Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
Make Harmony Between Technology and Nature, and Your Mind will Fly Free as a Bird
Annals of DAAAM International

STUDY OF THE CONDUCTION PHENOMENON INSIDE AN INHOMOGENEOUS HALF-SPACE OF EXPONENTIALLY INCREASING CONDUCTIVITY

NEMOIANU, I[osif] V[asile]; CAZACU, E[mil]; IONESCU, G[elu]; PALTANEA, V[eronica] & PALTANEA, G[heorghe]

Abstract: *The article treats the injection of a direct current through an above-ground circular plate earth electrode. A non-homogeneous soil of exponentially increasing conductivity is considered. The particularities of the problem allow the use of the separation of variables method for solving the homogeneous second order partial derivative equation (PDE) verified by the electric potential. The analytically obtained solution is used to calculate the magnitude variation of the electric field strength and of the Poynting's vector, respectively, and finally the formula of the earth electrode resistance is derived. By evaluating the limit of this relationship, the homogeneous case formula is obtained.*
Key words: *Earth electrode, Inhomogeneous soil*

1. INTRODUCTION

Analysis of inhomogeneous soil current flowing represents a real challenge due to the significant complexity of the PDE describing the phenomenon, when a non-uniform conductivity is taken into account. Therefore, authors mainly consider continuously varying conductivity soils as stratified ones (Ghourab, 2007; Colomina et al., 2002; Colomina et al., 2007), each layer being characterized by an uniform conductivity. This approach has major draw-backs, seriously affecting the accuracy of the solution. Many practical applications spanning from earthing to mineral and fossil deposit detection deal with continuous variations in conductivity, and therefore a mathematical procedure for solving the PDE could be beneficial for all the above-mentioned problems.

For example, the non-uniform absorption of underground water by a dry porous soil to its surface, leads to a continuous variation in conductivity, even for geologically uniform structures of the terrestrial crust (Ţugulea & Nemoianu, 2009).

Therefore, this article aims to study the injection of a direct current of intensity i through an above-ground circular plate of radius a into a soil characterized by the following variation function:

$$\sigma(z) = \sigma_0 \, e^{+\frac{z}{\lambda}} \tag{1}$$

where σ_0 is the conductivity at the surface of the ground, z is a spatial coordinate perpendicular to the separation plane.

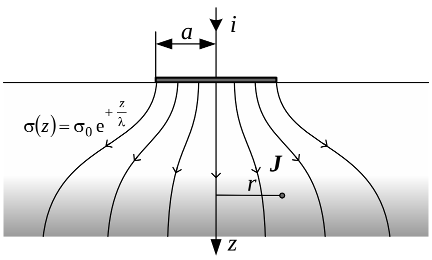

Fig. 1.Disc-shaped earth electrode injecting current into an inhomogeneous soil.

The in-depth variation in conductivity is described by the real constant λ (m), as depicted in Fig. 1.

The study begins from the steady-state local form of the charge conservation law:

$$\mathrm{div}\boldsymbol{J} = 0. \tag{2}$$

The left-hand side of (2) is expanded by substituting $\boldsymbol{J} = \sigma\boldsymbol{E}$.

$$\mathrm{div}(\sigma\boldsymbol{E}) = \boldsymbol{E}\cdot\mathrm{grad}\sigma + \sigma\mathrm{div}\boldsymbol{E} \tag{3}$$

and substituting also $\boldsymbol{E} = -\mathrm{grad}V$, we have

$$\Delta V = -\frac{\mathrm{grad}\sigma\cdot\mathrm{grad}\,V}{\sigma} \tag{4}$$

where $\Delta V = \mathrm{div}\mathrm{grad}V$.

