

OPTIMAL SOLUTIONS FOR INVERSE STRUCTURAL MODELS OF BIMOBILE SYSTEMS

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Abstract: *The inverse structure modeling of bi mobile mechanisms is based on the passive modular groups mentioned in the classical theory of mechanisms (Crossley, 1968; Pelecudi 1967). In function of the number of links and independent contours the inverse models may have a maximum number of links for their passive groups. Having in view the inverse models and the complex passive groups with a higher number of links and loops in the paper optimal solutions for bi mobile mechanisms is presented. Such bi mobile mechanisms applied to mechanical structures for robot arms and legs for mobile platforms ensure a higher functional precision.*

Key words: *Bi-mobile mechanism, robot arm, robot leg, Baranov truss, inverse model*



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This Publication has to be referred as: Comanescu, A[driana]; Comanescu, D[inu]; Dugaesescu, I[leana] & Ungureanu, L[iviu] (2014). Optimal Solutions for Inverse Structural Models of Bimobile Systems, Chapter 31 in DAAAM International Scientific Book 2014, pp.383-400, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-901509-98-8, ISSN 1726-9687, Vienna, Austria DOI: 10.2507/daaam.scibook.2014.31

1. Introduction

The paper, the result of many years of research is a synthesis of the past and present, future structural theory in the mechanisms science.

In the last part of the 20th century the mechanisms with two degrees of mobility become especially usefully for various systems in robotics and other equipment (Angeles, 2003). In the same time two concepts – direct and inverse models were developed (Voinea et al., 2000). For the direct model the parameters of any link and pair are principally expressed by those of the active pairs. For the inverse model the active pairs (actuators) parameters are determined relative to the effector parameters.

The effector extremity of such mechanisms may describe any curve in the certain domain of the mechanism.

The inverse structure modeling of bi mobile mechanisms is based on the passive modular groups mentioned in the classical theory of mechanisms (Artobolevski, 1977; Crossley, 1968; Manolescu et al., 1972; Pelecudi 1967).

In order to design structures for bi-mobile mechanisms the following steps are mentioned (Comanescu et al., 2010):

- to put into evidence the matrix of possible bases and effectors for the linkages;
- to eliminate the non-distinct solutions due to the symmetrical characteristics of the linkages;
- to verify the solutions for bases and effectors through the inverse structural model characterized by a zero instantaneous degree of mobility;
- the selection of the optimum structural-constructive solutions including a minimum number of passive modular groups;
- to place in the mechanism structure the active kinematic pairs (actuators);
- to create an optimal structural solution with a minimum number of modular groups.

A bi-mobile planar mechanism with an optimal structure used either for a robot arm or for a leg of a walking robot (Kakudou et al., 2013) must contain a minimum number of modular passive groups for its inverse model and also a minimum number of modular groups for the direct model.

2. Structural considerations

By using the 40 linkages with three independent loops and five degrees of freedom (Tab. 1) the bi-mobile planar mechanisms may be obtained when somebody nominates the basis and the effector. All these structural solutions have nine links, eleven lower pairs and three independent loops.

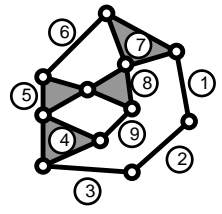
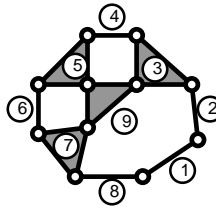
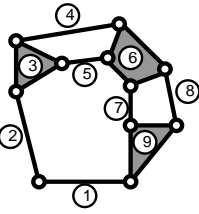
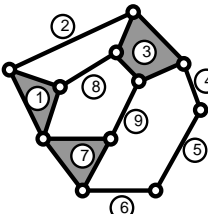
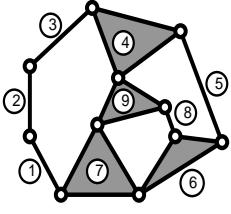
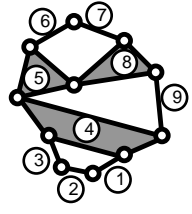
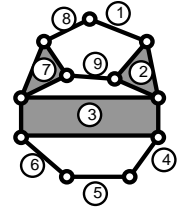
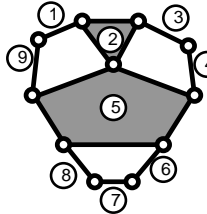
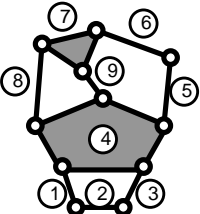
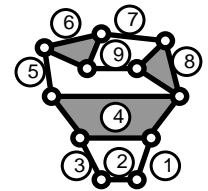
The mobility instantaneously becomes zero due to the placing a connection between the basis and the extremity of the effector and the inverse model has zero degree of mobility. This connection is equivalent to a lower pair with two constrains and a single mobility.

By excluding its basis the structure is composed by an even number of links, which determine passive modular groups [Comanescu et al., 2010].

