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TWO APPROACHES TO COMPUTATION OF STABILIZING PI CONTROLLERS

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Abstract: *The main aim of this contribution is to compare two recent approaches to computation of stabilizing Proportional-Integral (PI) controllers based on plotting the stability boundary locus. First of them, Tan's method, utilizes relatively straightforward computation of the boundary from the closed-loop characteristic polynomial while the second method employs the interesting features of Kronecker summation. The computation process is demonstrated on an illustrative example.*
Key words: *Stabilization, PI Controllers, Kronecker Summation, Stability Region*

1. INTRODUCTION

Contemporary control theory knows many advanced and sophisticated methods on control system design. However, the industrial practice still prefers the application of PI or PID controllers due to their simplicity, easy implementation and reliability. It has been reported that more than 95% of process control applications use PI/PID controllers (O'Dwyer, 2003); (Åström & Hägglund, 1995); (Åström & Hägglund, 2001). Thus, investigation on their suitable tuning is still topical.

The primary and most important request during control synthesis is the stability of closed loop. Among many others, potential techniques to calculation of stabilizing PI(D) controllers have been presented in (Söylemez *et al.*, 2003); (Tan & Kaya, 2003); (Tan *et al.*, 2006); (Fang *et al.*, 2009). Furthermore, possible real control application has been indicated in (Matušů *et al.*, 2010a); (Matušů *et al.*, 2010b).

This paper deals with comparison of Tan's method and Kronecker summation method for computation of stabilizing PI controllers. The exactingness of both approaches is demonstrated on an illustrative example for the third order plant.

2. PROBLEM FORMULATION

Suppose the classical feedback control loop with continuous-time controlled plant:

$$G(s) = \frac{b(s)}{a(s)} \tag{1}$$

and with PI controller:

$$C(s) = k_P + \frac{k_I}{s} = \frac{k_P s + k_I}{s} \tag{2}$$

The aim is to calculate all possible parameters of PI controller which guarantee stability of the closed control circuit. From graphical point of view, the task is to plot the stability boundary locus. Obviously, final results must always be the same independently on the used method. However the ways of their obtaining can differ as described in following sections.

3. TAN'S METHOD

The first approach has been proposed in (Tan & Kaya, 2003); (Tan *et al.*, 2006). Its idea is clear. One needs to substitute $j\omega$ instead of s in the plant (1) and decompose its numerator and denominator into even and odd parts:

$$G(j\omega) = \frac{b_E(-\omega^2) + j\omega b_O(-\omega^2)}{a_E(-\omega^2) + j\omega a_O(-\omega^2)} \tag{3}$$

Consequently, calculation of closed-loop characteristic polynomial and putting the real and imaginary parts to zero results in two expressions:

$$\begin{aligned} k_P &= \frac{\omega^2 a_O(-\omega^2) b_O(-\omega^2) + a_E(-\omega^2) b_E(-\omega^2)}{-\omega^2 b_O^2(-\omega^2) - b_E^2(-\omega^2)} \\ k_I &= \omega^2 \frac{a_E(-\omega^2) b_O(-\omega^2) - a_O(-\omega^2) b_E(-\omega^2)}{-\omega^2 b_O^2(-\omega^2) - b_E^2(-\omega^2)} \end{aligned} \tag{4}$$

Solving the equations simultaneously for various nonnegative frequencies and plotting the gained values into the (k_P, k_I) plane leads to the stability boundary locus. As will be shown in the illustrative example, it splits the plane into the stable and unstable areas. The test point within each region helps to decide which of them represents stability regions. Moreover, the potential problems with proper frequency gridding can be overcome using Nyquist plot based approach from (Söylemez *et al.*, 2003).

4. KRONECKER SUMMATION METHOD

An alternative approach to the previous one, based on interesting properties of Kronecker summation has been described in (Fang *et al.*, 2009). Due to the limited space, this paper exploits only the final rules while their detailed explanation and derivation is omitted.

