Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
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Annals of DAAAM International

AUTOMATIC CONTROL SYSTEM OF THE AIRCRAFT YAW'S ANGLE USING A PROPORTIONAL – INTEGRATOR – DERIVATIVE CONTROL LAW

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Abstract: *This paper presents a study of a system for the automatic control of the yaw's angle, with angular velocity feedback E.E. (execution element) and proportional – integrator – derivative (P.I.D.) control law. Starting from the block diagram of the system, one studies the stability and the performances of the system. For this purpose an algorithm has been successfully implementated in Matlab/Simulink. For three aircraft types, one obtained the transfer functions, poles, zeros, dimensional and non-dimensional transmission ratios, time variations of the yaw's angle, time variations of the direction deflection and so on.*
Key words: *aircraft, yaw, control law, direction deflection*

1. INTRODUCTION

The problem is to obtain a system which stabilizes the yaw's angle of an aircraft. This system has a proportional – integrator – derivative (P.I.D.) control law. The input of the system is the direction deflection and the output is the yaw's angle. Starting from the aircraft movement equation, the authors obtain, for three different aircraft types, the dimensional and non-dimensional transmission ratios. These allow the obtaining of the control law which stabilizes the output of the system (Lungu & Lungu, 2008). In this paper a new approach to the problem mentioned above is presented. The simulation's system is made in Matlab using a new algorithm developed by the authors.

2. THEORETICAL ISSUES

One considers the flight's regime described by matriceal equation

$$\begin{bmatrix}\dot\beta & \dot\omega_z & \dot\psi\end{bmatrix}^T = A\begin{bmatrix}\beta & \omega_z & \psi\end{bmatrix}^T + B\begin{bmatrix}z_1 & z_3 & \delta_d\end{bmatrix}^T, \tag{1}$$

with

$$A=\begin{bmatrix}a_{31} & a_{13} & 0\\ a_{31} & a_{33} & 0\\ 0 & 1 & 0\end{bmatrix}, B=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & b_d\\ 0 & 0 & 0\end{bmatrix}, \tag{2}$$

where ψ is the yaw's angle, $\omega_z=\dot\psi$ – way angular velocity, β – the sideslip angle, δ_d – the direction deflection and z_1, z_3 – disturbances, $z_1=\dot v_y+z_1'=0$.

The command law is has a P.I.D. form and may be expressed as

$$\delta_d = (1/\mathrm{D})\cdot\left[k_d^{\psi}(\overline{\psi}-\psi)-k_d^{\dot\psi}\dot\psi-k_d^{\ddot\psi}\ddot\psi\right]. \tag{3}$$

where $k_d^{\psi}, k_d^{\dot\psi}, k_d^{\ddot\psi}$ are the non-dimensional transmission ratios.

For the non-dimensional transmission ratios' calculus the closed loop transfer function must be obtained. After that, the transfer function takes Vîşnegradski form (Calise et al., 2002); it results

$$H_0(s)=\frac{\psi(s)}{\overline{\psi}(s)}=\frac{a_0}{s^3+a_2s^2+a_1s+a_0}=\frac{\omega_0^3}{s^3+A_2\omega_0s^2+A_1\omega_0^2s+\omega_0^3}, \tag{4}$$

where

$$A_2\omega_0=a_2=b_dk_d^{\ddot\psi}-a_{33}, A_1\omega_0^2=a_1=b_dk_d^{\dot\psi}-a_{31}, \quad \omega_0^3=a_0=b_dk_d^{\psi}; \tag{5}$$

$\omega_0=5$ is the own pulsation of the system PA – A (autopilot - aircraft) and A_1, A_2 – the assessed Vîşnegradski parameters (Lungu, 2010); $A_2=2.4, A_1\cong 1.5$. The coefficients a_{31}, a_{33}, b_d are calculated, for three aircraft types, using equation

$$a_{31}=-n_{31}; a_{33}=-n_{33}; b_d=-n_d, \tag{6}$$

where n_{31}, n_{33}, n_d have the values presented in Table 1.

