

## UTILITY FUNCTION IN ALTERNATIVE INVESTMENTS

COCULESCU, C[ristina]; DESPA, R[adu] & FOLCUT, O[vidiu]

**Abstract:** Explaining the investment in terms of expected utility has been an important step in substantiating investment theory portofoliu. Forma linear utility function was, however, that exclude certain decision situations (Allais's paradox) so that very shortly after the launch of Neuman's hypothesis - Morgenstem the specialists went to find a utility function to explain investment behavior as well.

**Key words:** utility function, investments, nonlinear utility, subjective probabilities

### 1. INTRODUCTION

Explaining the investment in terms of expected utility has been an important step in substantiating investment theory portofoliu. Forma linear utility function was, however, that exclude certain decision situations (Allais's paradox) so that very shortly after the launch of Neuman's hypothesis - Morgenstem the specialists went to find a utility function to explain investment behavior as well. The most important contributions in this regard are recent, belonging to the years' 80: if "weighted expected utility" owned by MacCrimmon in 1979 and Machina's in 1982, if "non-linear expected utility" of John Quiggin (1993) hypothesis Expected utility ordered "developed by Chew, Karni and Safra 1987, if the non-utility aditivitatii asteptate's" (Fishburne, 1988) or hypothesis based on expected utility investmental conduct (Machina, 1988).

### 2. WEIGHTED UTILITY HYPOTHESIS

This hypothesis was developed by Soo-Hong Chew and KRMacCrimmon in 1979 and was later resumed by Peter Fishburne in 1983. The expected form utility function of the two proposed which measures investors' preference between different investment alternatives is the following:

$$U(p) = \frac{\sum u(x_i)p_i}{\sum v(x_i)p_i} \quad (1)$$

Be a set of potential earnings  $x_1, x_2, x_3$  which assume known a set of distribution probabilities  $p_1, p_2, p_3$  with  $p_1 + p_2 + p_3 = 1 \rightarrow p_2 = 1 - p_1 - p_3$ . According above assumption, the utility function and probability related to their earnings potential corresponding distribution in:

$$U(p) = \frac{p_1 u(x_1) + (1 - p_1 - p_3) u(x_2) + p_3 u(x_3)}{p_1 v(x_1) + (1 - p_1 - p_3) v(x_2) + p_3 v(x_3)} = p_1 U^*(v(x_1) - v(x_2)) - (u(x_1) - u(x_2)) + p_3 U^*(v(x_3) - v(x_2)) - u(x_3) - u(x_2) = u(x_2) - U^* v(x_2) \quad (2)$$

This utility function representation variables are  $p_1$  and  $p_3$ , potential returns  $x_1, x_2, x_3$  is known. Derivativ partial order is the following utility function:

$$\frac{dp_3}{dp_1} = - \frac{U(v(x_1) - v(x_2)) - (u(x_1) - u(x_2))}{U^*(v(x_3) - v(x_2)) - (u(x_3) - u(x_2))} \quad (3)$$

Absence of variables  $p_1$  and  $p_3$  order partial derivatives 1 shows indifference curves  $U'$  and  $U''$ ,  $U'''$  the strength of this hypothesis comes from the fact Chew-MacCrimmon utility

function has the property that, for a set potential gains  $x_1, x_2, x_3$ , with probability distribution  $p_1, p_2, p_3$ , its related difference curves all intersect in a punt is outside the triangle of Maschak, where  $p_1 < 0$  and  $p_3 < 0$ . It could be inferred ko such utility function transitivity axiom no longer observe the set of axioms Neumann-Morgenstern.

### 3. HYPOTHESIS NONLINEAR UTILITY

Another approach to financial investments associated utility function belongs to Mark J. Machina in 1982. In the demarche, Engine started and it's the Allais paradox, noting that the expected utility function as proposed by his predecessors, observe complementarity axiom, and that of transitivity Arhimede. Machina noticed that meet the expected utility function only if the independence axiom that indifference curves associated with different kinds of investment have a nonlinear form .

