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MATHEMATICAL MODEL OF ASYMMETRIC HEAT FIELD IN A PLANE PLATE

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***Abstract:** In the paper we focused on study of unsteady heat conduction in a solid plane plate which is caused by double-sided heat action of the surrounding environments. In the first part of contribution, we formulated mathematic model of the process and we present its analytical solution describing temperature field in a heated or cooled body that we derived by use of Laplace transformation method. In the second part of contribution, we computed temperature fields for various cases of the process by mathematic software Maple. The obtained results we compared by simulation of the studied process by software Comsol Multiphysics.*
***Key words:** model, asymmetric temperature field, plane plate*

1. INTRODUCTION

Problems on the unsteady conduction of heat in solids are great practical importance. They occur by processing of metals, plastics, rubbers etc. The practise realisation of the material heat treatment is often cost and energy demanding. Therefore optimal technological procedures are searched these days.

For suggestion of the optimal technological procedure it is necessary course of the process to know. However, the heat transport is very complicated process that depends on many factors as are geometry parameters of the processed body, its physical and chemical properties, temperature of the surrounding environment etc. (Carslaw & Jaeger, 2008), (Charvátová, 2009). Furthermore, the experimental determination of the process course is in many cases very difficult. Therefore the theoretical tools have to be often used for description of the process course (Charvátová, 2007).

From this point of view, we focus on mathematic description of the non-stationary asymmetric heat conduction in a solid plane that is often occurred process during material processing. In the following text we formulate mathematic model of this process and results of its modeling by mathematic software Maple.

2. THEORY

We will solve the asymmetric problem of heat conduction in a solid plane plate made from isotropic material. Let us consider, that length and width of the plate are much longer than its thickness δ. The plate of initial temperature t_p is suddenly exposed double-sided heat action of surrounding environment, whereas we supposed that temperature of surrounding environment on the left side from plate t_{o1} is different to temperature on the right side from plate t_{o2}. Temperatures of both environments don‘t depend on time and in addition they differ from initial temperature of the plate.

Graphical description of the problem you can see in fig. 1. With respect to above mentioned assumptions, the heat transfer across the wall will be asymmetric in accordance with axis of the wall. We used the Fourier-Kirchhoff equation of heat conduction (1) with the initial and boundary conditions (2) – (4) for modelling of the problem (Lykov, 1967).

$$\frac{\partial t(x,\tau)}{\partial \tau} = a\frac{\partial^2 t(x,\tau)}{\partial x^2} \tag{1}$$

$$t(x,0) = t_p \tag{2}$$

$$t(0,\tau) = t_{o1} \tag{3}$$

$$t(\delta,\tau) = t_{o2} \tag{4}$$

The equation (2) is assumption of the initial uniform temperature distribution in a heated or cooled body. The conditions (3) and (4) assume that temperature of a wall margin is constant and it equals to surrounding temperature.

By use of Laplace transformation method we obtained analytical solution of the formulated problem. Temperature field in a plate during heating (cooling) $t(x, \tau)$ is given by equation

$$\begin{aligned} t(x,\tau) &= t_p + \frac{(x-\delta)(t_p - t_{o1})}{\delta} + \\ &+ 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\cdot\pi}\sin\left(\frac{(\delta - x)}{\delta}n\cdot\pi\right)\ e^{-\left((n\cdot\pi)^2\frac{a\cdot\tau}{\delta^2}\right)}(t_p - t_{o1}) + \\ &+ \frac{x(t_{o2}-t_p)}{\delta} + \\ &+ 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\cdot\pi}\sin\left(-\ \frac{x}{\delta}\cdot n\cdot\pi\right)\ e^{-\left((n\cdot\pi)^2\frac{a\cdot\tau}{\delta^2}\right)}(t_{o2} - t_p) \end{aligned} \tag{5}$$

where a is thermal diffusivity of the plate. It depends on thermal conductivity λ, density ρ and thermal capacity c_p of the solid body

$$a = \frac{\lambda}{\rho \cdot c_p} \tag{6}$$

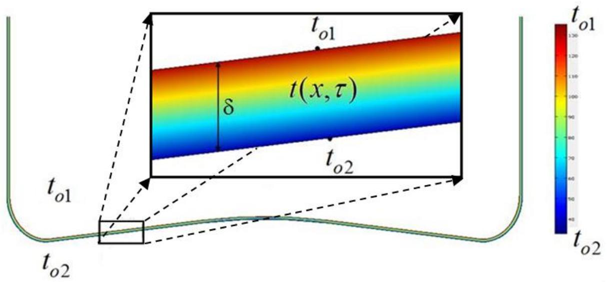

Fig. 1. Graphical sketch of the problem

3. COMPUTER MODELING OF THE ASYMMETRIC TEMPERATURE FIELD

We used mathematical software Maple for modeling of the studied problem. According to the relation (5), we programmed calculation of the temperature field in the heated (cooled) plate. In the fig. 2 we show various cases of the process course. The case a) represents asymmetric cooling. This process will occur when both temperatures of surroundings t_{o1} and t_{o2} will be lower then initial temperature of the plate t_p. The case b) represents asymmetric heating. It will occur when both temperatures t_{o1} and t_{o2} will be higher than initial temperature of the plate t_p. The model c) shows special case of the asymmetric thermal action of surroundings on the plate. In this case, initial temperature of the plate t_p is higher than temperature of surrounding acting from one side on the plate, but temperature of surrounding acting on the plate from opposite side is lower than initial temperature of the plate t_p.

