This type of wheel is conforming while the robot is moving and it does not create any barriers in motion. It is usually used as a support wheel for wheelframes with other types of wheels. Another type of wheels applied in mobile robotics are the omnidirectional wheels.

The advantage of this type of wheels is a larger motions variability of the robot. On the other hand, omnidirectional wheels are not able to overcome bigger barriers with the same wheel diameter, compared to the classical wheels.

3. GENERATING OF A ROBOT MODEL

There are two basic conditions defined for each wheel connected with a robot wheelframe:

- The wheel velocity in direction of the wheel axis with regard to the inertial coordinate system must have zero value. No skidding may occur.
- The motion velocity of the wheel in direction of rolling with regard to the inertial coordinate system must conform to the velocity of wheel rotation. No slipping may occur.

These conditions of each wheel of a wheelframe are expressed via the coordinate system of the robot. For further information about mathematical expression of the conditions please see (Šolc 2009).

An example of these conditions is demonstrated on a controlled wheel, see the Figure 3.

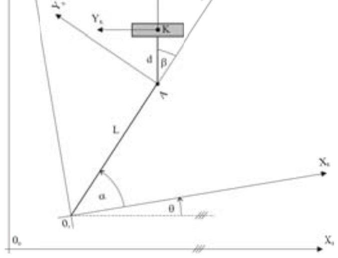

Fig. 3. Conditions calculation for one wheel

Mathematical expression of the condition for zero wheel skidding is shown in the equation 1.

$$v_{xr}\cos(\alpha+\beta)+v_{yr}\sin(\alpha+\beta)+\dot{\Theta}L\sin\beta=0 \tag{1}$$

Mathematical expression of the second condition, zero wheel slipping, is demonstrated by the equation 2.

$$-v_{xr}\sin(\alpha+\beta)+v_{yr}\cos(\alpha+\beta)+\dot{\Theta}\left[d+L\sin\beta\right]=-\dot{\beta}d-r\omega \tag{2}$$

From the quation 1 we can see, that some motions are for this wheel impossible. The wheelframe can be equipped with different number of wheels. For each wheel of the robot we gain following two equations.

These equations are modified into a matrix differential equation 3, which defines all the limitations of the robot motion.

$$\begin{bmatrix} C_{af} \\ C_{ac} \\ C_{v} \end{bmatrix}\begin{bmatrix} v_{xr} \\ v_{yr} \\ \dot{\Theta} \end{bmatrix}=\begin{bmatrix} C_{af} \\ C_{ac} \\ C_{v} \end{bmatrix}R_{R,0}\dot{X}=\begin{bmatrix} 0 \\ 0 \\ v \end{bmatrix} \tag{3}$$

This is a system of equations of particular conditions. C_{af} and C_{ac} is formed by limitations for controlled and fixed wheels and C_v abstracts the equations of conditions for all castor and omnidirectional wheels. By solution of this matrix equation we gain the mathematical model of a robot.

4. EXAMPLE

The process of generic creation of a model is further demonstrated on a simple example. We can see a draft of a differentially controlled wheelframe on the Figure 4.

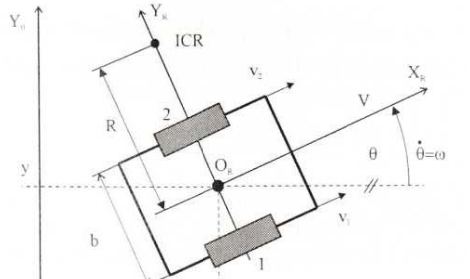

Fig. 4. Differentially controlled wheelframe

In order to express the equations of limitations for both wheels we have to allocate their position with respect to the coordinate system of the robot. Let us give following position to the wheel 1: L=b/2, α=-90°, β=0° and to the wheel 2: L=b/2, α=90°, β=0°. We express both conditions for each of these wheels and formulate these conditions in a form of the matrix equation 4.

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & -b/2 \\ 1 & 0 & -b/2 \end{bmatrix}\begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{x} \\ \dot{\Theta} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ -v_1 \\ v_2 \end{bmatrix} \tag{4}$$

By solution of this differential matrix we gain the well-known mathematical model of a differentially controlled wheelframe (Siegwart 2004). This model can be defined by the equation 5.

$$\begin{bmatrix} \dot{x} \\ \dot{x} \\ \dot{\Theta} \end{bmatrix}=\begin{bmatrix} \frac{v_1+v_2}{2}\cos\Theta \\ \frac{v_1+v_2}{2}\sin\Theta \\ \frac{v_1-v_2}{b} \end{bmatrix} \tag{5}$$

5. CONCLUSION

With the aid of this method it is possible to generate models of different wheel wheelframes with different number of wheels of different types. This method is universal.

The mentioned example of the model was already applied for control of an autonomous wheelchair (Uchiyama 2005). This type of chair is equipped with two fixed wheels and two castor wheels. Future plans are practical examinations of more complicated generic models of mobile robots wheelframes.

The wheelchair is equipped with a minicomputer Fox Board LX832 (***). This minicomputer operates with a wheelchair model which serves for controlling and data logging concerning the wheelchair position.

Fig. 5. Wheelchair

6. ACKNOWLEDGEMENTS

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