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ONE OF POSSIBLE APPROACHES TO CONTROL OF MULTIVARIABLE CONTROL LOOPS: SIMULATION VERIFICATION OF PROPOSED APPROCH TO CONTROL

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Abstract: *The paper describes possible approache to control of MIMO control loop. The designed method combinations classical way to ensure of autonomy of control loop via binding members and the use of the method of SISO branched control loops with measurement of disturbance to ensure of invariance of control loop via correction members. Simulation verification of proposed approach was carried out for three-variable loop.*
Key words: *MIMO, SISO, modelling, synthesis, simulation*

1. INTRODUCTION

At large numbers of controlled systems several variables have to be controlled at the same time. Typical examples of these systems are e.g. reactors, steam boilers, turbines, etc. In these cases there are not larger member of independent SISO (single-variable) control loop. These control loops are complex with several controlled variables where separate variables are not mutually independent. Mutual coupling of controlled variables is usually given by simultaneous action of each of input (manipulated and disturbance) variables of controlled plant to all controlled variables. These control loops are called MIMO (multi-variable) control loops and they are a complex of mutually influencing simpler control loops. (Balate, 2004).

2. MULTI-VARIABLE CONTROL LOOP

It is considered MIMO control loop with measurement of load disturbance according to Fig. 1. (Navratil & Balate, 2007).

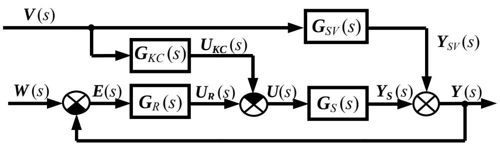

Fig. 1. MIMO control loop with measurement of disturbance

Legend: $\boldsymbol{G}_S(s)$, $\boldsymbol{G}_R(s)$, $\boldsymbol{G}_{SV}(s)$, $\boldsymbol{G}_{KC}(s)$ - transfer matrix of a controlled plant, controller, disturbance variables and correction members; $\boldsymbol{Y}(s)$ $[n\times1]$, $\boldsymbol{U}(s)$ $[n\times1]$, $\boldsymbol{V}(s)$ $[m\times1]$ - vector of controlled variables, manipulated variables and disturbance variables; $m \le n$

Transfer matrixes of controlled plant, disturbance variables, controller and correction members are considered in forms

$$\boldsymbol{G}_S(s)=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{SV}(s)=\begin{bmatrix} S_{V11} & S_{V12} & \cdots & S_{V1m} \\ S_{V21} & S_{V22} & \cdots & S_{V2m} \\ \vdots & \vdots & \cdots & \vdots \\ S_{Vn1} & S_{Vn2} & \cdots & S_{Vnm} \end{bmatrix} \qquad (1)$$

$$\boldsymbol{G}_R(s)=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{KC}(s)=\begin{bmatrix} KC_{11} & 0 & \cdots & 0 \\ 0 & KC_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & KC_{nn} \end{bmatrix} \qquad (2)$$

2.1 Autonomy and invariance of multi-variable control loop

At synthesis of MIMO control loop it is often required for control loop to be autonomous and invariant. In order to these conditions were ensured we start from a command transfer matrix $\boldsymbol{G}_{W/Y}(s)$ and disturbance transfer matrix $\boldsymbol{G}_{V/Y}(s)$

$$\boldsymbol{G}_{W/Y}(s) = \left[\boldsymbol{I}+\boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1}\boldsymbol{G}_S(s)\boldsymbol{G}_R(s) \qquad (3)$$

$$\boldsymbol{G}_{V/Y}(\mathrm{s}) = \left[\boldsymbol{I}+\boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1}\left[\boldsymbol{G}_{SV}(s)-\boldsymbol{G}_S(s)\boldsymbol{G}_{KC}(s)\right] \qquad (4)$$

For ensuring autonomy it is necessary that the matrix $\boldsymbol{G_S}(s)\,\boldsymbol{G_R}(s)$ is diagonal. This is possible if relation (5) is valid

$$\frac{R_{kl}}{R_{ml}}=\frac{s_{lk}}{s_{lm}} \qquad k,l,m=<1,\dots,n>,\ s_{lm}\neq 0 \qquad (5)$$

where s_{lk}, s_{lm} are algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G}_S(s)$ and R_{kl}, R_{ml} are separate members of a transfer matrix of controller $\boldsymbol{G}_R(s)$.

It is considered that diagonal (main) controllers R_{11}, R_{22},…, R_{nn} are usually known already from the first design of conception of control (Balate, 2004).

