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SPARSE TIME SERIES INTERPOLATION OF DAM DISPLACEMENTS

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Abstract: *In this paper work are presented the performances of three interpolation methods of sparse time series from the correlation point of view. The values of the time series represents the dam crust displacements measured for the same target (measuring) points. The two kinds of dam displacements time series are generate from two different measuring systems involved in dam monitoring process. One time series is a very large numbered and the other has just a few value and is a sparse time series. There is presented the comparison of the Fourier based correlation results between the large numbered time series and the sparse interpolated ones with the radial basis function, cubic spline and Fourier interpolation methods.*
Key words: *interpolation, sparse time series, RBF, Fourier transform.*

1. INTRODUCTION

Civil engineering structures are monitored by measuring their movements or displacements. The displacements are usually measured by two different independent systemts. The only common things are the target points (i.e. the measuring points) of which the displacements are measured.

Dam monitoring consists in measuring the dam crustal displacements at certain target points with two differents systems. The physical one involves displacements measuring with an optical coordiscope of an inverted pendulum built inside a dam plot (abutment). The surveying one involves a microtriangulation network buit up on a set of control points from which the displacements of the target points are measured with a surveying total station.

These two monitoring systems generate two time series of the displacements of the same target points. The inverted pendulum time series has a one measure per day resolution. The surveying displacements measuring epochs has a resolution of only 2 per year (2 values/365 days); thus is generate a sparse time series compared with the inverted pendulum time series (365 values/365 days) (Pytharouli & Stiros, 2008).

The goal of the dam monitoring is to avoid the dam cracking by very large values of the displacements. To ensure a high consistency of the displacements data the dam monitoring companies have to make reports that involve correlations between the two displacements time series (Pytharouli & Stiros, 2005). These correlations can be done with normalized Fourier correlation coefficient, $NFCC$, described (Grierson, 2006) by

$$NFCC(f(t),g(t)) = \max_x \left[\frac{\mathcal{F}^{-1}\left[\overline{F(-\nu)}\cdot G(\nu)\right](t)}{\max_x\left[\mathcal{F}^{-1}\left[\left|F(\nu)\right|^2\right](t)\right]^{0.5}\cdot \max_x\left[\mathcal{F}^{-1}\left[\left|G(\nu)\right|^2\right](t)\right]^{0.5}} \right] \quad (1)$$

where $f(x), g(x)$ are two functions, $F(k), G(k)$ are their Fourier transforms, t is the time, k is the frequency, $\mathcal{F}^{-1}$ is the inverse Fourier transform.

The correlation process can be done only when the time series are the same kind. In our case the displacements inverted pendulum time series have 2010 values and the surveying one have only 12 values, both belonging to the 2000-2005 time period. A good way to make the correlation possible is to interpolate the sparse time series (with 12 values) in a 2010 values time series.

In this paper are presented three interpolation methods of sparse time series: radial basis function (RBF) interpolation, Fourier interpolation and spline interpolation – the last one as reference method, and thus not emphasized.

2. METHODS

2.1 Radial basis function interpolation

Radial basis function (RBF) interpolation consists in finding the coefficients, $\lambda = (\lambda_1,\ldots,\lambda_n)$, for a base of radial functions and the coefficients, $c = (c_1,\ldots,c_l)$, for a set of fitting polynomial, $p = \{p_1,\ldots,p_l\}$, so that this interpolation function $s(x)$ defined below (Boer et al., 2007; Carr et al., 2003)

$$s(x) = p(x) + \sum_{i=1}^{n} \lambda_i \cdot \phi(\left|x - x_i\right|) \ , \ x \in R^n \quad (2)$$

has to pass through the values of definition (Carr et al., 2003)

$$s(x_i) = y_i \, , \ i = \overline{1,n} \ \text{ and } \ \sum_{j=1}^{n} \lambda_j \cdot p(x_j) = 0 \, , \quad (3)$$

where $(x_i; y_i)$ are the coordinates of N known points.

The thin plate radial function, $\phi(r) = r^2 \cdot \ln(r)$, was chosen for the studied case. These conditions, under the matrix form, can be written the following form (Carr et al., 2003)

$$\begin{pmatrix} R & P \\ P^T & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ c \end{pmatrix} = \begin{pmatrix} Y \\ 0 \end{pmatrix} \quad (4)$$

where we have: $R_{i,j} = \phi(\left|x_i - x_j\right|)$, $P_{i,l} = p_l(x_i)$, $Y_i = y_i$, $i, j = \overline{1,n}$, $l = \overline{1,m}$. The generated equations system has the solution given by

$$\begin{aligned} c &= \left[(P^T \cdot R^{-1} \cdot P)^{-1}\right] \cdot (P^T \cdot R^{-1} \cdot Y) \, , \\ \lambda &= (R^{-1} \cdot Y) - (R^{-1} \cdot P) \cdot [(P^T \cdot R^{-1} \cdot P)^{-1} \cdot (P^T \cdot R^{-1} \cdot Y)] \, . \end{aligned} \quad (5)$$

2.2 Fourier interpolation

We assume that a sparse time series, $g(t)$, has n values of definition and it have to be interpolated over $m(>> n)$ values. The Fourier interpolation first stage consists in zero padding the 12 values outside the definition time values in order to obtain a new 2010 time series, $zp\_g(t)$. After the zero padding the time series, $zp\_g(t)$, is Fourier transformed, $zp\_G(k)$ - k is the frequency. The Fourier transform frequencies, $zp\_G(k)$, are filtered with a low-pass band limited of size $W = m$.