Engine observe that for any investment option its associated probabilities  $p_1, p_2$  and  $p_3$ , indifference curves are increasing. Applying the principle of stochastic dominance for the expected utility function, Machina noted that for three types of investment  $p, q, r$  type ( $p_1$  with  $x_1, p_2$  with  $x_2, x_3$  with  $p_3$ ) an investor's order of preference for three different gain ceve  $x_1, x_2, x_3, x_3 >_{pref} x_2 > x_3$  indifference curves move to the left. Engine explain this by changing investor's share amounts invested in different types of gain  $x_1, x_2, x_3$  as the probability distribution varies ( $x_3$  investor will always prefer to win the three types and  $p_3$  as the probability of achieving this gain is greater the more he will invest a larger amount of investment in this version). Note that the grah investitoarul will always prefer investment option (or portfolio) which offers preferred gain ( $x_3$ ) with the highest probability ( $p_3$ ). Consequently  $r$  is the preferred choice of investment alternatives  $q$  or  $p$ . Considering the behavior of investors indifference curves can not have a linear form. Engine has found a solution to the paradox of Allains (utility function associated with a financial investment does not meet the independence axiom) of "local utility". Machina has shown that this linear approximation of a utility function depending on "local" meet  $u^p$  all four axioms: complementarity, transitivity, Archimedean axiom and axiom independence. Linear approximation of expected utility function is a utility function in the real sense, it is not the same for any probability distribution  $p$ . Local utility function is specific to each probability distribution  $p$ , experts believing it rather as a local index utility function determined by a linear approximation around a probability distribution  $p$ .

Machina to explain preference for an alternative placement to investors in light of the local linear function: if an investor  $u^p(q) >_{pref} u^p(p)$  where  $U(q) >_{pref} U(p)$ . In other words, preferably "local" to an investor for a specific investment option can be extended to the expected utility function.

Another interesting observation was related Machina's utility function as expected: according to his studies, the utility function is concave in  $x$  for any probability distribution  $p$ . This observation has contributed to further strengthening the theory

of risk aversion of investors explained in terms of expected utility.

#### 4. UTILITY FUNCTION IN TERMS OF SUBJECTIVE PROBABILITIES

Neumann's approach and the associated probabilities Morgenstern utility function were considered to be objective. He probability distribution approach is a "classic", considering that analysts associated probabilities of random events reflect exactly the evolution of these events. Classical principle of probability objective was developed by Pierre Simon de Laplace in 1795: *the probability of an event generating a random evolution is obtained by dividing the number of times that event the total number of attempts*

Classical theory of probability deficiencies that were later proven by empirical testing, "the principle of rational belief" under which likeness "physical" between events often leads to association and equal probability "insufficient rational principle" under which an analyst when which can not dissociate between events is likely to produce equal probability it associates acaestora These are the most important "critical" during the approach to classical random phenomena and their associated probability distribution. Symmetry probability distribution, and non counterintuitive nature -additive derived from the principle of rational failure led to the emergence of numerous theories that have tried to explain or offer solutions to these limitations of classical theory.

The first notable contribution in this direction belongs to Richard von Mises in 1928 and subsequently taken over by Hans Reichenbach in 1949. Frequency new approach introduces the concept of relative (or relative probability) associated with a random event: the probability of an event related specifically frequency may be associated with a relative occurrence of a specific event may be associated with a relative frequency appearance of a specific event for a infinite number of similar attempts.

Relative frequency idea was not new, from the law of large numbers developed by Jacob Bernoulli: if an event occurs  $k$  times in over a number of independent trials and then indented for any arbitrary extension of the number of trials, probability "objective". The event associated appearance can be approximated by  $\frac{k}{n}$ . In other words, if a set of tests in  $X$  event occurs  $k$  times then for any random number of attempts have extended the probability law of large numbers in an infinite number of attempts. Home criticizing this approach was linked to the unique nature of the event, to be associated with a probability of occurrence, the idea of repeat infinite and identical to a phenomenon is considered impossible in reality.

