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RELIABILITY AND EFFECTIVENESS OF SYSTEMS OF INDEPENDENT AUTOMATS

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Abstract: *Reliability, availability and production effectiveness of systems of independent automats supported by a few additional redundant units in cold, warm or hot standby regime with finite repair capacity is modelled as stochastic Markov process. Resulting system of Kolmogorov-Chapman differential equations is solved numerically. It is found, that the steady state is completely determined by the ratio of failure and the repair rate. However, the effectiveness measured by the volume of production is markedly influenced by further strategic maintenance factors like the renewal capacity and the number of redundant units.*
Key words: *reliability of production systems, Markov process, redundancy, effectiveness of production*

1. INTRODUCTION

Reliability, availability and effectiveness of production are the most important attributes of the quality of production systems (Boyadjiev et al., 2007; Gerstbakh, 2005). Methods of assessing the influence of failure probability on the overall volume of production are prominent topic of interest to factory management (Tolio et al., 2002; O`Connor, 2009). Failed units are usually repaired and the temporary overhaul of individual failed machine is covered by activation of redundant units (Bertsche, 2008; Dekýš et al., 2004). The present paper analyses the influence of failure rate, repair rate, renewal capacity and the influence of number of redundant units on the overall production. Redundant units are allowed to act in cold, warm or hot standby regime. The stochastic model adopted here is Markov process describing the probabilities of states by set of coupled Kolmogorov-Chapman differential equations (Meixner & Kolníková, 1984; Lisnianski & Levitin, 2003). Somewhat surprising are the following conclusions drawn from the present analysis:
- the overall production volume in steady state is linear function of both the ratio of failure and the repair intensity
- probability of individual states is almost insensitive to the number of redundant units when the repair capacity is nonzero
- dominant influence of the system configuration (*k,m,r*) on the production volume.

2. MODEL DESCRIPTION

We consider system of independent production units together with a few standby units. Let k is the number of possible positions for producing units, which under normal conditions is also the number of producing machines until the pool of redundant units and the renewal capacity is exhausted. Let m be the number of redundant units and r is the measure of the renewal capacity usually identified with the number of failed units, which can be repaired simultaneously. Total number of units is then $n=k+m$. The state of the system is represented by number of failed units, i.e. in state 0 all units are in up state, in state 1 one of k units is failed and its repair commences immediately and at the same time the failed unit is replaced without any delay by one of redundant units. Thus, the total production continues in full capacity of k active units. From the state 1 the system returns to state 0 after the repair of single failed unit is finished or eventually it can pass from state 1to state 2 if a failure of another unit happens, etc.

When there are no more redundant units, the production continues with decreased capacity proportional to $k-1$ until the repair of failed units is finished, etc. Ultimately, in state $n= k+m$ all units including the redundant units are failed and the production ceases until at least one of failed units is repaired. Thus, the system is at any time in one of $n+1$ possible states. In line with usual assumptions about Markov process only single failure or single renewal is admitted at any moment. Consequently, there are only transitions between neighbouring states. The transitions between states are described by the failure transition intensity λ_j and the repair transition intensity μ_j. Transition intensities multiplied by short time interval Δt represent the probability of the event of failure or repair at time interval $(t,\ t+\Delta t)$. For identical units the transitional failure intensities λ_j from state j to state $j+1$, the repair intensities μ_j for transition from state $j+1$ to state j and the number of actively producing units α_j are given in general by formulas

$$\lambda_j = \lambda\left[k + \mathrm{Min}(0, m-j)\right] + \nu\lambda\,\mathrm{Max}(0, m-j)$$
$$\mu_j = \mu\,\mathrm{Min}(j+1, r) \qquad (1)$$
$$\alpha_j = k + \mathrm{Min}(0, m-j)\,.$$

The first formula accounts for the possibility of cold, warm or hot standby regime of redundant units by including the factor $\nu\in\langle 0,1\rangle$, which transforms the failure rate λ of active units to failure rate λ_s of standby units. $Max(0,m\text{-}j)$ is number of disposable redundant units at the state j, $Min(0,m\text{-}j)$ is counter of missing working units after all redundant units have been used to substitute m or more failed units. The number of actively producing units α_j at state j is given by a combination of $Min(0,m\text{-}j)$ with k, the number of working positions. $Min(j+1,r)$ accounts for disposable repair capacity at state j, which is restricted from above by number r.