Aircraft type/ Flyght regime	Coefficients		
	n_{31}	n_{33}	n_d
Light aircraft (H=10 km; M=0.8)	5.76	0.22	3.18
Average aircraft (H=10 km; M=0.9)	69	0.89	29.6
Heavy aircraft (H=12 km; M=0.9)	50	0.9	19

Tab.1. Coefficients for three aircraft types

Using (1), (2) and (3), one obtains the block diagram of the system for the automatic control of the aircraft yaw's angle with P.I.D. control law – fig.1.

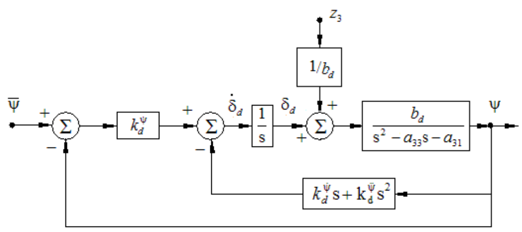

Fig.1 The state system for the automatic control of the yaw's angle with P.I.D. control law

The non-dimensional transmission ratios are obtained by solving of the equations system (5); one yields

$$\hat k_d^{\psi}=\frac{\omega_0^3}{b_d}, \hat k_d^{\dot\psi}=\frac{a_{31}-A_1\omega_0^2}{b_d}, \hat k_d^{\ddot\psi}=\frac{a_{33}+A_2\omega_0}{b_d}. \tag{7}$$

The obtaining of the dimensional transmission ratios, the following equations are used

$$k_d^{\psi}=\tau_a\hat k_d^{\psi}, k_d^{\dot\psi}=\hat k_d^{\dot\psi}, k_d^{\ddot\psi}=\tau_a\hat k_d^{\ddot\psi}, \tag{8}$$

where τ_a is the aerodynamic time constant.

Because the execution element has angular velocity feedback, the relationship that exists between the pilot command (u) and the direction deflection (δ_d) is

$$\frac{\delta_d}{u}=\frac{1}{\tau s} \tag{9}$$

and the form of the command u (Larin, 2003) is

$$u=\overline{\psi}-\sum_{i=1}^{n=3}k_{x_i}x_i(s), \tag{10}$$

where

$$x_1=\psi, x_2=\dot\psi, x_3=\ddot\psi, k_{x1}=1, k_{x2}=\frac{k_d^{\dot\psi}}{k_d^{\psi}}, k_{x3}=\frac{k_d^{\ddot\psi}}{k_d^{\psi}}. \tag{11}$$

The block diagram with execution element's visualization is the one from fig.2.

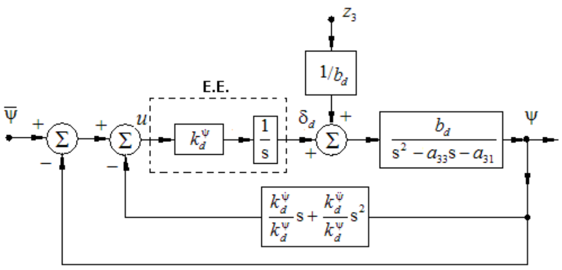

Fig.2 The block diagram with execution element's visualization

The open loop transfer function is obtained easily using (4)

$$H_d(s)=\frac{H_0(s)}{1-H_0(s)}=\frac{k_d^{\psi}b_d(s-a_{22})}{s^3+a_2 s^2+a_1 s}. \tag{10}$$

3. SIMULATION RESULTS

One is studing now the stability of the yaw angle's control system with P.I.D. control law for three different aircraft types and three flight regimes: light aircraft (H=10 km, M=0.8), average aircraft (H=10 km, M=0.9) and heavy aircraft (H=12 km, M=0.9) – Table 1.

One considers the case of lateral movement for three aircraft types and three flight regimes. The Matlab/Simulink model, associated to the block diagram of the system for yaw's angle control, is presented in fig.3. One has chosen, in the simulation of the Matlab/Simulink model, an integration step equal with 0.01.

