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NEW CONVOLUTIONAL APPROACH OF ESTIMATION OF INCERTAINTY OF COMPRESSIVE STRENGTH TEST RESULTS ON SOME ROCKS

REBRISOREANU, M[ircea] T[raian] I[on]; PENCEA, I[on]; DUMITRESCU, I[oan]; TRAISTA, E[ugen]; NIMARA, C[iprian] & POSTOLACHE, M[ihaela]

Abstract: *The paper addresses the uncertainty estimation of compressive test of rock based of a new convolutional approach. The authors have adopted the uniform probability density distribution of the compressive test as the only one fits the specificity of such a test. By convoluting the uniform density distribution have been calculated the standard deviation of the mean of a series of reproductive tests. The theoretical results were applied to estimate the uncertainty of the results of compressive tests done at University of Petroşani. The authors have proved that the adoption of their solution avoids the uncertainty overestimation as is the case for Gaussian density distribution.*
Key words: *uncertainty, convolution, compresive strength test*

1. INTRODUCTION

Incorrect assessment of the likely behavior of the building foundation and of the zone around a building can result in a considerable additional expenditure of time and money. For this reason a quasi-complete characterization of underground should be done to assist the decision of building or not. Unfortunately, the current standard approaches to the analysis of the relevant interactions between the in situ rock stresses and the strength of the surrounding rock only provide approximate evaluations of the risks (Braun, 2008; Hunter & Fell, 2007). Estimating the compressive strength of the underground is a complicate matter due to the test is a destructive one and is quite impossible to correlate the results obtained on a batch of samples to the bulk underground. As ISO/IEC 17025:2005 recommends, a numerical results should be presented together its uncertainty as a guaranty of the result quality. The term uncertainty should be understood as expanded uncertainty (U) (SR EN 13005, 2005) which means a quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. The association of a specific level of confidence with the interval defined by the expanded uncertainty requires assumptions regarding the probability density distribution associated to the measurand. The confidence level that may be attributed to this interval can be known only to the extent to which such assumptions can be justified (EN ISO 13005; Pencea et. all., 2009).

Taking into account, the specificity of rock compressive testing the authors have developed a new approach of uncertainty estimation based on the following hypotheses:

1. The test it reproducible but not repetitive
2. The density distribution associated to the compressive strength variable is of uniform type
3. The density distribution of mean variable is a multiple convolution of individual ones.

2. THEORETICAL CONSIDERATIONS

The result (r) of compressive test undergone by a specimen is considered as:

$$r = ro + x \tag{1}$$

where: r – the measured value; r_0 – the conventional true value; x- the accuracy of the measured value.

The x is assumed to have a uniform probability density distribution, respectively:

$$f(x) = \begin{cases} \frac{1}{2a}, & x \in [-a, +a] \\ 0, & x \in R \setminus [-a, +a] \end{cases} \tag{2}$$

where: a–the half length of the interval about the zero that encompasses x values;

The above assumption is designed to simplify the calculation and is very usefully because the standard deviation (SD) of x is identically with that of r. If one perform only one test then the SD associated to the result is $SD_1 = a/1.73$. In the most cases, the testing laboratories perform 3 to 10 reproductible tests and report the mean value and the SD of the mean estimated by:

$${SD_n}^2 = \left(\sum_{i=1}^{n} (x_i - \bar{x})^2 \right) / n(n-1) \tag{3}$$

where: $\bar{x}$ and ${SD_n}^2$ are well known experimental mean and experimental standard deviation (Jacobsson, 2005).

The rel.(3) is justified only in the case the probability distribution of the results is Gaussian which is not the case of compressive test of rock. To overpass this inconvenient we have calculated the density distribution of the composed variable $Y_n = X_1 + \ldots + X_n$ as a multiple convolution of identical uniform density distribution. Thus, Y_2 is:

$$Y_2(y) = \int_{(x)} f(x) \cdot f(y-x) \cdot dx = f \otimes f(y) = f^{2\otimes}(y) \tag{4}$$

where: $\otimes$ is the convolution operator (Mihoc & Firescu, 1966).