Time evolution of the probabilities $P_i(t)$ of individual states can be computed by the system of Kolmogorov-Chapman differential equations

$$\frac{d}{dt}\mathbf{P}(t) = \mathbf{A}\,\mathbf{P}(t) \qquad (2)$$

where $\mathbf{A}$ is the system matrix reflecting the probabilities of possible transitions between states, $\mathbf{P}^{\mathrm{T}}(t) = [P_0(t),\ P_1(t),\ \ldots,\ P_n(t)]$ is the vector of probabilities of individual states. When all units are initially in perfect state, the initial conditions are of form

$$\mathbf{P}^T(0) = \left[P_0(0), P_1(0), \ldots, P_n(0)\right] = \left[1, 0, \ldots, 0\right] \qquad (3)$$

In the case of generalized Birth and Death process the system matrix is of tridiagonal structure and the transitions are mapped by the following matrix for production chain of identical machines of parameters *k=5, m=3, r=3*, i.e. there are

$$\begin{bmatrix} -(5+3\nu)\lambda & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (5+3\nu)\lambda & -(5+2\nu)\lambda-\mu & 2\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (5+2\nu)\lambda & -(5+\nu)\lambda-2\mu & 3\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (5+\nu)\lambda & -5\lambda-3\mu & 3\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5\lambda & -4\lambda-3\mu & 3\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4\lambda & -3\lambda-3\mu & 3\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3\lambda & -2\lambda-3\mu & 3\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\lambda & -\lambda-3\mu & 3\mu \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & -3\mu \end{bmatrix}.$$

5 working positions occupied by 5 machines in perfect state in time *t=0*, number of redundant units is 3 as well as at most 3 failed units can simultaneously and independently undergo the repair. Then the number of disposable machines is *n = 8* and the number of states is *N=n+1=9*.

3. NUMERICAL RESULTS

The overall volume *V(t)* of production of the production chain at time *t* is directly proportional to the scalar product of the vector $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \ldots, \alpha_n]$ with $\mathbf{P}(t)$

$$V(t) = u\,h\,\boldsymbol{\alpha}\,.\,\mathbf{P}(t) = u\,h\sum_{i=0}^{n}\alpha_i P_i(t) \qquad (4)$$

where *u* is production of single unit per unit time, *h* is the observed time, α_i is number of actively producing units at state *i* and $P_i(t)$ is the probability of state *i*.

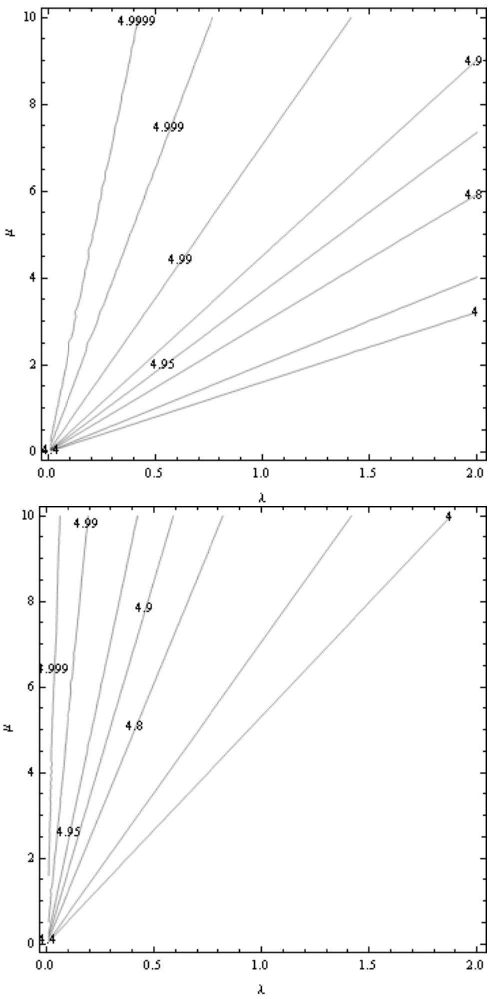

Fig. 1. Contours of production volume for k=5,m=3,r=3 (above) and for k=5,m=1,r=1 (lower)

Numerical results are presented here only for the steady state solution of the Kolmogorov – Chapman equations (1), as the state probabilities are typically rapidly decaying functions in time with tendency to converge to limiting values of the steady state solutions. Figure 1 illustrates the combinations of failure and repair rate necessary to achieve the desired level of production steady state for *k=5,m=3,r=3* as well as for *k=5,m=1,r=1* on the right. Unlike the time dependence of state probabilities, here the influence of the configuration is dominant. The relation (4) has been applied for unit time interval *h=1* and unit production *u =1* per hour so that the full production is 5 when *k=5* positions are open. It is obvious that constant levels of production are implied by linear relation between the failure and the repair rate. From Figure 1 immediately follows that the effectiveness of the system is markedly influenced also by such factors like the renewal capacity and the number of redundant units.

4. CONCLUSION

Somewhat surprising are the following conclusions drawn from the present analysis:

- the overall production volume in steady state is linear function of both the ratio of failure and the repair intensity
- probability of individual states is almost insensitive to the number of redundant units when the repair capacity is nonzero
- dominant influence of the system configuration (*k,m,r*) on the production volume.

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