For ensuring invariance it is necessary that the transfer matrix $G_{V/Y}(s)$ is zero. This is possible if relation (6) is valid

$$\boldsymbol{G}_{KC}(s)=\boldsymbol{G}_S^{-1}(s)\boldsymbol{G}_{SV}(s) \qquad (6)$$

At design of correction members *KC*, the task of which is to eliminate the influence of disturbance variable on control loop, internal couplings are omitted at MIMO control loop and thus *n* SISO branched control loop with measurement of a disturbance are gained. Connection of all these SISO control loops is the same and they differ only in separate transfers of controlled plants, controllers, correction members and disturbance variables (Balate, 2004). Transfer of correction members *KC* are gained by using the equation (6) in the following form

$$KC_{ii}=\frac{S_{Vii}}{S_{ii}} \qquad i=<1,\dots,n>,\ S_{ii}\neq 0 \qquad (7)$$

where S_{Vii} are separate members of transfer matrix of disturbance variables $\boldsymbol{G_{SV}}(\mathrm{s})$ and S_{ii} are separate members of transfer matrix of controlled plant $\boldsymbol{G_S}(\mathrm{s})$

2.2 Synthesis of multi-variable control loop

One of the possible approaches to MIMO control loops synthesis is described in the following part of the paper. Generally it is possible to divide this problem into three parts:

- Design of diagonal controllers by any synthesis method of SISO control loops, i.e. design of parameters of main controllers for *n* SISO control loops (R_{11}, R_{22}, …, R_{nn}). Here, it is considered that original diagonal transfer functions S_{ii} ($i = 1, \dots, n$) of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$ are modified to diagonal transfer functions $S_{ii,x}$ ($i = 1, \dots, n$). In these modified transfer functions influences of aside-from diagonal transfer functions of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$, i.e. S_{ij} ($i \neq j$, $i, j = 1, \dots, n$) on original diagonal transfer functions i.e. S_{ii} ($i = 1, \dots, n$) are included. Transfer functions $S_{ii,x}$, i.e $S_{11,x}$, $S_{22,x}$, $S_{33,x}$ etc. are determined from equation (8) by using relations (3) and (5).

$$S_{ii,x}=\sum_{j=1}^{n} S_{ij}\frac{s_{ij}}{s_{ii}} \qquad i,j=<1,\dots,n>,\ s_{ii}\neq 0 \qquad (8)$$

  where s_{ii}, s_{ij} are algebraic supplements of separate elements of a controlled plant transfer matrix $\boldsymbol{G_S}(s)$ and S_{ij} are separate members of a controlled plant transfer matrix $\boldsymbol{G_S}(s)$.
- Ensuring of autonomy of control loop via binding members (5) of transfer matrix of controller G_R(s).
- Ensuring of invariance of control loop via correction members KC (7) by using *n* SISO branched control loops with measuring of disturbance variables *v*.

3. SIMULATION VERIFICATION

Typical example of MIMO controlled plant is a steam turbine. In this case, the turbine with two controlled withdrawals which drives electric generator supplying determined part of electric network (it operates without phasing into power network) is considered (see Fig. 2).

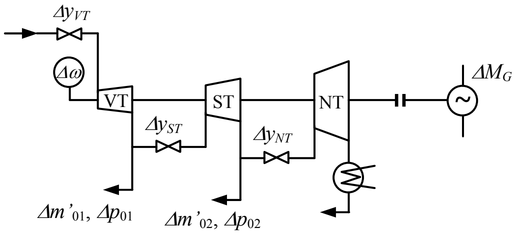

Fig. 2. Three-variable control loop of steam turbine

Legend: ΔM_G - change of electric load of turbo-generator, $\Delta\omega$ - change of angular speed of turbo-generator, $\Delta m'_{01}$, $\Delta m'_{02}$ - change of mass flow of withdrawn steam, Δp_{01}, Δp_{02} - change of steam pressure in corresponding withdrawals, Δy_{VT}, Δy_{ST}, Δy_{NT} - change of opening position of control valves of high-pressure, medium-pressure and low-pressure part of turbine

It is considered that controlled variables y are $\Delta\omega$, Δp_{01}, Δp_{02}, disturbance variables v are ΔM_G, $\Delta m'_{01}$, $\Delta m'_{02}$, manipulated variables u are Δy_{VT}, Δy_{ST}, Δy_{NT}.

3.1 Mathematical model of steam turbine

Resulting differential equations for creating mathematical model of the plant were gained already after deriving and using linearization (ALSTOM Power, 1998).

