

## THE DIFFERENTIAL MODELING OF END-EFFECTOR'S POSE ERRORS APPLIED TO ROBOTS CALIBRATION

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**Abstract:** *The present paper proposes a method for the differential modeling of robots end-effector's pose errors and its application in robots calibration. This study leads to a relevant improvement of industrial robots performances. Differential modeling is used to design a robotic application in order to determine the end-effector's distribution errors in the robot's workspace. It can thus be determined the optimal workspace for the given application. Considering the fact that the measurement of the end-effector's position and orientation is performed for many points in the robot's workspace, the equation system is sometimes over estimated. This fact leads to approximate solutions, using the smallest squares method.*

**Key words:** *modelling, pose error, calibration, robot*

### 1. INTRODUCTION

A high level of robots accuracy can be quoted as the essential request in many advanced robotic applications. Without any changes of the mechanical structure, the robots pose capacity can be improved successfully, using different calibration procedures. Considering only the aspect regarding the static position and orientation, a robot's pose accuracy is determined by: geometrical factors, like the variation of an element's length and joint's orientation; non-geometrical factors, like element's elasticity and clearances in joints, compliance, eccentricities of gearings and their beatings, thermo alterations of the element's length; uncertainty in locating the base coordinates system, in comparison with the world coordinate system. In literature, is appreciated the contribution of different sources to these errors, concluding the fact that errors due to the geometrical factors represent (90÷95)% of robots and manipulators pose errors. (Elatta et al., 2004) gives an overview of the existing work on robots calibration and (Hefele, J., 2002) investigates a new photogrammetric approach to determine the pose of the robot's end-effector in real-time in order to calibrate the robots. (Maas, H.G., 2006) analyze the requirements of a measurement technique for industrial robots calibration. For this, the author propose a photogrammetric robot calibration system. (Ye, S.H. et al., 2006) presents a novel iterative algorithm which can also get parameters deviations at the same time. It was a method of differential kinematics to solve link parameters deviations and approaching real values step-by-step. The present paper, propose a method for the differential modeling of an robot's end-effector's pose errors and its application in robots calibration.

The major problem is that the differences between the robot's nominal geometry, obtained by designing, in concordance with robot's functions and its real geometry, affected by manufacturing tolerances and assembling errors, occur during robot's assembling. Generally, the nominal geometrical model is simple. Differences between the nominal geometrical model and the real one appear, also, because of some non-geometrical errors (joints elasticity, errors of the reduction ratio etc.). For example, joints and links flexibility is responsible for (8÷10) % of end-effectors position and

orientation errors, much smaller than joints movements. Therefore, a robot's geometrical calibration implies four stages: modeling the robot's functions, measuring the end-effector's pose and the relative pose of elements in the robot joints, identifying the differences between robot's nominal and real geometry and the performing the corrections in the robot's program.

### 2. ERRORS GEOMETRICAL PARAMETERS

In literature, there are different ways of modeling a robot's pose errors. Most authors use Hartenberg-Denavit parameters ( and ) in modeling the geometrical errors of relative position parameters between a robot's elements. It is known the fact that the small geometrical errors of these four parameters can lead to important variations of pose parameters. For example, if the axis is of successive rotation, joints are parallel; the common normal which defines the distance  $a_1$  between the two axes can be arbitrarily localized. If the two axes have a very small deviation from parallelism, this distance can vary very much as the size and the position are depending on its localization. According to this reason, it is necessary an external parameter, the angle, which defines a system rotation ( $i$ ) in a new system ( $i'$ ), around the  $y_i$  axis (fig. 1). In angle's absence (called twist angle), non-parallelism of axis must be set off by an artificial correction of lengths  $a_i$  and  $d_i$ , even if these parameters were initially correct. The rotation angle is used only for the parallel axis of successive rotation joints. For the nominal geometrical model,  $\theta_i = 0$ . The geometrical model allows establishing the pose vector of the robot's end-effectors, as function of Hartenberg-Denavit geometrical parameters:

$$(1)$$

where  $\mathbf{p}_i$  are  $R^n$  vectors, for the robot's "n" joints. The additional geometrical parameter  $\theta_i$  can or cannot be used in the geometrical model. Further on, it is analyzed the Hartehberg-Denavit (HD) geometrical errors influences, as well the nongeometrical errors influences, generated by the clearance from the robot's joints, on his end-effectors pose errors.

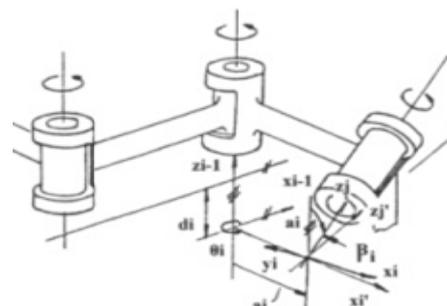


Fig.1. Errors parameters

It is known the fact that for describing the relative shape and position among a robot's successive joints, in the matrix method, is assigned to each element "i", a (x, y, z) coordinate system (fig.2). In this way, the definition of element's "i" relative position, in comparison with element "i-1", is made by a homogeneous transformation matrix,  ${}^{i-1}A_i^N$ , where:

$d_i, \theta_i, a_i, \alpha_i$  are HD nominal geometrical parameters. The superior index N shows the fact that this is a nominal transformation matrix. The nominal parameters  $d_i$  and  $\theta_i$  are variable at translation and rotation joints. Generally, the nominal position and orientation of the reference system attached to the robot's element "n",  $(x, y, z)_n$ , related to the base reference system  $(x, y, z)_0$ , can be written as a matrix product such as:  ${}^0T_n^N = \prod_{i=1}^n A_i^N$ . The robot's "n" element is the end-

effector. This equation describes an ideal situation (nominal transformation). In fact, each element's real position and orientation differ from the nominal one, due to the errors generated by the clearances in joints and element's dimensional errors. In (Vacarescu et al., 2008), is proposed a method for the differential modeling of the pose errors for the end-effector of the upper human limb, modeled through an open kinematic chain with 7 DOF. This is used for develop an arm prosthesis and improve its performance by geometrical calibration. Same method can be used in robotics domain for geometrical calibration.