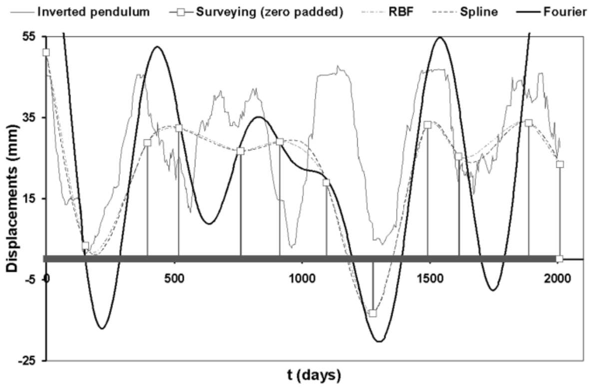

Fig. 1. Displacements initial time series and interpolated sparse time series, for taget point 5.

The filtering result, $bf\_zp\_G(k)$, is inverse Fourier transformed. The final time series result has m values, interpolated from the n sparse values (Grierson, 2006). The overall Fourier interpolation of sparse time series $g(t)$ can be summarize by the equation below

$$iF\_g(t) = = \mathcal{F}^{-1}\left[\frac{m}{W}\cdot bf\_zp\_G(k)\right] = \mathcal{F}^{-1}\left[\frac{m}{W}\cdot bf\left[\mathcal{F}[zp\_g(t)]\right]\right]. \quad (6)$$

3. RESULTS AND DISCUSSIONS

The data used for the interpolation methods testing were gathered from the five target points of the median plot of Drăgan dam – Cluj County, Romania. The five target points are distributed on the vertical axis of the mentioned plot. The entire database consists in five time series with $m = 2010$ displacements values from the inverted pendulum measuring system and five sparse time series with $n = 12$ displacements values from the surveying measuring system (figure 1).

The interpolation results for target point 5 are presented in the figure 1. The RBF interpolation polynom is of second degree. The spline interpolation has a cubic interpolant function.

In order to emphasize the benefits of the three interpolation methods in the dam monitoring process were done two sets of correlations.

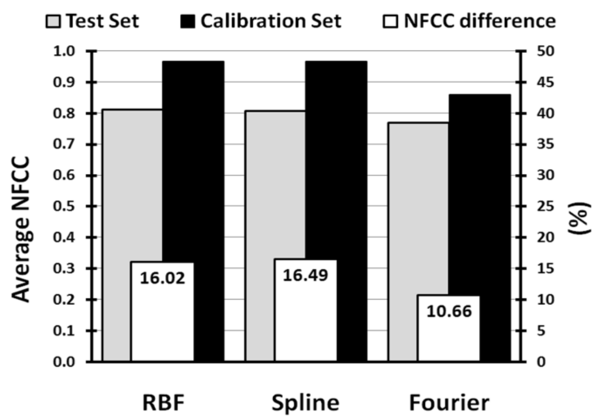

Fig. 2. Bar graph of the correlation results: the test set (the grey bars), calibration set (the black bars), the correlation results differences between the calibration set and test set (the white bars, on secondary axis).

The first set are the correlations between the displacements time series measured by inverted pendulum (2010 values) and a sparse time series as a partion of the first one (12 values) – figure 2 with the black bars. This set was intended to see the interpolation performances within the same measuring system – this set it will be denoted as the calibration set.

The second set are the correlations between the displacements time series measured by inverted pendulum (2010 values) and the sparse time series (12 values) measured by surveying method – figure 2 with the white bars. This set was intended to see the interpolation performances between the two different measuring systems – this set it will be denoted as the test set.

The bar graph in figure 2 presents the average values of *NFCC* - on the primary axis - and the differences between these values for the calibration set and test set, for the five target points, an the secondary axis.

In both cases – calibration and test sets – RBF and Spline interpolation methods have high and sensible equal *NFCC* values. The Fourier interpolation *NFCC* values are lower than RBF and spline interpolation ones: in calibration set case with 12.28% and in the test set case with only 4.96%.

4. CONCLUSIONS

Three interpolation methods of sparse time series for correlation of dam crustal displacements are presented. The correlation methos is based on Fourier transform (eqn. 1).

From the correlation point of view, the best interpolation performances have the RBF and cubic spline methods in the both calibration and test set cases – as these methods has the highest *NFCC* values (figure 2). Despite these good performances, the most robust correlation interpolation method is the Fourier one, as it has the lowest *NFCC* difference , 10.66%, between the calibration and test set cases (figure 2), while the RBF and spline interpolation methods have 16.02% and 16.49% *NFCC* difference values (figure 2). This means that if one must choose a robust correlation sparse time series interpolation method of displacements measured with two different systems, then the Fourier interpolation method is a better choice than RBF and spline interpolation methods.

Future work can involve 2D sparse interpolation of dam horizontal displacements time series.

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