It is quite easily to derive the Y_n density distribution as the convolution of Y_{n-1} and Y density distributions, respectively:

$$Y_n(y) = Y_n(y) \otimes f(y) = f^{n\otimes}(y) \tag{5}$$

The density distribution of mean variable associated to a batch of n reproducible tests (Mn=Yn/n) is denoted by $f_{Mn}(m)$ and is calculated by:

$$f_{Mn}(m) = n \cdot f_{Yn}(nm) \tag{6}$$

where: m- the mean value of n reproducible tests

The standard deviation of the results of n reproducible tests was calculated by:

$$SD_{Mn}^2 = \int_{-a}^{+a} m^2 f_{Mn}(m)\, dm \tag{7}$$

By our knowledge, there is not a general method to calculate the convolution of n identical uniform density distributions neither for different ones. Because we address only five reproducible tests we present here only the f_{M5} and its associated SD_{m5}, respectively:

$$f_{M5}(m) = \begin{cases} \frac{25(23a^4 - 150a^2m^2 + 625m^4)}{12(2a)^5}, & m \in \left[-\frac{a}{5}, \frac{a}{5}\right] \\ \frac{25(11a^4 + 10a^3m^2 - 150a^2m^2 + 250a^1m^3 - 125m^4)}{6(2a)^5}, & \frac{a}{5} < |m| \le \frac{3a}{5} \\ \frac{3125(a - |m|)^4}{24(2a)^5}, & \frac{3a}{5} < |m| \le a, \\ 0, & m \notin [-a, a] \end{cases} \tag{8}$$

$$SD_{M5} = 0.208 * a \quad (9)$$

The theoretical density distribution of mean given by rel.(8) shows that even for 5 tests the mean variable behave similar to a Gauss-Laplace (N(0,1)) one in the vicinity of theoretical mean and has polynomial profile for the extreme values.

3. EXPERIMENTAL

For a case study we choose a compressive test of an igneous plutonic rock. The compressive tests of the rock were done 5 times on specimens having the layered texture perpendicular to the direction of applied force (F) and 5 times on specimens having layered texture parallel with F. The tests were performed with a hydraulic Universal Press, type Ulanov GUR 08, having 60tf maximum compressive force.
The initial shape of specimens are parallelipiped with the base about 50x50 mm^2 and 100 mm height (Table 2 and Table 3).

4. RESULT AND DISCUSSIONS

In the Table 1 and Table 2 are presented the dimensions of the specimen bases (l_1, l_2), the broken force F and the compressive strength in MPa and in daN/cm^2.
The uncertainty budget of R consists from F, l_1, l_2 facctors and of structural inhomogeneity of sample. The contribution of F, l_1 and l_2 to the relative SD (RSD) could be estimated according to error propagation law as:

$$RSD_R^2 = RSD_F^2 + RSD_{l1}^2 + RSD_{l2}^2 \quad (10)$$

where: $RSD_F^2 = SD_F^2/F^2$; $RSD_{l1}^2 = SD_{l1}^2/l_1^2$ ibid RSD_{l2}^2

The SD_F^2 was estimated as:

$$SD_F^2 = SD_E^2 + SD_O^2 = 200\text{daN} \quad (11)$$

where: SD_E^2 is the certified SD of the equipment and SD_O^2 is the operator contributions. The SD_{l1} and SD_{l1} were estimated at 0.01 mm.

No.	F[daN]	l_1[mm]	l_2[mm]	R(MPa)	R(daN/cm^2
1	35750	47,6	47,3	158,78	1587,84
2	42250	48,4	48,3	180,73	1807,32
3	46700	48,4	48,4	199,35	1993,55
4	41750	48,4	48,3	178,59	1785,93
5	44750	48,4	48,3	191,43	1914,26

Tab. 1. Compressive test data for perpendicular case

No.	F[daN]	l_1[mm]	l_2[mm]	R(MPa)	R(daN/cm^2)
1	39400	48,6	48,2	168,19	1681,95
2	46150	48,6	48,6	195,39	1953,89
3	44250	48,3	48,4	189,29	1892,87
4	39000	48,6	48,2	166,49	1664,87
5	42850	48,4	48,4	182,92	1829,20

