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STATIC SYSTEM FOR THE AUTOMATIC CONTROL OF THE AIRCRAFT YAW'S ANGLE WITH PROPORTIONAL – DERIVATIVE CONTROL LAW

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Abstract: *This paper presents a study of a static system for the automatic control of the yaw's angle, with rigid feedback execution element (E.E.) and proportional – derivative (P.D.) control law. Starting from the block diagram of the system, the authors obtain, for three aircraft types, the transfer functions, poles, dimensional and non-dimensional transmission ratios, time variations of the yaw's angle, time variations of the direction deflection and so on. It was made for this purpose, an algorithm that is successfully implemented in Matlab/ Simulink.*
Key words: *aircraft, yaw, control law, feedback*

1. INTRODUCTION

The problem is to obtain a system which stabilizes the yaw's angle of an aircraft. This system has a proportional – derivative (P.D.) control law. The input of the system is the direction deflection and the output is the yaw's angle. Starting from the movement equation of the aircraft, the authors obtain, for three different aircraft types, the dimensional and non-dimensional transmission ratios. These allow the obtaining of the control law which stabilizes the output of the system (Lungu & Lungu, 2008). In this paper a new approach to the problem mentioned above is presented. The simulation's system is made in Matlab using a new algorithm developed by the authors.

2. THEORETICAL ISSUES

One considers the flight's regime described by matriceal equation

$$\begin{bmatrix}\dot\beta & \dot\omega_z & \dot\psi\end{bmatrix}^T = A\begin{bmatrix}\beta & \omega_z & \psi\end{bmatrix}^T + B\begin{bmatrix}z_1 & z_3 & \delta_d\end{bmatrix}^T, \qquad (1)$$

with

$$A=\begin{bmatrix}a_{31} & a_{13} & 0\\ a_{31} & a_{33} & 0\\ 0 & 1 & 0\end{bmatrix}, B=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & b_d\\ 0 & 0 & 0\end{bmatrix}, \qquad (2)$$

where ψ is the yaw's angle, $\omega_z=\dot\psi$ – way angular velocity, β – the sideslip angle, δ_d – the direction deflection and z_1, z_3 – disturbances, $z_1=\dot v_y+z_1'=0$. From the two above equations one yields

$$\ddot\psi = a_{31}\psi + a_{33}\dot\psi + b_d\delta_d + z_3. \qquad (3)$$

This equation is equivalent with the following one

$$\begin{bmatrix}\dot\psi\\ \ddot\psi\end{bmatrix} = \begin{bmatrix}0 & 1\\ a_{31} & a_{33}\end{bmatrix}\begin{bmatrix}\psi\\ \dot\psi\end{bmatrix} + \begin{bmatrix}0\\ b_d\end{bmatrix}\delta_d + \begin{bmatrix}0\\ 1\end{bmatrix}z_3. \qquad (4)$$

The command law is a P.D. one and may be expressed as

$$\delta_d = k_d^{\psi}(\overline{\psi}-\psi) - k_d^{\dot\psi}\dot\psi = k_d^{\psi}\overline{\psi} - \begin{bmatrix}k_d^{\psi} & k_d^{\dot\psi}\end{bmatrix}\begin{bmatrix}\psi\\ \dot\psi\end{bmatrix}, \qquad (5)$$

where k_d^{ψ} and $k_d^{\dot\psi}$ are the transmission ratios.

Replacing (5) in (4) one obtains the equation of the autopilot – aircraft

$$\begin{bmatrix}\dot\psi\\ \ddot\psi\end{bmatrix} = \begin{bmatrix}0 & 1\\ a_{31}-b_dk_d^{\psi} & a_{33}-b_dk_d^{\dot\psi}\end{bmatrix}\begin{bmatrix}\psi\\ \dot\psi\end{bmatrix} + b_dk_d^{\psi}\begin{bmatrix}0\\ 1\end{bmatrix}\overline{\psi} + \begin{bmatrix}0\\ 1\end{bmatrix}z_3. \qquad (6)$$

The above equation may be modelled by the following block diagram (Lungu, 2010)

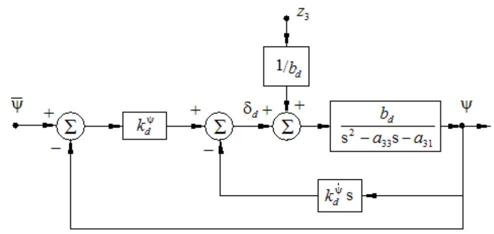

Fig.1. The static system for the automatic control of the yaw's angle with P.D. control law