### 3. THE MODELING OF END-EFFECTOR'S POSE ERRORS GENERATED BY THE GEOMETRICAL ERRORS OF HD PARAMETERS

Therefore, in the geometrical error's absence, the transformation matrix between the systems {n} and {0} is given by the relation (2) where  $A_i^n$  represents the homogeneous transformation HD nominal matrix, between {i-1} and {i}, being (for successive rotation joints) like:

$$A_i^n = \begin{bmatrix} c\theta_i & -c\alpha_i \cdot s\theta_i & s\alpha_i \cdot s\theta_i & a_i \cdot c\theta_i \\ s\theta_i & c\alpha_i \cdot c\theta_i & -s\alpha_i \cdot c\theta_i & a_i \cdot s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where  $s\theta_i$  and  $c\theta_i$ ,  $s\alpha_i$  and  $c\alpha_i$  represents  $\sin\theta_i, \cos\theta_i$  and  $\sin\alpha_i, \cos\alpha_i$ . Due to the geometrical errors, noted by  $\Delta a_i, \Delta \alpha_i, \Delta d_i, \Delta \theta_i$  (and possibly  $\Delta \beta_i$ ), the matrix (2) register an elementary variation  $\delta A_i$ , called the transformation differential matrix between {i-1} and {i}:  $\delta A_i = \delta A_i \cdot A_i^n$  where  $\delta A_i$  is a differential operator. Its elements are function of the geometrical errors  $\Delta a_i, \Delta \alpha_i, \Delta d_i, \Delta \theta_i$ . The differential operator  $\delta A_i$  is defined by the matrix:

$$\delta A_i = \begin{bmatrix} R_i^e & P_i^e \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -e_{iz}^e & e_{iy}^e & \delta_{ix}^e \\ e_{iz}^e & 0 & -e_{ix}^e & \delta_{iy}^e \\ -e_{iy}^e & e_{ix}^e & 0 & \delta_{iz}^e \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

where:

$$\begin{aligned} e_{ix}^e &= c\theta_i \cdot \Delta \alpha_i; \\ e_{iy}^e &= s\theta_i \cdot \Delta \alpha_i; \\ e_{iz}^e &= \Delta \theta_i; \\ \delta_{ix}^e &= c\theta_i \cdot \Delta a_i - d_i \cdot s\theta_i \cdot \Delta \alpha_i; \\ \delta_{iy}^e &= s\theta_i \cdot \Delta a_i + d_i \cdot c\theta_i \cdot \Delta \alpha_i; \end{aligned} \quad (4)$$

$$\delta_{iz}^e = \Delta d_i$$

The differential matrix of the end-effector 's pose errors is expressed by the relation:

$$\delta T = \text{Transl}(dx, dy, dz) \cdot \text{Rot}(x, d\lambda) \cdot \text{Rot}(y, d\rho) \cdot \text{Rot}(z, d\theta) \quad (5)$$

where:  $dx, dy, dz$  - are the vector's component parts of the robot's end-effector's position errors;  $d\alpha, d\rho, d\theta$  - are the vector's component parts of the robot's end-effector's orientation errors. The differential first rank model (linear) of the errors is characterised by the matrix equation (6):

$$\delta T^I = \begin{bmatrix} 0 & -\delta\theta & \delta\rho & dx \\ \delta\theta & 0 & -\delta\lambda & dy \\ -\delta\rho & \delta\lambda & 0 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \sum_{i=1}^n \delta A_i \quad (6)$$

In relation (6), the left equation member is known by measuring the end-effector's position and orientation errors, with a measurement system on the end-effector's pose, and in the right equation member are known the values  $\theta_i$  and  $d_i$  displayed by transducers from joints. Using these initial dates, through the identification of the corresponding terms of the matrix in relation (6), are established the geometrical parameter's errors  $\Delta a_i, \Delta \alpha_i, \Delta d_i, \Delta \theta_i$  (and eventually  $\Delta \beta_i$ ). Using these values, is corrected the robot's geometrical model, and is obtained a decreasing of the end-effector's pose error. The correct transformation matrix is obtained:  $A_i^C = A_i^N + \delta A_i$ . Generally, are made some iterations, until the desired accuracy is achieved.

### 4. CONCLUSION

Generally kinematic model-based calibration is considered as a global calibration method that improves robots accuracy across the whole volume of robots workspace. Kinematic calibration consists of four sequential steps: modelation, measurement, identification and compensation or correction. The differential modeling of the end-effector's pose errors allows the improvement of an industrial robot's performances, by geometrical calibration. The method is used by authors in various domains: robotics, developing prosthesis and also biometric measurements of the human spine.

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