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ROBUST CONTROLLERS FOR ANISOCHRONIC DELAYED SYSTEMS

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Abstract: *A way for treating general delayed systems with uncertain delays in both the numerator and denominator is shown. The proposed procedure is demonstrated by an example. A simple controller is derived via algebraic theory and the structured singular value, which treats uncertain time delay in both the numerator and denominator of an anisochronic system. The overall performance is verified by simulations and compared with standard tool for robust control design.*
Key words: *robust control, linear fractional transformation, time delay, algebraic synthesis, structured singular value*

1. INTRODUCTION

The aim of this paper is an application of robust control design to general anisochronic plant with time delay in both the numerator and denominator, and the algebraic approach together with the structured singular value is used as a tool.

The algebraic theory [e.g. Kučera (1972)] is well known and its importance is growing due to the simplicity of controller derivation and the fact that some crucial properties of the resulting feedback loop can be easily influenced by the choice of the controller structure, which is not hard to do within the scope of this approach. The structured singular value denoted μ [see Doyle (1982)] provides a measure of robust stability and performance that can take into account many aspects of controller design. However, the standard tools for μ synthesis are not able to design controllers with a predefined structure. The algebraic approach provides methodology for synthesis of very simple controllers (PI, PID), yet with an excellent functionality compared with the *D-K* iteration.

Many procedures has been developed for control of time delay systems including LFT approaches using multiplicative uncertainty or internal model control (IMC) dealing with design in the ring of meromorphic functions [e.g. Zítek and Kučera (2003)].

In this paper, a general scheme for treating anisochronic delayed systems via LFT will be shown alongside with an example of application to such a system with time delay in both the numerator and denominator. The controller design is performed using algebraic μ-synthesis [see Dlapa et al. (2009)] as well as a comparison study with a standard tool - the *D-K* iteration. The overall performance is verified by simulation of step response for different values of time delays and for two degrees of freedom feedback loops (2DOF).

2. MODELING OF DELAYED SYSTEMS VIA LFT

Consider general delayed system with uncertain time delays in both the numerator and denominator:

$$P(s) \equiv \frac{(b_0 + b_1 s + \ldots + b_n s^n)e^{-\tau_b s}}{s^n + a_0 e^{-\tau_0 s} + a_1 s e^{-\tau_1 s} + \ldots + a_{n-1} s^{n-1} e^{-\tau_{n-1} s}}$$

$$\tau_b \in [0, T_{db}],\ \tau_i \in [0, T_{di}]\ i = 0, 1, \ldots, n\text{–}1 \qquad (1)$$

This plant can be (with some conservatism) expressed via LFT in Fig. 1. Perturbations δ_{delb}, $\delta_{deli} \in \mathbf{C}$ satisfy conditions

$$|\delta_{delb}| < 1\,,\ |\delta_{deli}| < 1 \qquad (2)$$

And for weights W_{delb} and W_{deli} the following inequalities must be held for all $\omega \in \mathbf{R}$:

$$|W_{delb}(j\omega)| > |1 - e^{j\omega T_{db}}| \qquad (3)$$

$$|W_{deli}(j\omega)| > |1 - e^{j\omega T_{di}}|,\ i = 0,1,\ldots,n-1 \qquad (4)$$

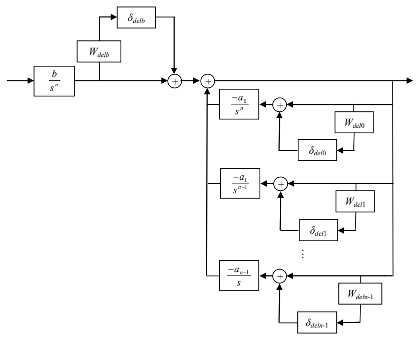

Fig. 1. LFT interconnection of general delayed system

3. ALGEBRAIC μ-SYNTHESIS

The plant (1) can be treated by the interconnection in Fig. 2 with sensitivity function as performance indicator.

Perturbation matrix has the form:

$$\mathbf{\Delta}_{del} \equiv \begin{bmatrix} \delta_{delb} & 0 \\ 0 & \mathbf{\Delta}_{dela} \end{bmatrix},\ \mathbf{\Delta}_{dela} \equiv \begin{bmatrix} \delta_{del0} & 0 & \cdots & 0 \\ 0 & \delta_{del1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \delta_{deln-1} \end{bmatrix},$$

$$|\delta_{delb}| < 1\,,\ |\delta_{deli}| < 1\,,\ \delta_{delb},\ \delta_{deli} \in \mathbf{C},\ i = 0,\ldots,n\text{–}1 \qquad (5)$$

For stability and performance the following theorem holds:
Theorem 1 [Doyle (1982)]: For Δ_{del} defined by (5) is the loop in Fig. 2 well posed, internally stable and $\|\mathbf{F}_L[\mathbf{F}_U(\mathbf{G}, \mathbf{\Delta}_{del}), K]\|_\infty \le 1$ iff

$$\sup_{\substack{\omega \in R \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G}, K)(j\omega)] \le 1 \qquad (6)$$

with $\mathbf{\Delta} \equiv \begin{bmatrix} \delta_P & 0 \\ 0 & \mathbf{\Delta}_{del} \end{bmatrix}$, $|\delta_P| < 1$, $\delta_P \in \mathbf{C}$.

