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ROBUST CONTROL OF THE INVERTED PENDULUM

MARHOLT, J[iri] & GAZDOS, F[rantisek]

Abstract: *This paper deals with the design of feedback control system for unstable processes. For this purpose, the polynomial approach is employed. The method is presented and verified on the unstable system of the inverted pendulum PS600. The resultant controllers enable stabilization of the pendulum rod in the unstable top position in a robust way.*
Key words: *unstable systems, feedback control*

1. INTRODUCTION

Unstable systems, such as various types of reactors, distillation columns, combustion systems etc, are often parts of industrial technologies (Chidambaram, 1997; Padma Sree & Chidambaram, 2006). Control of such systems is difficult and can even be dangerous (Stein, 2003). In this paper the polynomial approach to control system design is employed (Kučera, 1993; Hunt, 1993; Anderson, 1998) for the challenging task. This approach is able to solve common control problems as well as special problems of optimal, robust control and filtering. Basic feedback configuration together with robust setting of closed-loop poles has been utilized in this work in order to guarantee not only stable behaviour but also ensure satisfactory performance in the presence of disturbances and modelling errors. The methodology is presented and tested on the unstable system of the inverted pendulum PS600 (***, 2004) where the resultant controllers enable stabilization of the pendulum rod in the unstable top position in a robust way.

2. SYSTEM OF INVERTED PENDULUM

The unstable and nonlinear system of the inverted pendulum PS600 is depicted in Fig. 1.

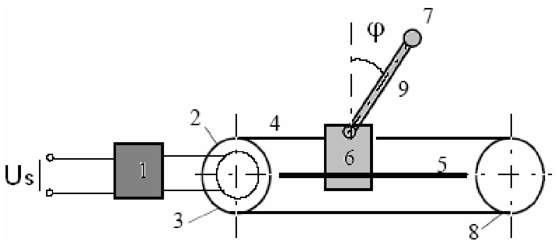

Fig. 1. Scheme of the inverted pendulum PS600

It has one input – control voltage of a DC motor which can change position of the cart, and two outputs – cart position and angle of the pendulum rod. In the picture, the numbers refer to the following parts: 1 – servo amplifier, 2 – motor, 3 – drive wheel, 4 – transmission belt, 5 – metal guiding bar, 6 – cart, 7 – pendulum weight, 8 – guide roll and 9 – pendulum rod. Basic control tasks include position control of the cart and angle control of the pendulum rod. For details, see e.g. (***, 2004). This system can be described by these differential equations:

$$m\ddot{r} + F_r\dot{r} + m_K l\ddot{\phi}\cos\phi - m_K l(\dot{\phi})^2\sin\phi = F \tag{1}$$

$$\Theta\ddot{\phi} + C\dot{\phi} + m_K \lg\sin\phi - m_K l\ddot{r}\cos\phi = 0 \tag{2}$$

where F represents the force produced by the DC motor (input signal), output signals are r – cart position, $\dot{r}$ – cart speed, φ – pendulum angle, $\dot{\varphi}$ – pendulum angular speed. Symbol g is the gravity acceleration constant, F_r represents velocity proportional friction of the cart and C is the pendulum friction (***, 2000; Marholt & Gazdoš, 2009). Following substitutions were used in the equations:

$$\Theta = \Theta_S + m_K l^2\,,\ m = m_V + m_K \tag{3}$$

where m_V is the cart weight, m_K is the pendulum weight, l is the distance between gravity and rotation centres of the pendulum and Θ_s represents the inertia moment of the pendulum rod with respect to the centre of gravity. All the used constants were either taken from the producer (***, 2000) or identified by experiments (Chalupa & Bobal, 2008; Marholt & Gazdos, 2009).

For the purpose of controller design, nonlinear differential equations (1)-(2) were linearized in the operating point π (top unstable position of the pendulum rod). Transfer function of the pendulum angle then takes the form (Marholt & Gazdos, 2009):

$$G^{\pi}_{\phi/F}(s) = \frac{0{,}5164s}{s^3 + 1{,}689s^2 - 22{,}27s^1 - 32{,}93} \tag{4}$$

3. CONTROL SYSTEM DESIGN

The classical feedback control configuration presented in Fig. 2 was employed for the control system design,

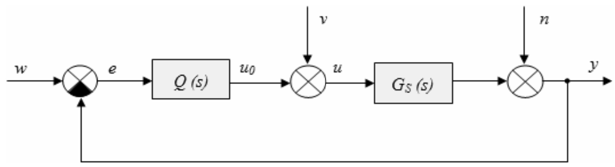

Fig. 2. Control system configuration

where the used variables correspond to the following signals: w – reference signal; e - control error; u – control input; v, n – disturbances; y – controlled output and $Q(s)$, $G_S(s)$ represent transfer functions of a controller and controlled system respectively and they are defined using polynomials in the complex Laplace variable “s” as:

$$Q(s) = \frac{q(s)}{p(s)}\,,\ G_S(s) = \frac{b(s)}{a(s)} \tag{5}$$

Control system requirements were formulated as stability, asymptotic tracking of the reference signal, disturbance attenuation and inner properness. For the reference and disturbance from the class of step functions and assuming the pendulum transfer function in the form of (4), these conditions will be fulfilled if the following formulas hold (Kučera, 1993).