The planar structures with zero degree of mobility are mentioned [Manolescu et al., 1972) and named Baranov trusses. Any structure with zero degree of mobility has an odd number of links. In the literature [Artobolevski, 1977; Manolescu, 1972] there are mentioned the passive modular groups with 2, 4 and 6 elements.

In the case of the previously mentioned linkages (Tab. 1) the inverse models are constituted by the following passive modular groups with 2+2+2+2, 2+4+2, 2+2+4, 4+2+2, 2+6, 6+2 or 8 elements.

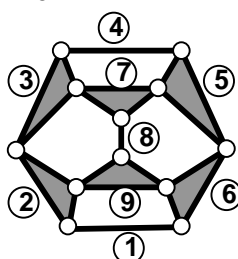
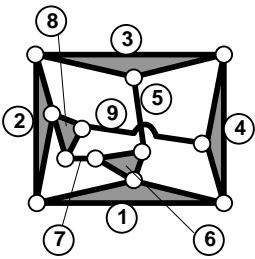
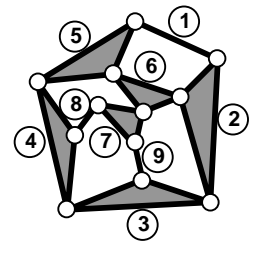
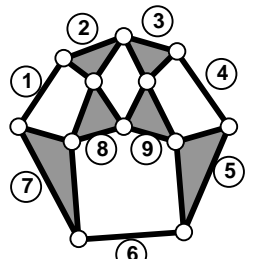
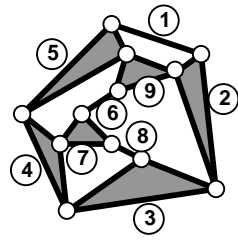
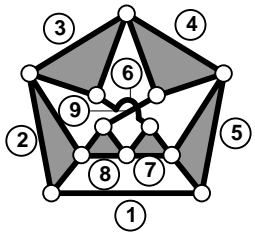
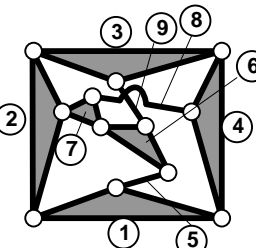
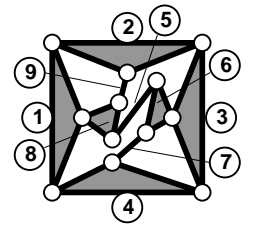
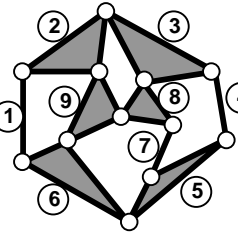
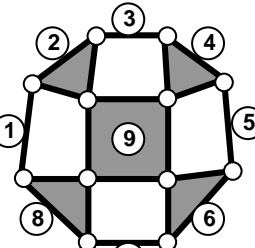
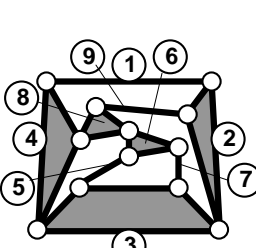
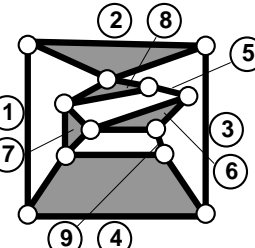
L1 	L2 	L3 	L4 	L5
L6 	L7 	L8 	L9 	L10
L11 	L12 	L13 	L14 	L15
L16 	L17 	L18 	L19 	L20
L21 	L22 	L23 	L24 	L25
L26 	L27 	L28 	L29 	L30