The axis-symmetric configuration presented by the geometry of the problem recommends the use of the cylindrical system of coordinates (r, φ, z), where $\partial\sigma/\partial r = 0$, $\partial\sigma/\partial\varphi = 0$, and $\partial V/\partial\varphi = 0$, and therefore

$$\mathrm{grad}\sigma = \frac{1}{r}\left(r\frac{\partial\sigma}{\partial r}\boldsymbol{u_r} + \frac{\partial\sigma}{\partial\varphi}\boldsymbol{u}_\varphi + r\frac{\partial\sigma}{\partial z}\boldsymbol{u_z} \right) = \frac{\partial\sigma}{\partial z}\boldsymbol{u_z} \tag{5}$$

$$\begin{aligned} \mathrm{grad}\,V &= \frac{1}{r}\left(r\frac{\partial V}{\partial r}\boldsymbol{u_r} + \frac{\partial V}{\partial\varphi}\boldsymbol{u}_\varphi + r\frac{\partial V}{\partial z}\boldsymbol{u_z} \right) = \\ &= \frac{\partial V}{\partial r}\boldsymbol{u_r} + \frac{\partial V}{\partial z}\boldsymbol{u}_z \end{aligned} \tag{6}$$

$$\Delta V = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} \tag{7}$$

where $\boldsymbol{u}_r$, $\boldsymbol{u}_\varphi$ and $\boldsymbol{u}_z$ are the unit vectors of the cylindrical system of coordinates.

2. CURRENT INJECTION INTO A EXPONENTIALLY INCREASING CONDUCTIVITY SOIL

Taking now into account the assumed variation of conductivity given by (1), and by substituting the gradients given by (5) and (6), the right-hand side of (4) becomes with (7):

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} + \frac{1}{\lambda}\frac{\partial V}{\partial z} = 0 \tag{8}$$

The homogeneous PDE given by (8) is solved using the separation of variables method, by expressing the potential function as a product of two independent single-variable functions $V(r, z) = R(r)\cdot Z(z)$.

We get the following independent PDEs:

$$\frac{\mathrm{d}^2 R}{\mathrm{d} r^2} + \frac{1}{r}\frac{\mathrm{d} R}{\mathrm{d} r} - p^2 R = 0 \tag{9}$$

$$\frac{\mathrm{d}^2 Z}{\mathrm{d} z^2} + \frac{1}{\lambda}\frac{\mathrm{d} Z}{\mathrm{d} z} + p^2 Z = 0 \tag{10}$$

Equations (12) and (13) are solved, and the following general solutions are obtained:

$$R_p(r) = C_{1p}\,\mathrm{I}_0(pr) + C_{2p}\,\mathrm{K}_0(pr) \tag{14}$$

$$Z_p(z) = D_{1p}\cdot \mathrm{e}^{\frac{-1+\sqrt{1-4p^2\lambda^2}}{2\lambda}z} + D_{2p}\cdot \mathrm{e}^{\frac{-1-\sqrt{1-4p^2\lambda^2}}{2\lambda}z} \tag{15}$$

where C_{1p}, C_{2p}, D_{1p} and D_{2p} are integration constants, p the separation parameter, and I_0 and K_0 are the zero order Bessel functions of the first and second kind, respectively.

3. ELECTRIC POTENTIAL, ELECTRIC FIELD STRENGTH AND EARTH RESISTANCE FORMULAS

By imposing that the potential function should be constant on the earth electrode's surface and null at infinity, we get:

$$V(r,z) = \frac{2\,i}{\pi\sigma_0}\frac{\lambda}{a}\int_0^{\infty} \mathrm{J}_0(kr)\mathrm{J}_1(ka)\,\mathrm{e}^{-\frac{1+\sqrt{1+4k^2\lambda^2}}{2\lambda}z}\cdot \frac{\mathrm{d}k}{1+\sqrt{1+4k^2\lambda^2}} \tag{16}$$

On the earth's surface ($z = 0$) the electric field strength is null, and for $r \le a$ becomes:

$$E_r(r,0) = \frac{2\,i}{\pi\sigma_0}\frac{\lambda}{a}\int_0^{\infty} \mathrm{J}_1(kr)\mathrm{J}_1(ka)\frac{k\,\mathrm{d}k}{1+\sqrt{1+4k^2\lambda^2}} \tag{17}$$

