Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
Make Harmony Between Technology and Nature, and Your Mind will Fly Free as a Bird
DAAAM Symposium

# GENERALIZATION OF THE METHOD OF FINITE DIFFERENCES

**ARYASSOV, G[ennady]; BARASHKOVA, T[atjana]; GORNOSTAJEV, D[mitri] & PETRITSHENKO, A[ndres]**

***Abstract:*** *The purpose of this paper is to generalize the method of finite differences described in (Kollats, 1969). For obtaining of interpolation polynomials, the matrixes and method of uncertain coefficients are used. The essential simplification of the calculation formulae is received; in particular case they are the L. Kollats' formulas. The accuracy of the used approach is estimated in same way as it is made in the classical method of finite differences. The use of matrix symbolics gives the convenient tool for realization of calculations by computers. The numerical results are presented as well. The received results can be applied to the solution of boundary value problems of various classes and to increase the accuracy of the finite elemet method.*

***Key words***: *Method of Grids, Interpolation Polynomials,* ***Matrix*** *Equations, Method of Uncertain Coefficients*

## 1. INTRODUCTION

The theory of the method of finite differences is based on the theory of the approximation of functions, when values of them in discrete points are known. For this purpose, the interpolation polynomials obtained by the method of the uncertain coefficients are applied. Such approximation is possible to execute without resorting to the finite difference schemes (Jensen, 1972). The method of uncertain coefficient can be used as for the traditional method of grids as for the "improved method of grids", which has been developed (Kollats, 1969) for solution of partial differential equations especially. The method of grids allows to reduce a task of continuous analysis to a problem of solution of system of the algebraic equations. The accuracy of the used interpolation polynomials is established by the well-known formulas from literature.

## 2. APPROXIMATION BY METHOD OF UNCERTAIN COEFFICIENTS

Let's consider the closed interval [*a,b*] shown in Fig. 1 and which is a part of wider interval [*A,B*]. We set a task to approach the given function $y = f(x)$ by the method of uncertain coefficients. The arbitrarily located interval [*a,b*], along an axis *x* with length $l = b-a$, is divided to $n$ equal parts with length $h = (b-a)/n$.

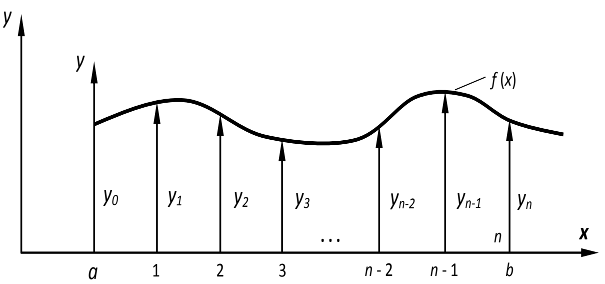


Fig. 1. The closed interval of [a,b]

It is required to find coefficients $a_i$ of the interpolation polynomial $P_n(x)$ of degree $n$

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n \qquad (1)$$

Using dimensionless argument $\xi$ given in Eq. (2)

$$x = a + \frac{b-a}{n}\xi\ ;\ 0 \le \xi \le n\ ;\ \xi = \frac{n(x-a)}{b-a} \qquad (2)$$

the polynomial $P_n(x)$ in Eq. (1) can be rewritten in matrix notation as follows

$$P_n^*(\xi) = [\xi]^T \{\alpha\} \qquad (3)$$

where $\alpha_i$ are the coefficients of the interpolation polynomial $P_n^*(\xi)$.

Substitution of the integer values of the dimensionless argument $\xi$ and corresponding values of function *y* to the Eq. (2), the system of equations for calculation of $\alpha$ are obtained

$$P_n^*(n) = [W_n]\{\alpha\} = \{y\}\ , \qquad (4)$$

where $[W_n]$ - Vandermonde matrix.

Once the coefficients $\{\alpha\}$ from Eq.(4) are determined, we can rewrite Eq. (3) as follows

$$P_n^*(\xi) = \{\xi\}^T [W_n]^{-1}\{y\} = \left\{1\ \xi\ \xi^2 ...\ \xi^n\right\}^T [W_n]^{-1}\{y\} \quad (5)$$

*Example.* Approximate a broken line consisting of straight lines $y = 0$ and $y = -12(x-2)$ by the fourth degree polynomial shown in Fig. 2.

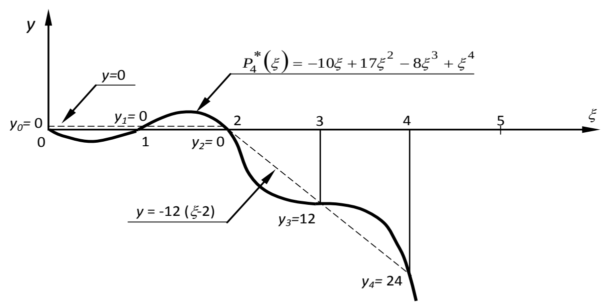


Fig. 2. The interpolation of broken line

It should be noticed, that the beginning of coordinates of interpolation polynomials can be changed arbitrarily, but changes in the order of nodes and corresponding ordinates is not allowed in any event.

