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THE OIL PRESSURE LAW IN JOURNAL BEARINGS DUE TO OF THE SHAFT CONICAL MOTION

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Abstract: *From the unaligned journal bearings the pressure distribution is much different in comparison with the aligned case, becoming strongly non-symmetric, so it is obvious that this pressure produces not only a load capacity, but a moment too. This must be taken into account when calculating the stiffness and damping coefficients of unaligned journal bearings. The angular displacements and velocities produce strong effects on the moment attitude angle. The pressure distribution law is obtained by solving the Reynolds equation in the case of a conical motion of the shaft in a journal bearing.*
Key words: *pressure distribution, journal bearing.*

1. INTRODUCTION

It is well known that journal bearings have high load capacity, low friction, and minimal wear and, therefore, they appear to be ideal candidates to replace roller bearings as rotor support elements in high speed turbo machinery.

Taking into account both hydrodynamic and squeeze effects, they could also be safety used as dampers in some aircraft engine applications or in high speed turbo machinery handling cryogenic liquids. . (Baskharone et al., 1991)

The object of this paper is to develop a theoretical model for the problem of an unaligned journal bearing, considering first the stationary case, and then the squeeze effects, induced by the angular velocities of the shaft.

In the end of this work the pressure distribution law is obtained into an explicit form for both, stationary and unstationary cases.

2. NOMENCLATURE

B - width of bearing;
$\overline{B}$ - dimensionless width of bearing, B/D;
d - diameter of journal;
D - diameter of bearing;
e - journal lateral eccentricity;
h - film thickness;
$\overline{h}$ - dimensionless film thickness, $2h/J$;
J - diametral clearance, $D - d$;
n - shaft speed;
x, y, z- cartesian co-ordinates (fixed $xOyz$ – frame);
x_b, y_b- co-ordinates of point M_b on the bearing contour;
x_s, y_s- co-ordinates of point M_s on the shaft contour;
$\overline{z}$ - dimensionless axial co-ordinate, $2z/B$;
X,Y,Z- cartesian co-ordinates (rotated $XOYZ$–frame);
α, β - angular displacements;
$\dot{\alpha}, \dot{\beta}$ - angular velocities;
$\dot{\alpha}^*, \dot{\beta}^*$ - dimensionless angular velocities, $\left(\dot{\alpha}, \dot{\beta}\right)/\left(\pi \cdot n\right)$;
ε - eccentricity ratio, $2e/J$;
Ψ - dimensionless clearance, J/D;
η - dynamic viscosity;
θ - angular co-ordinate.

3. THE MATHEMATICAL MODEL

The narrow journal bearing with shaft angular displacements is shown in Figure 1. (Someya,1989)

The following assumptions are made:

1. The lubricant is a Newtonian liquid of constant viscosity (isothermal flow).

2. The surfaces of the journal and the sleeve are rigid.

3. The order of angular displacements is given by the ratio between diametral clearance and the width of the bearing: $\alpha, \beta \approx \Psi / \overline{B}$.

4. Journal and the bush are not aligned (the conical motion of the shaft is present, see Figure 1). (San Andres, 1992)

Taking a fixed initial cartesian frame–$xOyz$, with point O placed at the bearing central plane and Oz being the shaft axis in the aligned case, the angular tridimensional position of the unaligned journal can be described in a $XOYZ$–cartesian frame with the same origin but axes rotated with Euler's angles: α and β . (San Andres, 1993)

The connection between these frames is given by two rotation matrices:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 1 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (1)$$

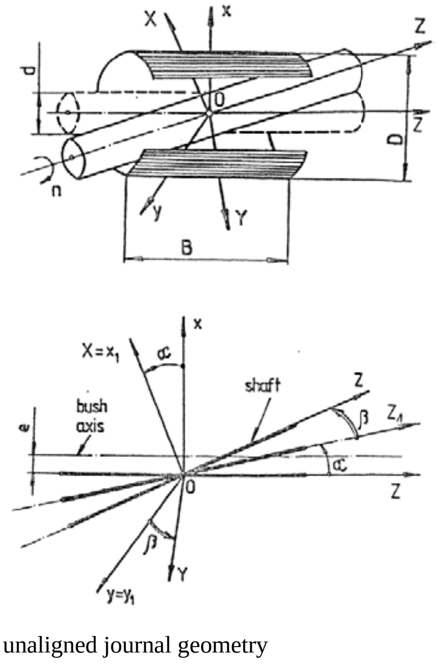

Fig. 1. The unaligned journal geometry

In the *XOYZ*–frame, the equation describes the journal contour is:

$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 \tag{2}$$

and taking into account both relation (1) and the assumption that angles α and β are considered small perturbations, the *xOyz*–shaft equation becomes:

$$(x + y\alpha\beta - z\alpha)^2 + (y + z\beta)^2 = \left(\frac{d}{2}\right)^2 \tag{3}$$