For the transmission ratios' calculus one obtains the closed loop transfer function and writes it as a Vîşnegradski function (Calise et al., 2002); it results

$$H_0(s) = \frac{\psi(s)}{\overline{\psi}(s)} = \frac{b_dk_d^{\psi}}{s^2+\left(b_dk_d^{\dot\psi}-a_{33}\right)s+\left(b_dk_d^{\psi}-a_{31}\right)} = \frac{b_dk_d^{\psi}}{s^2+2\xi\omega_0 s+\omega_0^2}. \quad (7)$$

Imposing values for ξ and ω_0, from (7), one results the two non-dimensional transmission ratios

$$\hat k_d^{\psi} = \frac{a_{31}+\omega_0^2}{b_d},\ \hat k_d^{\dot\psi} = \frac{a_{33}+2\xi\omega_0}{b_d}. \qquad (8)$$

To find out the dimensional transmission ratios one proceeds as follows

$$\hat k_d^{\psi} = k_d^{\psi}, \hat k_d^{\dot\psi} = \frac{\hat\delta_d}{\frac{d\hat\psi}{d\hat t}} = \frac{\delta_d}{\frac{d\psi}{d\hat t}} = \frac{\delta_d}{\frac{d\psi}{d(t/\tau_a)}} = \frac{1}{\tau_a}\frac{\delta_d}{\dot\psi} \Rightarrow k_d^{\dot\psi} = \tau_a\hat k_d^{\dot\psi}, \quad (9)$$

where τ_a is the aerodynamic time constant.

The open loop transfer function is obtained easily using (7)

$$H_d(s) = \frac{H_0(s)}{1-H_0(s)} = \frac{b_dk_d^{\psi}}{s^2+\left(b_dk_d^{\dot\psi}-a_{33}\right)s-a_{31}}. \qquad (10)$$

For the obtaining of the block diagram with execution element's visualization, one takes into account the dependence that exists between direction deflection δ_d and the pilot's commnad u (Larin, 2003)

$$\frac{\delta_d}{u} = k_d^{\psi}, \qquad (11)$$

and the equation

$$u = \overline{u} - \sum_{i=1}^{2} k_{x_i}x_i = \overline{\psi} - \psi - \frac{k_d^{\dot\psi}}{k_d^{\psi}}\cdot\dot\psi, \qquad (12)$$

where $\overline{u}$ is the pilot desired command and $\overline{\psi}$ – the imposed value of the yaw's angle.

3. SIMULATION RESULTS

One is studing now the stability of the yaw angle's control system with P.D. control law for three different aircraft types and three flight regimes: light aircraft (H=10 km, M=0.8), average aircraft (H=10 km, M=0.9) and heavy aircraft (H=12 km, M=0.9). The coefficients that occurs in (4) may be calculated with equation

$$a_{31} = -n_{31}\,; a_{33} = -n_{33}\,; b_d = -n_d\,, \tag{13}$$

where n_{31}, n_{33}, n_d have the values presented in Table 1.

Aircraft type/ Flyght regime	**Coefficients**		
	n_{31}	n_{33}	n_d
Light aircraft (H=10 km; M=0.8)	5.76	0.22	3.18
Average aircraft (H=10 km; M=0.9)	69	0.89	29.6
Heavy aircraft (H=12 km; M=0.9)	50	0.9	19

Tab.1. Coefficients for three aircraft types

One considers the case of lateral movement for three aircraft types and three flight regimes (Table 1). The Matlab/Simulink model, associated to the block diagram of the system for yaw's angle control, is presented in fig.2. One has chosen, in the simulation of the Matlab/Simulink model, an integration step equal with 0.01.