The first equation is equation of moment balance

$$\begin{aligned}518.4\Delta\dot{\omega} &= -63.3\Delta\omega + 656.9\Delta p_{01} + 4611.7\Delta p_{02} + 1007.3\Delta y_{VT} \\ &+ 200.6\Delta y_{ST} + 121.5\Delta y_{NT} - \Delta M_G\end{aligned} \tag{9}$$

next equations are equations of flow through flow spaces

$$1.865\Delta\dot{p}_{01} = -1.610\Delta p_{01} + 0.167\Delta p_{02} + 1.523\Delta y_{VT} - 0.361\Delta y_{ST} - \Delta m'_{01} \tag{10}$$

$$13.45\Delta\dot{p}_{02} = 1.563\Delta p_{01} - 10.517\Delta p_{02} + 0.361\Delta y_{ST} - 0.222\Delta y_{NT} - \Delta m'_{02} \tag{11}$$

It is possible to re-write the equations (9) - (11) into better form by introducing relative values, with regard to starting stable state-operational (calculated point), at which relation of values can be generally written in the form $\varphi_X = \Delta X / (X)_0$ where

$$\begin{array}{lll}(\omega)_0 = 628.3\,[\text{rad/s}] & (p_{01})_0 = 14\,[\text{bar}] & (p_{02})_0 = 1{,}55\,[\text{bar}] \\ (y_{VT})_0 = 19.15\,[\text{mm}] & (y_{ST})_0 = 59.9\,[\text{mm}] & (y_{NT})_0 = 69.8\,[\text{mm}] \\ (M_G)_0 = 39789\,[\text{Nm}] & (m'_{01})_0 = 6.94\,[\text{kg/s}] & (m'_{02})_0 = 6.94\,[\text{kg/s}]\end{array} \tag{12}$$

hence

$$\begin{aligned}325710.7\dot{\varphi}_{\omega} &= -39771.4\varphi_{\omega} + 9196.6\varphi_{p_{01}} + 7148.1\varphi_{p_{02}} + 19289.8\varphi_{y_{VT}} \\ &+ 12015.9\varphi_{y_{ST}} + 8480.7\varphi_{y_{NT}} - 39789\varphi_{M_G}\end{aligned} \tag{13}$$

$$26.110\dot{\varphi}_{p_{01}} = -22.540\varphi_{p_{01}} + 0.259\varphi_{p_{02}} + 29.165\varphi_{y_{VT}} - 21.624\varphi_{y_{ST}} - 6.94\varphi_{m'_{01}} \tag{14}$$

$$20.848\dot{\varphi}_{p_{02}} = 21.882\varphi_{p_{01}} - 16.301\varphi_{p_{02}} + 21.624\varphi_{y_{ST}} - 15.496\varphi_{y_{NT}} - 6.94\varphi_{m'_{02}} \tag{15}$$

The Laplace transform of an output (controlled) variable is given by the following relation

$$\begin{bmatrix}\varphi_{\omega} \\ \varphi_{p_{01}} \\ \varphi_{p_{02}}\end{bmatrix} = \boldsymbol{G}_S(s)\begin{bmatrix}\varphi_{y_{VT}} \\ \varphi_{y_{ST}} \\ \varphi_{y_{NT}}\end{bmatrix} + \boldsymbol{G}_{SV}(s)\begin{bmatrix}\varphi_{M_G} \\ \varphi_{m'_{01}} \\ \varphi_{m'_{02}}\end{bmatrix} \tag{16}$$

where

$$\boldsymbol{G}_S(s) = \begin{bmatrix}\frac{0.73s^2 + 1.59s + 1.11}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{0.46s^2 + 0.74s + 0.09}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{0.32s^2 + 0.32s + 0.04}{12.3s^3 + 21.8s^2 + 10.7s + 1} \\ \frac{1.68s + 1.31}{1.51s^2 + 2.48s + 1} & \frac{-1.25s - 0.96}{1.51s^2 + 2.48s + 1} & \frac{-0.01}{1.51s^2 + 2.48s + 1} \\ \frac{1.76}{1.51s^2 + 2.48s + 1} & \frac{1.56s + 0.040}{1.51s^2 + 2.48s + 1} & \frac{-1.12s - 0.97}{1.51s^2 + 2.48s + 1}\end{bmatrix} \tag{17}$$

$$\boldsymbol{G}_{SV}(s) = \begin{bmatrix}\frac{-1.51s^2 - 2.48s - 1}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{-0.09s - 0.15}{12.3s^3 + 21.8s^2 + 10.7s + 1} & \frac{-0.09s - 0.08}{12.3s^3 + 21.8s^2 + 10.7s + 1} \\ 0 & \frac{-0.40s - 031}{1.51s^2 + 2.48s + 1} & \frac{-0{,}005}{1.51s^2 + 2.48s + 1} \\ 0 & \frac{-0.420}{1.51s^2 + 2.48s + 1} & \frac{-0.501s - 0.432}{1.51s^2 + 2.48s + 1}\end{bmatrix} \tag{18}$$

3.2 Synthesis of three-variable control loop

In the next part of the paper the principal described in the paragraph 2.2 is used. First transfers of main controllers R_{11}, R_{22}, R_{33} are determined for modified diagonal transfer functions $S_{11,x}$, $S_{22,x}$ a $S_{33,x}$ (8) then autonomy of control loop by using relation (5) is being solved and in the end fulfilment of the condition of invariance (approximate invariance) of control loop is ensured by using equation (7). At design of parameters of main controllers the following SISO methods of synthesis were used, i.e. Ziegler Nichols step response method (Balate, 2004), method of desired model (Balate, 2004), polynomial method of synthesis for 1DOF configuration - pole placement method (Prokop et al., 2006).