The paper (Fang *et al.*, 2009) has proved that each couple (k_P, k_I) satisfying:

$$\det\left(M \oplus M\right) = 0 \tag{5}$$

defines the stability boundary. Symbol $\oplus$ represents Kronecker summation (Bernstein, 2005) and M is a square matrix:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \ \cdots & 0 \\ 0 & 0 & 1 & 0 \ \cdots & 0 \\ \vdots & 0 & 0 & 1 \ \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \ \ddots & 0 \\ 0 & 0 & 0 & \cdots \ 0 & 1 \\ -\frac{f_0(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_1(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_2(k_P,k_I)}{f_n(k_P,k_I)} & \cdots \ \cdots & -\frac{f_{n-1}(k_P,k_I)}{f_n(k_P,k_I)} \end{bmatrix} \tag{6}$$

where the last row coefficients follow from the closed-loop characteristic polynomial:

$$\begin{aligned} P_{CL} &= a(s)s + b(s)\left(k_P s + k_I\right) = \\ &= f_n(k_P,k_I)s^n + \cdots + f_1(k_P,k_I)s + f_0(k_P,k_I) \end{aligned} \tag{7}$$

Similarly to the previous method, the determination of the stabilizing area(s) can be done using a test point within each region.

5. ILLUSTRATIVE EXAMPLE

Consider a third order controlled system given by transfer function:

$$G(s) = \frac{5}{s^3 + 2s^2 + 3s + 4} \tag{8}$$

First, the Tan's method has been applied. For plant (8), the relations (4) take the final form:

$$\begin{aligned} k_P &= 0.4\omega^2 - 0.8 \\ k_I &= -0.2\omega^4 + 0.6\omega^2 \end{aligned} \tag{9}$$

Solution of (9) leads directly to pairs of PI controller coefficients which constitute stability boundary locus.

Alternatively, for Kronecker summation method, the matrix (6) is:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5k_I & -4-5k_P & -3 & -2 \end{bmatrix} \tag{10}$$

So, one has to calculate the pairs (k_P, k_I) which fulfil (5). Again, these values define the curve splitting the plane into stable and unstable regions.

Both approaches result in the stability boundary locus as depicted in fig. 1. Moreover, the fact that the inner space represents the region of stability can be easily tested using arbitrary point (k_P, k_I) from relevant part and calculating corresponding closed-loop characteristic polynomial.

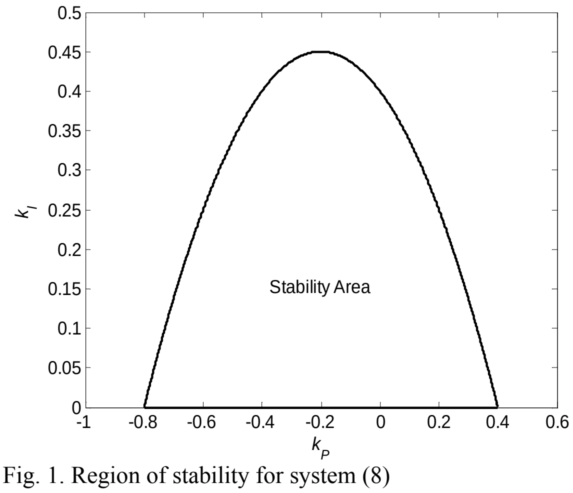

Fig. 1. Region of stability for system (8)

Moreover, both methods can be further embellished via so-called sixteen plant theorem (Barmish *et al.*, 1992); (Barmish, 1994) in order to make them usable for robust stability of closed loops with PI controllers and interval plants (Tan *et al.*, 2006); (Matušů, 2008).

Obviously not all possible stabilizing combinations from fig. 1 would comply with requirements under real control conditions (negative gain, performance specifications, etc.). However, selection of the final controller according to user demands is another task (Matušů *et al.*, 2010b).

6. CONCLUSION

The paper has been focused on analysis of two recent techniques to determination of stabilizing PI controllers in order to highlight the practical computational differences of the methods. This comparison has been done by means of simple example in which the third order controlled system has been stabilized

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