Besides, *xOyz*–bearing equation is:

$$(x - e)^2 + y^2 = \left(\frac{D}{2}\right)^2 \tag{4}$$

where e is the standard lateral eccentricity. Now considering that $y \cdot \alpha \cdot \beta$ in equation (3) as a second order term which can be neglected, and introducing an angular θ co-ordinate (measured from the fixed *Ox*–axis in the journal sense of rotation) the position of points M_b and M_s (on the bearing, respectively on the shaft contour at the same journal radius) is described by: (Suciu & Parausanu, 1996)

$$\begin{aligned} x_b - e &= \frac{D}{2}\cos\theta; \qquad y_b = \frac{D}{2}\sin\theta \\ x_s - z\alpha &= \frac{d}{2}\cos\theta; \quad y_s + z\beta = \frac{d}{2}\sin\theta \end{aligned} \tag{5}$$

Thus, the oil film thickness can be expressed as:

$$h = \sqrt{(x_b - x_s)^2 + (y_b - y_s)^2} \tag{6}$$

Introducing equations (5) in (6) and neglecting under square root the term $[e\sin\theta - z(\alpha\sin\theta + \beta\cos\theta)]^2$, having the same order with the term $[e\sin\theta]^2$ which is usually neglected in the hydrodynamic journal bearing theory, the film thickness is given by:

$$h \cong \frac{J}{2} + e\cos\theta - z\alpha\cos\theta + z\beta\sin\theta \tag{7}$$

or in dimensionless form:

$$\bar{h} = \frac{2h}{J} = 1 + \varepsilon\cos\theta - \frac{\bar{B}}{\Psi}\bar{z}\alpha\cos\theta + \frac{\bar{B}}{\Psi}\bar{z}\beta\sin\theta \tag{8}$$

where: $\bar{z} = 2 \cdot z/B$; $\bar{B} = B/D$ and $\Psi = J/D$. The pressure is obtained by solving the Reynolds equation which for the unaligned narrow journal bearing has the form:

$$\begin{aligned} \frac{\partial}{\partial z}\left[h^3 \frac{\partial p}{\partial z}\right] &= 12\eta z\left(\dot{\beta}\sin\theta - \dot{\alpha}\cos\theta\right) + \\ &+ 12\pi \cdot n\eta\left[(z\alpha - e)\sin\theta + z\beta\cos\theta\right] \end{aligned} \tag{9}$$

where the first term in the right member gives the squeeze film effects induced by the angular velocities $\dot{\alpha}$ and $\dot{\beta}$. When equation (9) is integrated twice along the axial co-ordinate z under boundary conditions:

$$p = 0 \text{ for } z = \pm\frac{B}{2} \tag{10}$$

the pressure distribution has the form:

$$p = \frac{6\pi \cdot n\eta\bar{B}^2}{\Psi^2}\mathrm{E}_1 + 3\pi \cdot n\eta\,\mathrm{E}_2(\mathrm{E}_3 - \mathrm{E}_4) \tag{11}$$

where

$$\mathrm{E}_1 = \frac{\varepsilon\sin\theta + (1 + \varepsilon\cos\theta)\dfrac{\left(\alpha + \dot{\beta}^*\right)\sin\theta + \left(\beta - \dot{\alpha}^*\right)\cos\theta}{\beta\sin\theta - \alpha\cos\theta}}{(1 + \varepsilon\cos\theta)^3(1 + K\bar{z})^2} \times \left(1 - \bar{z}^2\right) \tag{12}$$

$$\mathrm{E}_2 = \frac{\left(\alpha + \dot{\beta}^*\right)\sin\theta + \left(\beta - \dot{\alpha}^*\right)\cos\theta}{(\beta\sin\theta - \alpha\cos\theta)^3} \tag{13}$$

$$\mathrm{E}_3 = 2\ln(1 + K\bar{z}) - \left(\frac{1 + K}{1 + K\bar{z}}\right)^2\left[1 + \bar{z} - \frac{K}{2}\left(1 - \bar{z}^2\right)\right]\ln(1 + K) \tag{14}$$

$$\mathrm{E}_4 = \left(\frac{1 - K}{1 + K\bar{z}}\right)^2\left[1 - \bar{z} - \frac{K}{2}\left(1 - \bar{z}^2\right)\right]\ln(1 - K) \tag{15}$$

and

$$K = \frac{\bar{B}}{\Psi} \cdot \frac{\beta\sin\theta - \alpha\cos\theta}{1 + \varepsilon\cos\theta} \tag{16}$$

4. CONCLUSION

The final aim of the authors is to improve the calculation of the critical rotor speed by including the flexibility of the bearing oil film due to the angular displacements of the shaft.

So, this analysis occurs as a complete preliminary hydrodynamic step, which will be followed by another rotor-dynamic step.

In this work the pressure distribution is set into an explicit form for both, stationary and non-steady cases, the “pressure mountain” becoming strongly non-symmetric.

This work is just a preliminary announcement of some results regarding unaligned journal bearings.

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