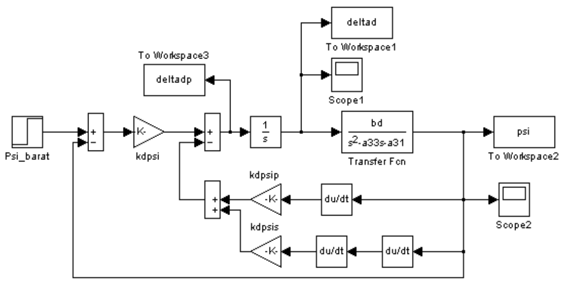

Fig.3 Matlab/Simulink model for the yaw's angle control system

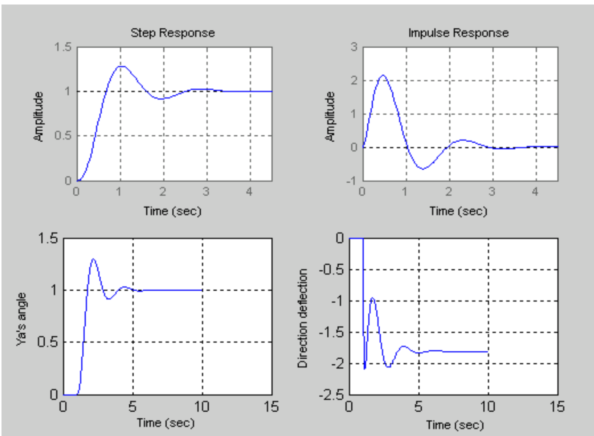

Fig.4 Characteristics for a light aircraft

Using the program developed by the authors, one obtains, for every aircraft's type, the step response of the system (using instruction *step*), Dirac impulse of the system (using instruction *impulse*), time variation of the yaw's angle ψ (instruction *plot*) and time variation of the direction deflection δ_d (instruction *plot*) (Ghinea, Fireteanu, 2001).

These characteristics are fig.4 (for a light aircraft), fig.5 (for an average aircraft) and fig.6 (for a heavy aircraft).

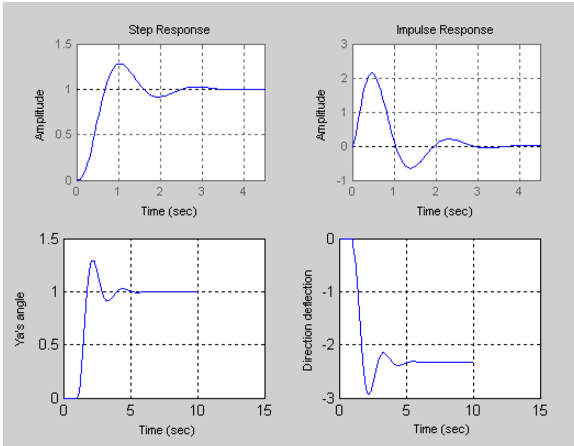

Fig.5 Characteristics for an average aircraft

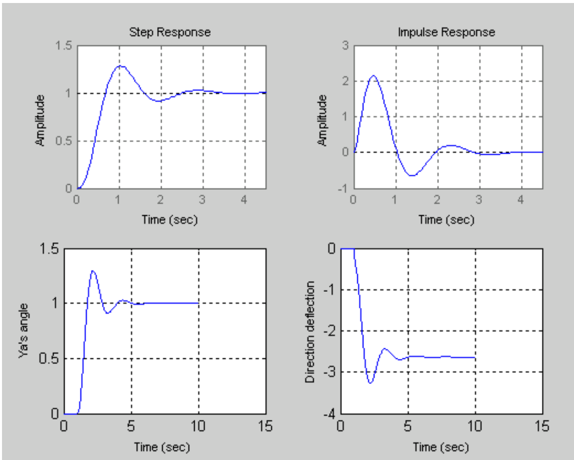

Fig.6 Characteristics for a heavy aircraft

As one can see, the system is a stable one with very good properties (the transient regime is small). Because the denominators are the same, the poles for light, average or heavy aircrafts are the same $(-9.428, -1.285 \pm 3.406i)$. From the analysis of the system's poles one concludes that the system is stable.

4. CONCLUSION

A system which stabilizes the yaw's angle of an aircraft has been obtained. By determination of the dimensional and non – dimensional transmission ratios, one has projected the control law. The simulation of the system is made in Matlab/Simulink using a new Matlab algorithm developed by the authors. Indicial responses, impulse responses and time variations of the yaw's angle and direction's deflection have been obtained for three aircraft types and three flight regimes.

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