Most of the "critics" classical approach to probability of generating risk-generating events have turned against "objectivity": In tests conducted studies and also appeared many argue that is not a random phenomenon a phenomenon that can be measured using objective probabilities. In any phenomenon considered to occur randomly we always find a number of factors which determine to a certain extent the evolution of this phenomenon. For example theoretically throw a coin is considered to be a random phenomenon, the probability of a girl is an objective. Argument critics classical theory in this case was related to the fact that although at first sight, throwing a coin is a random phenomenon, there are many factors that can be known in advance (weight side, the room air currents, etc.) and which influences in some extent the probability of one of the girls.

In other words, any associated probability of a random phenomenon contains a subjective component. The quality of this case lies in the analyst of determine well as the degree of subjectivity in the analysis. The major difficulty in this approach lies in failure probability distribution develop a mathematical model to consider personal interpretations (mostly subjective) of this new vision of analysts.

Knight Frank distinction between certainty, risk and uncertainty pecum and Neumann's expected utility function and faded in front Morgenstern arguments of proponents of the idea that the probability of a phenomenon associated with a subjective component to a certain extent depending on the analysts' personal qualities or decision makers.

First appeared on the idea that psychological investment behavior to occupy a particular place the decision on investment, an investor personal how the risks that it assumes that gain or opportunities and introducing market offers a pronounced bias note in the analysis.

Another similar model that separates the subjective component of the probabilities of Finetty developed by Bruno. The central argument of both theory based on a sample basis: at the track a number of people who participate have a solid knowledge of performance horses, race difficulty etc. besides these professionals of the May race betting and a number of people who do not know much about horse racing, and they often earn, in their decision, the same people include their views, their personal beliefs about a phenomenon (Fishburn, 1994). To determine the degree of subjectivity in this case was sufficient to analyze the behavior of who know nothing about this phenomenon.

#### 5. CONCLUSION

Undoubtedly the introduction of expected utility gains offered by a portfolio of securities (regardless of its location) in the equation for investment decision represents a step forward in understanding the investment behavior and mechanisms of functioning of financial markets. Taking into consideration only the profits, without regard to its utility for someone who is willing to invest in financial markets, an assumption proved incomplete and insufficient for investment decision.

Meaning that you have printed Neumann and Morgenstern conceptualization investment decision in conditions of risk and uncertainty was, with few exceptions and adjustments, to be a good one. Later psychological and behavioral dimension associated probability (considered to be partially goals) have been completed and added new investment theory. Unanimously accepting the idea that expected utility matters in investment decision, subsequent efforts of specialists focused on finding new ways assigned utility function, to explain how decision makers with better behavior towards risk exposures.

This paper was developed within the research Project "Innovation and the Growth of Competitiveness – Main Vectors of Social-Economic Progress of Romania", no. 91\_071/2007-2010, PNCDI\_II.

#### 6. REFERENCES

- Arrow, K.; Lind, R.C., (1970). *Uncertainty and the Evaluation of Public Investment Decisions*, AER
- Coculescu, C. (2006). *Modeling and Simulation of Economic Processes*, Juristic Universe Editing Press, ISBN 973-7643-66-6, Bucharest, Romania
- Coculescu, C., Despa, R., Folcuț, O., (2008). New Tendencies in Software Applications for Modeling and Simulation, The 31<sup>st</sup> International Convention MIPRO 2008, Conference on Computers in Education, in *Proceedings – Vol. IV, CE – Computers in Education*, pp. 151\_154, ISBN 978-953-233-039-7, May 26-30, 2008, Opatija, Croatia
- Fishburn, P.C., (1994). *Utility and Subjective Probability*, R.J. Aumann and S. Hart, editors, Handbook of Game Theory, Vol. II. Amsterdam: North-Holland
- Swamidass, P.M. (2002). *Innovations in Competitive Manufacturing*, AMACOM, ISBN 978-0814471401
- Ritchie, B., Marshall, D., (1993). *Business Risk Management*, Chapman & Hall, Londra