- stability of the control system:

$$a(s)p(s)+b(s)q(s)=d(s) \tag{6}$$

  with $d(s)$ a stable polynomial,

- asymptotic tracking of the reference signal and disturbance attenuation:

$$p(s)=s^2\cdot\tilde{p}(s) \tag{7}$$

- inner properness (polynomial degrees):

$$\deg q=\deg a\ ,\ \deg\tilde{p}=\deg a-2\ ,\ \deg d\ge 2\cdot\deg a-1 \tag{8}$$

The stable characteristic polynomial $d(s)$ was chosen as:

$$d(s)=n(s)(s+\alpha)^2 \tag{9}$$

where $\alpha\succ 0$ is a tuning constant and the polynomial n is computed from $a(s)$ using the spectral factorization:

$$a^*(s)\cdot a(s)=n^*(s)\cdot n(s) \tag{10}$$

(the asterisk denotes complex conjugate polynomial $x^*(s)=x(-s)$). This will ensure stability of the control system together with the connection to the original process behaviour.

Controlling unstable systems can be a real hazard (Stein, 2003). Therefore it is desirable to ensure satisfactory behaviour not only in the nominal conditions but also in case of inaccurate mathematical models and unexpected disturbances, i.e. the control system must be robust (Grimble, 1993). In this work this is ensured using the robust setting of the tuning parameter α from (9). This constant is obtained from the minimization of the H_∞ norm of the closed loop sensitivity function defined as:

$$\in=\frac{a(s)\cdot p(s)}{d(s)} \tag{11}$$

Dependence of the infinity norm H_∞ of the sensitivity function upon the parameter α is depicted in Fig. 3.

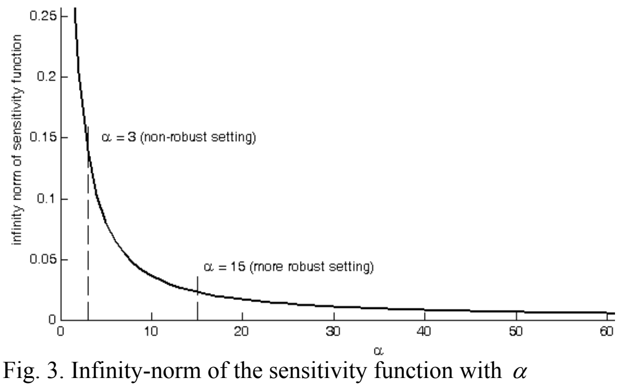

Fig. 3. Infinity-norm of the sensitivity function with α

From the graph it can be seen that the higher the value of α the less sensitive the control loop is, however, from the practical point of view it is not desirable to have too high value of α as it means bigger control action and overshoots. The value of α around 15 can be seen as a trade-off between robustness and practicability since approx. from this value the sensitivity does not change significantly. The resultant controller obtained from (6) together with (7)-(10) for $\alpha=15$ then takes the form of (12).

$$Q(s)=\frac{34.7s^3+458.1s^2+2842.4s+122.6}{s^2(s+29.3)} \tag{12}$$

Besides $\alpha=15$, also $\alpha=3$ is presented to demonstrate the influence of choosing a robust and non-robust value of this constant. The results can be seen in Fig. 4. During the control, step disturbance of the amplitude -0.07 [rad] influenced the controlled output in the time $t\in(15;\ 15.5)$ [s].

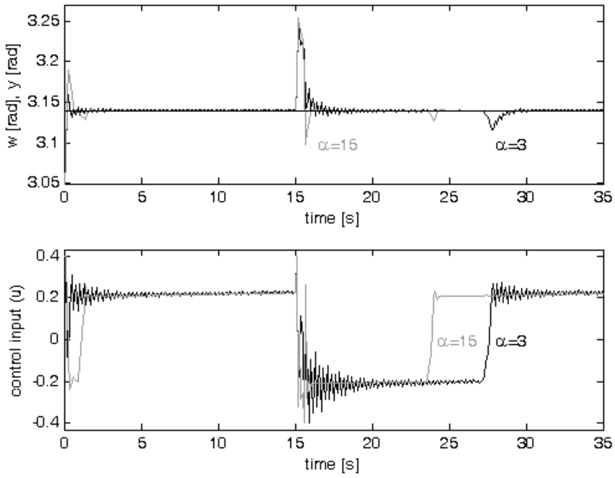

Fig. 4. Controlled output and control input responses

4. CONCLUSION

In this paper, control of the unstable nonlinear system of the inverted pendulum was presented. For this purpose, polynomial approach was employed and the control loop was tuned to be robust using optimization of the tuning parameter α with the help of the sensitivity function. The contribution can be seen from the presented comparison of robust and non-robust setting of this constant in the case of unexpected disturbances. Influence of this parameter on the control input and overshoots is also apparent. The proposed solution represents a reasonable trade-off between the robustness and practicability and can be generalised for stabilization and control of other unstable processes in a robust way. This is the goal of our future research where also the limitation on control input, not considered in this work, will be addressed.

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