Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
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Annals of DAAAM International

ON THE GEOMETRY OF THE PENSTOCK LOWER BEND FOR LARGE FLOW FRANCIS TURBINES

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Abstract: *Hydropowers stations with large Francis turbines, have, generally, one penstock for each turbine. The penstocks are manufactured from metal sheets, bended and welded. The more complex part of each penstock is the penstock lower bend. This part is made from metal shels bordered by circles of different diameters, placed in inclined plans, one from each others. But, these important parts of the hydropowers stations can be manufactured in situ, in the base of a combination between technical and mathematical elements. This kind of combination was used, until now successfully in manufacturing of welded spiral casing and draft tubes with oval cross section for hydraulic turbines. The paper that we propose presents the mathematical elements of this method.*
Key words: *Francis turbines penstock technology*

1. MATHEMATICAL ELEMENTS

From mathematical point of view, any bended sheet of metal is a deployment surface (Bărglăzan M., 1999).

Such a shell – considered as a ruled surface - is obtained by shifting in space a straight line. Some ruled surfaces are not deployable.

To be deployable, a surface must accomplish a certain condition (Sundar Varada Raj P., 1995, Şriro I.I., 1961).

So, if in a xOyz reference frame a deployable surface has the equation (1).

$$z = f(x,y)\ , \tag{1}$$

then this surface in deployable if satisfied the differential equation (2)

For the case of the spiral casing of certain hydraulic machines, this equation was considered from long ago, and was used including to patented technologies, and, also, the problem was studied (Vertan Gh. et al., Man E.T. et al., 2010).

$$\frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} = \left(\frac{\partial^2 z}{\partial x\, \partial y} \right)^2 . \tag{2}$$

2. THE DETERMINATION OF THE CURRENT POINT COORDINATED ON THE SPATIAL CONTOUR OF THE SHELLS

For a current ruled surface for a penstock lower bend with oval sections $A_1A_nB_nB_1$ in a xOyz reference frame (figure 1 „a").

The end of the shell placed in xOy plane is defined by the known values l_1 and R_1 (figure 1 „b"). The other end is placed in x_1Oy plane, which has the angle δ with xOy plane.

This second end is defined by the values l_2 and R_2 (figure 1 „c").

In the plane xOy, a current point from the contour A_i is defined through the angle α and has the coordinates:

$$\begin{cases} x_{Ai} = l_1 + R_1 \cdot \cos(\alpha) \\ y_{Ai} = R_1 \cdot \sin(\alpha) \\ z_{Ai} = 0 \end{cases} . \tag{3}$$

At the same shell, in x_1Oy plane, a current point B_i from the contour is determined by the angle β, which, in the plane x_1Oy has the abscissa

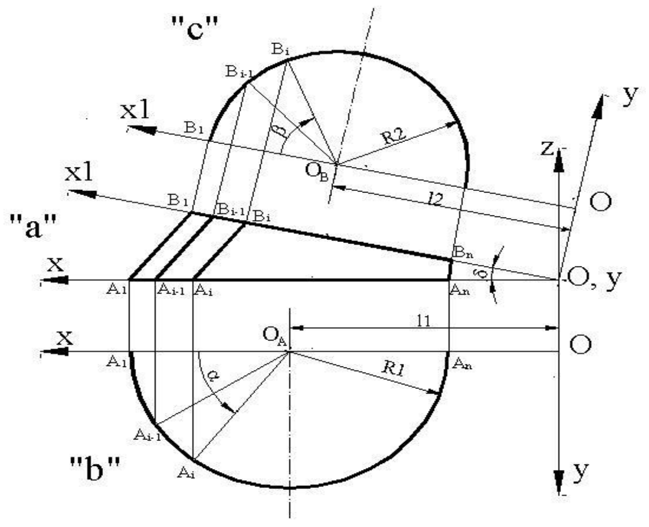

Fig. 1. Symmetrical shell of penstock lower bend

$$x'_{Bi} = l_2 + R_2 \cdot \cos(\beta) \quad (4)$$

and in the system xOyz has the coordinates

$$\begin{cases} x_{Bi} = \left(l_2 + R_2 \cdot \cos(\beta)\right) \cdot cos(\delta) \\ y_{Bi} = R_2 \cdot \sin(\beta) \\ z_{Bi} = \left(l_2 + R_2 \cdot \cos(\beta)\right) \cdot sin(\delta) \end{cases} . \quad (5)$$

3. THE DETERMINATION OF THE CURRENT GENERATOR OF THE SURFACE

For any shell of the penstock lower bend the vital problem consist in determining the point B_i, as function of the current point A_i, in such a matter that the straight line A_iB_i represents the current generator of the surface supported by the curves $A_1A_iA_n$ and $B_1B_iB_n$.