All mentioned cases of asymmetric heating and cooling of the plate we also simulated by software Comsol Multiphysics under the same conditions. In the fig. 3 we show simulated temperature fields in the plate by its combined cooling and heating. As you see the temperature distribution is in accordance with temperature field computed by software Maple (Fig. 2 c))

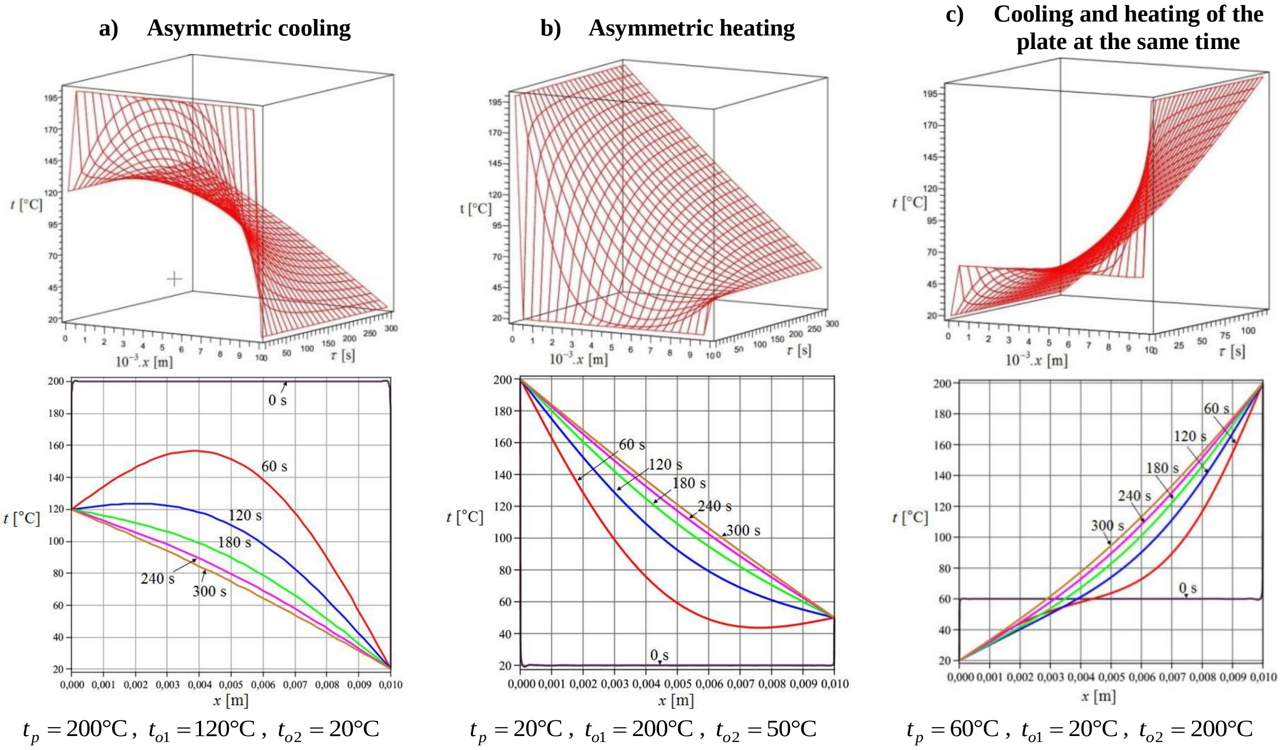

Fig. 2. Modeling of asymmetric temperature fields by software Maple. In all cases we used these material properties: λ = 0.2 W.m^{-1}.K^{-1}, ρ = 930 kg.m^{-3}, c_p = 1800 J.kg^{-1}.K^{-1}

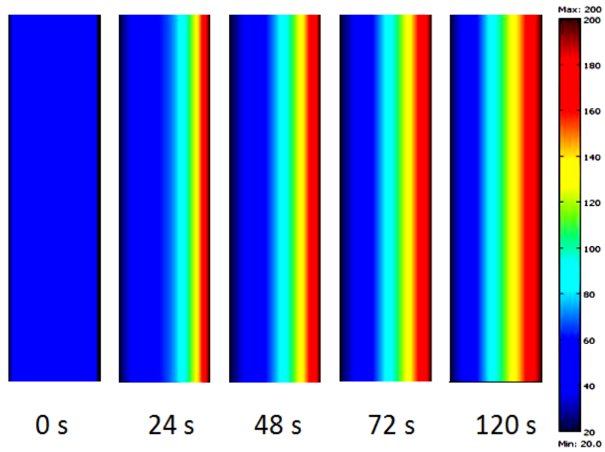

Fig. 3. Simulation of the combined cooling and heating of the plate by software Comsol Multiphysics

4. CONCLUSION

In the paper we used theoretical tools of chemical engineering for description of the thermal processes in the solid bodies. For this purpose we formulated mathematic model of the asymmetric heating and cooling of the plane plate and derived analytical solution that describes non-stationary heat conduction in the plate. We confirmed accuracy of the solution by computer simulation.

Although the mathematic model is only simplification of the real process, it can be used for obtaining of the useful basic information about given process course. Therefore We will used our mathematic model and its analytical solution for real technological computing of asymmetric heat conduction in the plane.

5. ACKNOWLEDGEMENTS

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