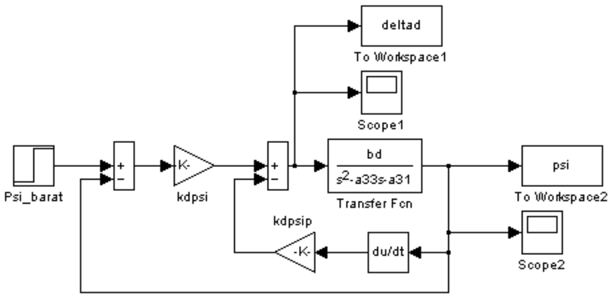

Fig.2. Matlab/Simulink model of the system

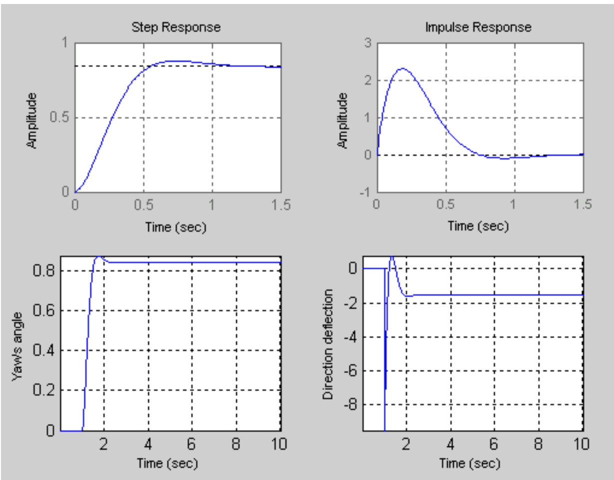

Fig. 3. Characteristics for a light aircraft

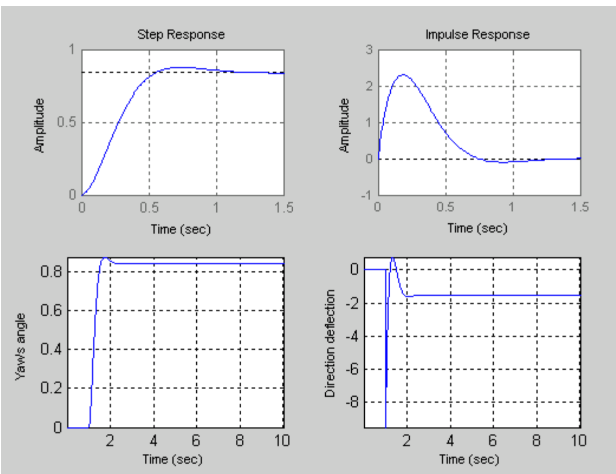

Fig. 4. Characteristics for an average aircraft

Using the program developed by the authors, one obtains, for every aircraft's type, the step response of the system (using instruction *step*), Dirac impulse of the system (using instruction *impulse*), time variation of the yaw's angle ψ (using instruction *plot*) and time variation of the direction deflection δ_d (using instruction *plot*) (Ghinea, Fireteanu, 2001). These characteristics are fig.3 (for a light aircraft), fig.4 (for an average aircraft) and fig. 5 (for a heavy aircraft).

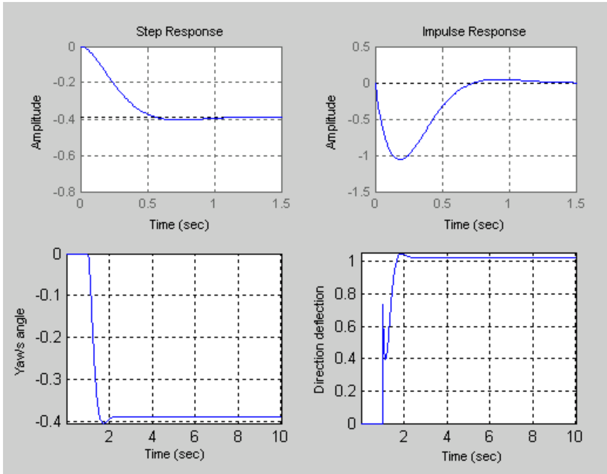

Fig. 5. Characteristics for a heavy aircraft

As one can see, the system is a stable one with very good properties (the transient regime is small). The poles of the system are presented below (because the denominators are the same, the poles for light, average or heavy aircrafts are the same). From the analysis of the system's poles one concludes that the system is a stable one.

Aircraft type	**Poles of the systems**
Light aircraft	$4.242 \pm 4.243\,i$
Average aircraft	$4.242 \pm 4.243\,i$
Heavy aircraft	$4.242 \pm 4.243\,i$

Tab.2. The poles of the three systems

4. CONCLUSION

A system which stabilizes the yaw's angle of an aircraft has been obtained. By determination of the dimensional and non – dimensional transmission ratios, one has projected the control law. The simulation of the system is made in Matlab/Simulink using a new Matlab algorithm developed by the authors. Indicial responses, impulse responses and time variations of the yaw's angle and direction's deflection have been obtained for three aircraft types and three flight regimes. In all cases, one may notice the stability of the system and small time response.

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