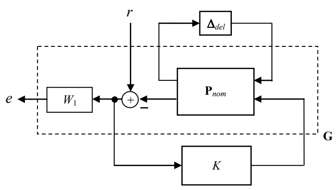

Fig. 2. Closed-loop interconnection

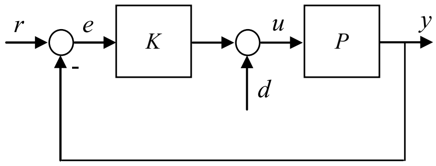

Fig. 3. Feedback loop

Define sensitivity function as transfer function from reference r to error e in Fig. 3:

$$S \equiv \frac{1}{1+PK} \tag{7}$$

Now, as a consequence of Theorem 1, a sufficient condition for robust stability and performance of the feedback loop in Fig. 3 can be formed for sensitivity function S and the family of plants (1).

Corollary 1: For the set of plants (1) feedback loop in Fig. 3 is internally stable and $\|SW_1\|_\infty \le 1$ if conditions (3), (4) and (6) hold.

The algebraic synthesis can be applied to the nominal plant

$$P_0(s) \equiv \frac{b_0 + b_1 s + \ldots + b_n s^n}{s^n + a_0 + a_1 s + \ldots + a_{n-1} s^{n-1}} \tag{8}$$

which can be transformed to:

$$P_0(s) = \frac{\dfrac{b_0 + b_1 s + \ldots + b_n s^n}{(\alpha_1 + s)(\alpha_2 + s)\cdot\ldots\cdot(\alpha_n + s)}}{\dfrac{s^n + a_0 + a_1 s + \ldots + a_{n-1} s^{n-1}}{(\alpha_1 + s)(\alpha_2 + s)\cdot\ldots\cdot(\alpha_n + s)}} = \frac{B}{A},\ A, B \in \mathbf{R}_{\mathrm{PS}} \tag{9}$$

Then the controller is obtained as a solution to the Diophantine equation:

$$AD_K + BN_K = 1, \qquad D_K, N_K \in \mathbf{R}_{\mathrm{PS}} \tag{10}$$

Equation (10) is often called the Bezout identity, and all feedback controllers N_K/D_K are given by

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT}, \quad N_{K_0}, D_{K_0} \in \mathbf{R}_{\mathrm{PS}} \tag{11}$$

where N_{K_0}, $D_{K_0} \in \mathbf{R}_{\mathrm{PS}}$ are particular solutions of (10) and T is an arbitrary element of $\mathbf{R}_{\mathrm{PS}}$.

The controller (11) derived as a solution to (10) safeguards that the nominal feedback loop in Fig. 4 is BIBO stable, which is important for appropriate theorems related to the structured singular value. If the nominal feedback system has a pole in the right half plane then these theorems cannot be used. However, this is not the case if the BIBO stability is held.

The aim of synthesis is to design a controller which satisfies condition:

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G}, K)(\omega, \alpha_1, \ldots, \alpha_{n+n_1+n_2}, t_1, \ldots, t_{n_2})] \le 1,\ \omega \in (-\infty, +\infty) \tag{12}$$

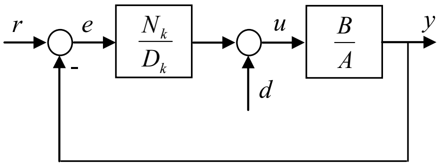

Fig. 4. Nominal feedback loop

where $n + n_1 + n_2$ is the order of the nominal feedback system, n_1 is the order of particular solution K_0, t_i are arbitrary parameters in $T = \dfrac{t_0 + t_1 s + \ldots + t_{n_2} s^{n_2}}{(\alpha_{n_1+1} + s)\cdot\ldots\cdot(\alpha_{n_1+n_2} + s)}$ and μ_Δ denotes the structured singular value of LFT on generalized plant $\mathbf{G}$ and controller K with

$$\mathbf{\Delta} \equiv \begin{bmatrix} \mathbf{\Delta}_{del} & 0 \\ 0 & \delta_P \end{bmatrix},\ \delta_P < 1,\ \delta_P \in \mathbf{C} \tag{13}$$

where $\mathbf{\Delta}_{del}$ denotes the perturbation matrix (5) and δ_P is a complex number corresponding with the robust performance condition.

Tuning parameters are positive and constrained to real axis since parameters of transfer function have to be real and due to the fact that non-real poles cause oscillation of nominal feedback loop.

A crucial problem of the cost function in (12) is the fact that many local extremes are present. Hence, in most cases, local optimization does not yield a suitable or even stabilizing solution. This can be overcome via evolutionary optimization, using Differential Migration (Dlapa, 2009).

4. EXAMPLE – UNSTABLE DELAYED SYSTEM

Consider the set of anisochronic systems with time delay in the numerator and denominator:

$$P(s) \equiv \frac{3e^{-\tau_1 s}}{5s - e^{-\tau_2 s}},\ \tau_1 \in [0, 4],\ \tau_2 \in [0, 0.8] \tag{14}$$

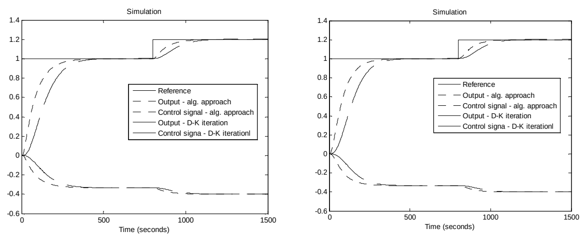

Fig. 5. Simulation for with 2DOF structure ($\tau_1 = 4$, $\tau_2 = 0.8$ and $\tau_1 = 2$, $\tau_2 = 0.4$)

Simulation for 2DOF structure and stepwise reference signal is in Fig. 5. It is apparent that the *D-K* iteration has a non-zero steady state error, which is not the case of the algebraic approach.

5. ACKNOWLEDGMENTS

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