The magnitude of the Poynting's vector on the earth's surface $S(r, 0) = E_r(r, 0)\cdot H(r, 0)$, for $r > a$ may be calculated by considering the simple formula of the magnetic field intensity given by the Ampère's law $H(r, 0) = i/(2\pi r)$. We get:

$$S(r,0) = \frac{i^2}{\pi^2\sigma_0}\frac{\lambda}{a}\frac{1}{r}\int_0^{\infty} \mathrm{J}_1(kr)\mathrm{J}_1(ka)\frac{k\,\mathrm{d}k}{1+\sqrt{1+4k^2\lambda^2}}\,. \tag{18}$$

Integration of (18) with respect to r over the interval (a, ∞) gives the electromagnetic power P transferred to the conducting soil. Finally, the earth electrode resistance is obtained:

$$R = \frac{P}{i^2} = \frac{1}{\pi\sigma_0 a}\int_0^{\infty} \mathrm{J}_0(ka)\mathrm{J}_1(ka)\frac{2\lambda}{1+\sqrt{1+4k^2\lambda^2}}\,\mathrm{d}k\,. \tag{19}$$

Let us define now the normalized earth electrode resistance R_{norm}, by dividing the right-hand side of (19) by a quantity having the dimension of resistance, namely $(\pi\sigma_0 a)^{-1}$. For radius of the circular plate $a = 0.5$ m, the right-hand side integral of (19) is numerically evaluated, for a set of discrete values of parameter λ. The variation graph of the normalized earth electrode resistance is shown in Fig. 2.

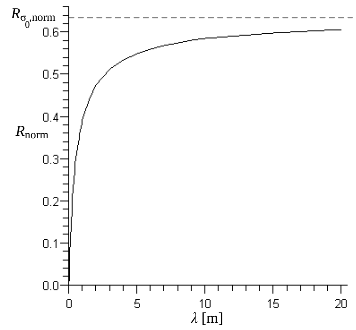

Fig. 2. Graph of the normalized earth electrode resistance vs. λ.

Taking the limit of R_{norm} for $\lambda \to \infty$, an already reported in the scientific literature (Nemoianu, 1964) formula of the earth electrode resistance in the uniform σ_0 conductivity case is obtained:

$$R_{\sigma_0,\text{norm}} = \int_0^{\infty} \frac{\mathrm{J}_0(ka)\mathrm{J}_1(ka)}{k}\,\mathrm{d}k = \frac{2}{\pi} \tag{20}$$

4. CONCLUSION

As expected, the smaller the value of parameter λ (corresponding to a rapid rise in conductivity inside the soil), the smaller the value of the earth electrode resistance is obtained. In this case, due to a significant z-direction conductivity increase, the initial half-space may be approximated with a high conductivity plate of finite thickness. With even smaller values of this thickness ($\lambda \to 0$) the resistance of the plate becomes unimportant, and practically a superconducting sheet is obtained. Unlike this case, for increasing values of λ, the earth electrode resistance rises rapidly asymptotically, approaching the homogeneous conductivity value, that was already reported in the scientific literature.

Authors estimate that further developments of this study may consider the time-harmonic case (ac injected current), which also takes into account the skin effect phenomenon.

5. REFERENCES

Colominas, I.; Navarrina, F. & Casteleiro, M. (2002). A numerical formulation for grounding analysis in stratified soils, *IEEE Trans. on Power Delivery*, vol. 17, no.2, pp. 587-595, 0885-8977

Colominas, I; Navarrina, F. & Casteleiro, M. (2007). Numerical Simulation of Transferred Potentials in Earthing Grids Considering Layered Soil Models, *IEEE Trans. on Power Delivery*, vol. 22, no.3, pp. 1514-1522, 0885-8977

Ghourab, M.E. (2007). Evaluation of earth resistivity for grounding systems in non-uniform soil structure, *European Transactions on Electric Power*, vol.6, no.3, pp. 197-200, 1430-144X

Nemoianu, C. (1964). *The skin-effect phenomenon in the case of time-harmonic conduction currents flowing in straight line conductors of discontinuous varying cross section*, PhD Thesis, Polytechnics Institute of Bucharest, Bucharest, (in Romanian)

Ţugulea, A. & Nemoianu, I.V. (2009). Time-harmonic electromagnetic field diffusion into an exponentially decreasing conductivity half-space, *Rev. Roum. Sci. Techn.– Électrotechn. et Énerg.*, 54, 1, pp. 21-27, 0035-4066