Essentially, the problem consist in determination the angle β as function of angle α and the rest geometrical elements, such as: l_1, R_1, δ, l_2, R_2:

$$\beta = arctg\left(\frac{l_2 \cdot sin(\alpha)}{l_1 \cdot cos(\alpha)}\right) + \\ + arctg\left(\frac{R_2 \cdot sin(\alpha)}{\sqrt{l_2^2 \cdot cos(\alpha) + \left(l_1 \cdot cos(\alpha) + R_1\right)^2 - .. \\ ... - R_2 \cdot sin^2(\alpha)}}\right) \quad (6)$$

for any $\alpha \in \left(0, \frac{\pi}{2}\right)$.

4. DETERMINATION OF THE CONTOUR OF THE SHELL AND THE BENDINGS LINES

In this purpose there is used the mathematical notion of geodetic curve, whose length is, generally, determined by an integral equation.

The shell surface is decomposed in n quadrilateral surfaces, going from an initial generator A_1B_1 to the final one A_nB_n.

The number "n" is determined to be convenable from technological point of view, to result finally "n" bending lines, uniform distributed on the whole surface of the shell.

Then, each quadrilateral is decomposed in two triangles.

Ones are the triangles $A_{i-1}A_iB_{i-1}$ and $B_{i-1}B_iA_i$, (figure 1 "a") in the first hypothesis.

In the second hypothesis, the considered triangles are $A_{i-1}B_{i-1}B_i$ and $A_{i-1}A_iB_i$ (figure 1 "a").

To find the length of the sides of this triangles means that the wholw surface of the shell is decomposed in triangles with known sides.

Going from the initial generator, each triangle is developed, obtaining both the developing of the shell and the corresponding positions of the bending lines.

Each of the four triangles, considered on the surface of the spatial shell (figure 1 "a"), is helded in plan and has two sides exactly calculable.

One of this is the generator $A_{i-1}B_{i-1}$ or A_iB_i, and the second is the arc curve $A_{i-1}A_i$ or $B_{i-1}B_i$. And every of this four triangles has a third side ($A_{i-1}B_i$ or $B_{i-1}A_i$), which, on the curve surface of the shell is a geodetic curve.

Considering that all the quadrilaters which compose a particular shell are decomposable in two triangles, following the procedure presented above, results that all points A_i and B_i (i = 2, …, n), exception A_1 and B_1, placed on the initial generator, has the coordinate approximate.

The approximation is higher for the points A_n and B_n, which has maximum errors, the coordinates values being higher than the real ones.

In order to obtain acceptable results, the shell decompose in n_c > n quadrilaters, growing up their number, n_c, until the difference between the coordinates calculated through both hypothesis are less than the accepted tolerance.

From all this points A_i and B_i will be withhelded a "n" number uniform distributed on the contour of the shell; the number "n" will be chosen to be convenient from the technologic point of view.

5. CONCLUSIONS

The absolute value of max(|α-β|) differs form one shell to another. In industrial cases, this difference is about 1^o to 5^o.

If is considered α = β, then results small differences from the correct variant, but the bending lines induces technological difficulties, the shells differing from the correct form.

The method based on the exposed brieffly calculus offer the coordinates of the points representing the contour of each particular shell and, also, the bending lines; this allowed realizing the shells at the site of the hydroelectric power plant, which reduce considerably the costs.

Determination, in the limits of accepted tolerance, of the coordinates of the points that represent the contour of each shell and, also, the ends of the bending lines, ensure the correct cutting off and the bending of each shell.

This particularity ensure the technological procces of realizing the penstocks lower bends in the site of the hydroelectric power plant.

6. ACKNOWLEDGEMENTS

This paper is supported by CNCSIS – UEFISCSU, project number 679/2009 PNII – IDEI code 929/2008, *Transmisii hidrodinamice inteligente* (*Intelligent Hydrodynamic Transmitions*), and Grant CNMP 21047/1467/2007.

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