ON THE GEOMETRY OF THE PENSTOCK LOWER BEND FOR LARGE FLOW FRANCIS TURBINES


Abstract: Hydropower stations with large Francis turbines, have, generally, one penstock for each turbine. The penstocks are manufactured from metal sheets, bended and welded. The more complex part of each penstock is the penstock lower bend. This part is made from metal sheets bordered by circles of different diameters, placed in inclined planes, one from each others. But, these important parts of the hydropower stations can be manufactured in situ, in the base of a combination between technical and mathematical elements. This kind of combination was used, until now successfully in manufacturing of welded spiral casing and draft tubes with oval cross section for hydraulic turbines. The paper that we propose presents the mathematical elements of this method.

Key words: Francis turbines, penstock technology

1. MATHEMATICAL ELEMENTS

From mathematical point of view, any bended sheet of metal is a deployment surface (Bărgălzan M., 1999).

Such a shell – considered as a ruled surface - is obtained by shifting in space a straight line. Some ruled surfaces are not deployable.

To be deployable, a surface must accomplish a certain condition (Sundar Varada Raj P., 1995, Şriro I.I., 1961).

So, if in a xOyz reference frame a deployable surface has the equation (1).

\[ z = f(x,y), \quad (1) \]

then this surface in deployable if satisfied the differential equation (2).

\[ \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} = \left( \frac{\partial^2 z}{\partial x \partial y} \right)^2. \quad (2) \]

2. THE DETERMINATION OF THE CURRENT POINT COORDINATED ON THE SPATIAL CONTOUR OF THE SHELLS

For a current ruled surface for a penstock lower bend with oval sections \( A_1 A_2 B_1 \) in a xOyz reference frame (figure 1 „a”).

The end of the shell placed in xOy plane is defined by the known values \( l_1 \) and \( R_1 \) (figure 1 „b”). The other end is placed in \( x_1 Oy \) plane, which has the angle \( \delta \) with xOy plane.

This second end is defined by the values \( l_2 \) and \( R_2 \) (figure 1 „c”).

In the plane xOy, a current point from the contour \( A_i \) is defined through the angle \( \alpha \) and has the coordinates:

\[ x_{A_i} = l_1 + R_1 \cdot \cos(\alpha), \]
\[ y_{A_i} = R_1 \cdot \sin(\alpha), \]
\[ z_{A_i} = 0 \quad (3) \]

At the same shell, in \( x_1 Oy \) plane, a current point \( B_i \) from the contour is determined by the angle \( \beta \), which, in the plane \( x_1 Oy \) has the abscissa

Fig. 1. Symmetrical shell of penstock lower bend
\[ x_{Bi} = l_2 + R_2 \cdot \cos(\beta) \]  
\[ y_{Bi} = R_2 \cdot \sin(\beta) \]  
\[ z_{Bi} = \left( l_2 + R_2 \cdot \cos(\beta) \right) \cdot \sin(\delta) \]  
(4)  
(5)  

3. THE DETERMINATION OF THE CURRENT GENERATOR OF THE SURFACE

For any shell of the penstock lower bend the vital problem consist in determining the point \( B_n \), as function of the current point \( A_n \), in such a matter that straight line \( AB \) represents the current generator of the surface supported by the curved surfaces \( A_1A_2A_3 \) and \( B_1B_2B_3 \).

Essentially, the problem consist in determining the angle \( \beta \) as function of angle \( \alpha \) and the rest geometrical elements, such as: \( l_1, R_1, \delta, l_2, R_2 \):

\[
\beta = \arctg \left( \frac{l_2 \cdot \sin(\alpha)}{l_1 \cdot \cos(\alpha)} + R_2 \cdot \sin(\alpha) \right) + \arctg \left( \frac{R_2 \cdot \sin(\alpha)}{\sqrt{\frac{l_2^2}{2} \cdot \cos(\alpha) + l_1^2 \cdot \cos(\alpha) + R_1^2}} \right) - \ldots - R_2 \cdot \sin^2(\alpha) 
\]

for any \( \alpha \in \left[ 0, \frac{\pi}{2} \right) \)

4. DETERMINATION OF THE CONTOUR OF THE SHELL AND THE BENDING LINES

In this purpose there is used the mathematical notion of geodetic curve, whose length is, generally, determined by an integral equation.

The shell surface is decomposed in \( n \) quadrilateral surfaces, going from an initial generator \( A_1B_1 \) to the final one \( A_nB_n \).

The number “\( n \)” is determined to be conveneable from technological point of view, to result finally “\( n \)” bending lines, uniform distributed on the whole surface of the shell.

Then, each quadrilateral is decomposed in two triangles.

Ones are the triangles \( A_1A_2B_2 \) and \( B_2B_3A_3 \) (figure 1 “a”) in the first hypothesis.

In the second hypothesis, the considered triangles are \( A_1B_2B_3 \) and \( A_1A_2B_1 \) (figure 1 “a”).

To find the length of the sides of this triangles means that the whole surface of the shell is decomposed in triangles with known sides.

Going from the initial generator, each triangle is developed, obtaining both the developing of the shell and the corresponding positions of the bending lines.

Each of the four triangles, considered on the surface of the spatial shell (figure 1 “a”), is hiled in plan and has two sides exactly calculable.

One of this is the generator \( A_1B_1 \) or \( A_2B_2 \) and the second is the arc curve \( A_1A_2 \) or \( B_2B_3 \). And every of this four triangles has a third side \( (A_1B_2 \) or \( B_1A_3 \)), which, on the curve surface of the shell is a geodetic curve.

Considering that all the quadrilaterals which compose a particular shell are decomposable in two triangles, following the procedure presented above, results that all points \( A_i \) and \( B_i \) (\( i = 2, \ldots, n \)), exception \( A_1 \) and \( B_1 \), placed on the initial generator, have the coordinate approximate.

The approximation is higher for the points \( A_n \) and \( B_n \), which has maximum errors, the coordinates values being higher than the real ones.

In order to obtain acceptable results, the shell decompose in \( n_2 > n \) quadrilaterals, growing up their number, \( n_2 \), until the difference between the coordinates calculated through both hypothesis are less than the accepted tolerance.

From all this points \( A_i \) and \( B_i \) will be withhielded a “\( n \)” number uniform distributed on the contour of the shell; the number “\( n \)” will be chosen to be convenient from the technologic point of view.

5. CONCLUSIONS

The absolute value of \( \text{max}(\alpha - \beta) \) differs form one shell to another. In industrial cases, this difference is about \( 1^\circ \) to \( 5^\circ \).

If considered \( \alpha = \beta \), then results small differences from the correct variant, but the bending lines induces technological difficulties, the shells differing from the correct form.

The method based on the exposed briefly calculus offer the coordinates of the points representing the contour of each particular shell and, also, the bending lines; this allowed realizing the shells at the site of the hydroelectric power plant, which reduce considerably the costs.

Determination, in the limits of accepted tolerance, of the coordinates of the points that represent the contour of each shell and, also, the ends of the bending lines, ensure the correct cutting off and the bending of each shell.

This particularity ensure the technological proces of realizing the penstocks lower bends in the site of the hydroelectric power plant.

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7. REFERENCES