## 3. INTERPOLATION OF DERIVED FUNCTIONS

From the Eq. (5) follows, that for calculation of derivative of interpolation polynomial, it is sufficient to differentiate only the matrix-line $\{\xi\}^T$. For example, the third derivative of the polynomial in Eq. (5) will be defined as

$$\frac{d^3P_n^*(\xi)}{d\xi^3}\left[0\;0\;0\;6\;\;24\cdot\xi\;\;120\cdot\xi^2\ldots\, n(n-1)(n-2)\xi^{(n-3)}\right]^T\left[W_n\right]^{-1}\{y\} \quad (6)$$

It is important to know the values of derivatives in the nodes of interpolation. These values are obtained easily from the Eq. (8), if it is supposed, that $\xi$ accepts consistently the values as: $\xi$ = *0, 1, 2, …, n.* At $n$ = 4 the third derivative from $P_n^*(\xi)$ will be

$$\frac{d^3P_4^*(\xi)}{d\xi^3}=[-]_4''[W_4]^{-1}\{y\} \quad (7)$$

where $[-]_4''$ is a square matrix. The lower index of the matrix specifies the polynomial order, and upper index specifies the derivative order. The foregoing formulae for differentiation of functions, which are given in discrete points, are generalization of the classical formulae of numerical differentiation. The accuracy of the method can be established in same way, as it is made in the classical methods (Korn & Korn, 1968).

The generalized matrix $\left[O_n^{(m)}\right]$, which simultaneously carries out operations of interpolation and differentiation of function *{y}* both simultaneously, is given by a vector according to Eq. (7) as

$$\left[O_n^{(m)}\right]=[-]_n^{(m)}[W_n]^{-1} \quad (8)$$

We shall confine ourselves to consider only the differential equation with zero regional conditions. In the case of general boundary conditions it is required to apply the matrixes, which are interpolated on Ermit. So we have

$$[g(\xi)]y''+[r(\xi)]y'+\lambda[s(\xi)]y=\{f(\xi)\} \quad (9)$$

with *y(0)*=0 and *y(n)=0,* where $[g(\xi)]$, $[r(\xi)]$ and $[s(\xi)]$ are diagonal matrixes with the corresponding values of functions $g(\xi), r(\xi)$ and $s(\xi)$ in points or nodes of interpolation, $f(\xi)$ is free function in the right part in the same points or nodes. Taking into account Eq. (7) and Eq. (8), the system of the linear algebraic equations in Eq. (9) in a matrix form is

$$[D]\{y\}=\{f\} \quad (10)$$

where $[D]=[[g(\xi)]][O_n'']+[r(\xi)][O_n']+\lambda[s(\xi)]$ is a matrix operator of given differential equation.
The solution of Eq. (10) can be found with inverse matrix as

$$\{y\}=[D]^{-1}\{f\} \quad (11)$$

## 4. NUMERICAL RESULTS

For an illustration we consider Euler problem shown in Fig. (3). The boundary conditions at the ends of the beam are $v(0)=v(l)=0$.

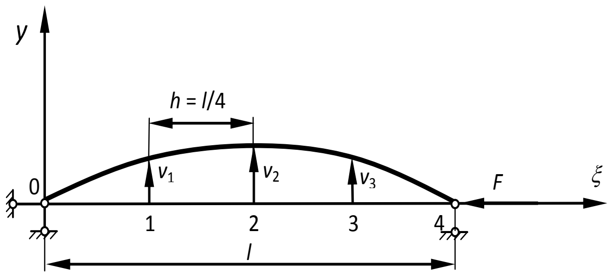


Fig. 3. Beam loaded by the axial force *F*, when *(n*=4*)*

Using the operator $[O_2'']$ in the Eq. (10) and don't taking into account the overlapping of intervals, we receive the critical force value $F_{kp}=9.38EI/l^2$ with an error 5.2 %.

Applying the operator $[O_4'']$ (10), we receive $F_{kp}=9.395EI/l^2$ with an error 5.0 %. Increasing the division numbers to $n$=8 and using operator $[O_2'']$, we receive $F_{kp}=9.79EI/l^2$ with an error 0.81 %.
Applying operator $[O_4'']$ in the Eq. (8) in case of double number of nodes and using the overlapping of intervals, the value of critical force will be $F_{kp}=9.87544EI/l^2$ with an error 0.05 %.

## 5. CONCLUSION

The received formula allows to approximate the functions and their derivatives not resorting to differences as it is made in a classical method of grids. The use of overlapping of interpolation intervals allows to increase an accuracy of the solution. The calculation results show that it is possible to adjust the accuracy of the solution either by changing the degree of the interpolation polynomial or with the help of overlapping of intervals. This is the main difference not only from usual, but also from the "improved" method of grids. The received results can be applied to the solution of boundary value problems and to increase the accuracy of finite element-finite difference method. Especially, it is suggested to use the given approach for calculation of stresses in threaded joints (Aryassov & Petritshenko, 2008) and the eigenvalues of orthotropic plates (Aryassov & Petritshenko, 2009).
In future, the given approach will be extended to the two dimensional boundary value problems as well.

## 6. REFERENCES


Aryassov, G; Petritshenko, A. (2008). Analysis of stress distribution in roots of bolt threads. *Proceedings of the 18th International DAAAM Symposium*, Katalinic, B. (Ed), pp. 0035-0036, ISSN 1726-9679, Trnava, Nov. 2008, DAAAM International, Vienna

Aryassov, G; Petritshenko, A. (2009). Study of free vibration of ladder frames reinforced with plate. J. *Solid State Phenomena*, Vols. 147-149, (Jan. 2009), pp. 368-373, ISSN 1662-9779

Jensen, P, S. (1972). Finite difference techniques for variable grids. *Computer & Structures*, Vol. 2, Iss. 5, (Oct. 1972), pp. 17-29, doi:10.1016/0010-4485(72)90029-2

Kollats, L. (1969). *Numerical methods of the decision of the differential equations*. Machinostrojenije, Moscow

Korn, G; Korn, T. (2000). *Mathematical Handbook for Scientists and Engineers*. Dovers Publications, Inc. ISBN 0486411478