Transfer matrix of controllers $\boldsymbol{G}_R(s)$ with utilization of a chosen synthesis method (polynomial method of synthesis) and transfer matrix of correction members $\boldsymbol{G}_{KC}(s)$ are given by the equations (19) and (20).

$$\boldsymbol{G}_R(s) = \begin{bmatrix}\frac{0.81s + 0.09}{s} & \frac{-0.39s^2 + 0.22s + 0.03}{s(s + 2.09)} & \frac{-0.08s^2 + 0.02s + 0.003}{s(s + 1.06)} \\ \frac{1.09s + 0.13}{s} & \frac{0.312s - 0.22}{s} & \frac{-0.11s^2 + 0.02s + 0.006}{s(s + 1.06)} \\ \frac{1.52s + 0.18}{s} & \frac{0.44s^2 - 0.37s + 0.04}{s(s + 2.09)} & \frac{0.33s - 0.11}{s}\end{bmatrix} \tag{19}$$

$$\boldsymbol{G}_{KC}(s) = diag(KC_{ii});\ i = 1,...,3;\ KC_{11} = \frac{-2.06s^2 - 3.39s - 1.37}{s^2 + 2.18s + 1.52};\ KC_{22} = \frac{0.32s + 0.25}{s + 0.77};\ KC_{33} = 0.45 \tag{20}$$

Simulation course of three-variable control loop of a steam turbine with utilization of chosen SISO synthesis method is presented on the Fig. 3. To simulation of the control loop the program MATLAB/SIMULINK was used (Karban, 2006).

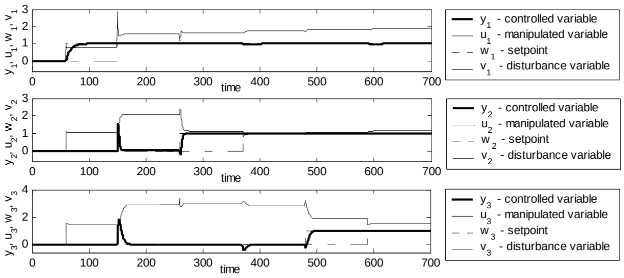

Fig. 3.Simulation course of control loop with utilization of polynomial method of synthesis

Variables on the previous figure (Fig. 3.) correspond to variables in the three-variable control loop of steam turbine (see Fig. 2 and its detailed description).

It is obvious from result of simulation of control loop presented on the Fig. 3 and from other simulation experiments (Navratil & Balate, 2007) that the condition of autonomy and invariance was fulfilled. Autonomy of control loop was ensured via binding members R_{ij}, which are aside-from-diagonal elements of transfer matrix of controller $\boldsymbol{G}_R(s)$. Invariance (approximate invariance) of control loop was ensured by means of correction members KC_{ii} which are considered for elimination of influence of diagonal disturbance variables v.

4. CONCLUSION

This paper dealt with a description of one of possible methods to control of multivariable control loop. By means of described method it is possible to ensure of autonomy and invariance (approximate invariance) of MIMO control loop simultaneously. It is considered that MIMO controlled plant has the same number of input and output signals. Next, it is considered that correction members serve to elimination influences only diagonal disturbance variables of transfer matrix of disturbance variables.

Simulation verification of described method presented on three-variable control loop of steam turbine indicates that it is possible to utilize this method for control of MIMO systems.

5. ACKNOWLEDGEMENT

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6. REFERENCES

ALSTOM Power, Ltd. (1998) *Materials of the company from the project OTROKOVICE*. (in Czech)

Balate, J. (2004). *Automatic Control*, 2nd edition, BEN - technical literature, ISBN 80-7300-148-9, Praha. (in Czech)

Karban, P. (2006). *Computing and simulation in program Matlab and Simulink*. Computer Press, ISBN 80-251-1301-9, Brno. (in Czech)

Navratil, P. & Balate, J. (2007). One of possible approaches to control of multivariable control loop, *8th International IFAC Symposium on Dynamics and Control of Process Systems*. Vol.1., pp. 207-212, Cancun, Mexico, June 2007.

Prokop, R.; Matusu, R. & Prokopova, Z. (2006). *Automatic control theory - linear continuous dynamic systems*. TBU in Zlin, ISBN 80-7317-369-2, Zlin. (in Czech)