Tab. 2. Compressive test data for parallel case

No. test	R(daN/cm2)	RSD_R	SD_R
1	1587,84	0,003176	5,0
2	1807,32	0,003104	5,6
3	1993,55	0,003094	6,2
4	1785,93	0,003105	5,5
5	1914,26	0,0031	5,9

Tab. 3. RSD_R and SD_R for perpendicular case

No. test	R(daN/cm2)	RSD_R	SD_R
1	1681,95	0,003106	5,2
2	1953,89	0,003083	6,0
3	1892,87	0,003101	5,9
4	1664,87	0,003107	5,2
5	1829,20	0,0031	5,7

Tab. 4. RSD_R and SD_R for parallel case

Based on the data in Table 2 and the uniform density distribution (UDD) of M5 variable have been calculated SD_{M5} = 44 daN/cm^2. The extended uncertainty U(95%) for uniform density distribution f_{M5} correspond to m = 0.6a = 2.88*SD_R, respective U_{M5} (95%) = 125 daN/cm^2. In practice, many times the experimentalists use the Gaussian density distribution. In this case the SD of mean is SDm_G = 69 daN/cm^2 and as a consequence Um_G (95%) = 138 daN/cm^2.
In the frame of the same considerations for the parallel case we obtained**:** SD_{M5} = 31 daN/cm^2; U_{M5} (95%) = 90daN/cm^2; SDm_G = 49 daN/cm^2 and Um_G (95%) = 98 daN/cm^2
Using the data presented in Table 3 and Table 4 one can report the compressive strength of the underground rock, with 95% confidence degree, for perpendicular case as: $R_{\perp\mathbf{UDD}}$ = 1817 ± 125 daN/cm^2 or $R_{\perp G}$ = 1817 ± 138 daN/cm^2. The same for parallel case: $R_{\|\mathbf{UDD}}$ = 1805 ± 90 daN/cm^2 or $R_{\|G}$ = 1805± 98 daN/cm^2
The uncertainties of $R_\perp$ and $R_\|$ estimated by both type of density distributions are of the same order but the convolutional approach make more sense and reduce significantly the uncertainty estimated value.
On the other hand, it is quite difficult to estimate the overall R and U_{M5} (95%) of the sample because the variable associated to R is a ten times convolution of the uniform density distribution associated to the variable X.

5. CONCLUSIONS

The UDD associated to the compressive strength variable is more fitted to the case then Gaussian one.
The multi-convolutional approach for calculation of the density distribution of mean variable is the only one way to derive it.
The association of Gaussian density distribution to the compressive strength variable provides an over estimation of the uncertainty at list with 10%.
Our endeavor to derive the general expression for n convoluted identical UDD based on Fourier transformation did not succeed because of complicated expression of inverse Fourier transformation.
We consider that there are other many tests that need a multi-convolutional approach of uncertainty estimation based on uniform probability density distributions.

6. REFERENCES

Braun. R. **(**2008**),** *Consideration of 3D Rock Data for Improved Analysis of Stability and Sanding,* OIL GAS European Magazine 2, ISSN 0179-3187/08/II, Urban-Verlag, Germany

Hunter, G.; Fell, R. (2007). *The deformation behavior of rocks*, ISBN 85841 372 8, New South Wales Ed., Sydney, Australia

Jacobsson, L. (2005). *Triaxial compression test of intact rock*, ISSN 1651-4416 SKB P-05-217, *Available on: www.skb.se, Accessed on: 2010-06-01*

Mihoc, G.; Firescu, D.*(1966). Statistical mathematics,* Didactical and Pedagogical Ed., Bucharest, Romania

Pencea, I.; Sfat, C.E.; Bane, M.; Parvu, S.I. (2009). *Estimation of the contribution of calibration to the uncertainty budget of analytical spectrometry*, Metalurgia, No. 5, pp. 21-27

***, (2005). SR EN ISO/CEI 17025- *Generala requirement for the competence of testing laboratories*, ASRO Ed., Bucharest, Romania

***, (2005). SR EN ISO 13005 –*Guide to the Expression of Uncertainty in Measurement*), ASRO Ed., Bucharest, Romania