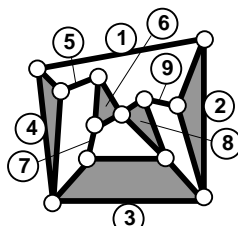
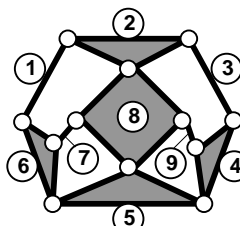
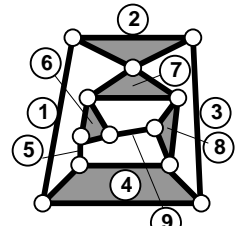
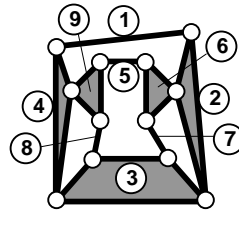
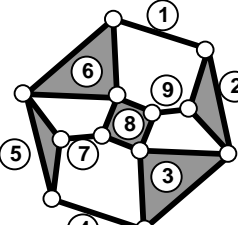
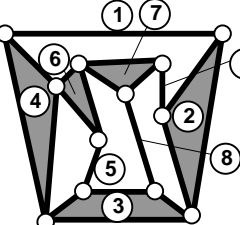
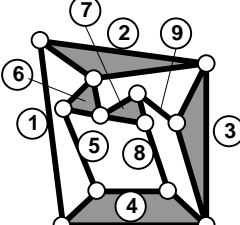
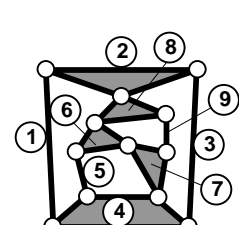
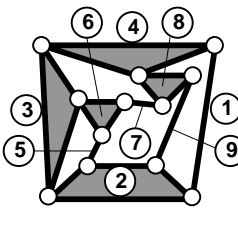
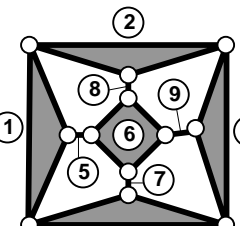
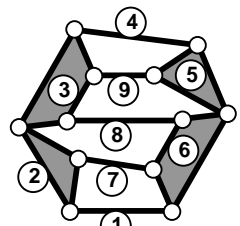
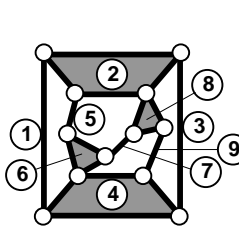
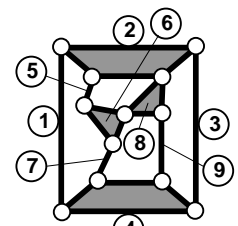
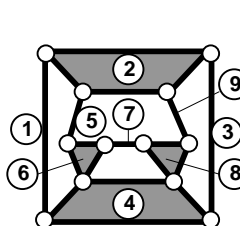
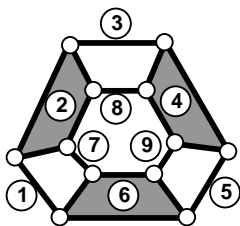
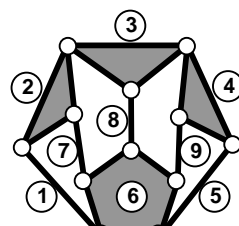
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<p>L36</p> 	<p>L37</p> 	<p>L38</p> 	<p>L39</p> 	<p>L40</p> 

Tab. 1. Three loops planar linkages with five degrees of freedom

The optimal solutions have only one passive modular group with 8 elements connected at the adopted basis. Such groups with 8 elements may be obtained from Baranov trusses with nine elements and four independent loops [Tab. 2] by eliminating a link.

The BT1-BT5 Baranov trusses [Comanescu et al., 2010] have respectively 3, 5 and 7 elements and 1, 2 and 3 independent loops and are eliminated in Tab. 1.

<p>BT 6</p> 	<p>BT 7</p> 	<p>BT 8</p> 	<p>BT 9</p> 
<p>BT 10</p> 	<p>BT 11</p> 	<p>BT 12</p> 	<p>BT 13</p> 
<p>BT 14</p> 	<p>BT 15</p> 	<p>BT 16</p> 	<p>BT 17</p> 

<p>BT 18</p> 	<p>BT 19</p> 	<p>BT 20</p> 	<p>BT 21</p> 
<p>BT 22</p> 	<p>BT 23</p> 	<p>BT 24</p> 	<p>BT 25</p> 
<p>BT 26</p> 	<p>BT 27</p> 	<p>BT 28</p> 	<p>BT 29</p> 
<p>BT 30</p> 	<p>BT 31</p> 	<p>BT 32</p> 	<p>BT 33</p> 

Tab. 2. Baranov trusses with nine elements and four loops

3. Optimal solutions for the inverse models

In some researches of many years there are analyzed the matrix of possible bases and effectors for the linkages mentioned in Tab. 1.

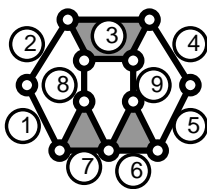
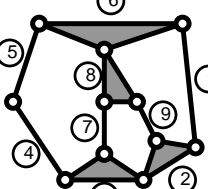
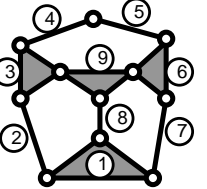
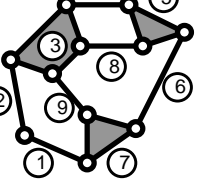
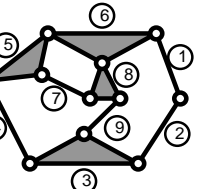
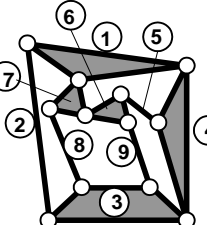
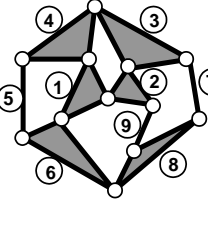
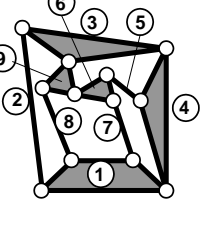
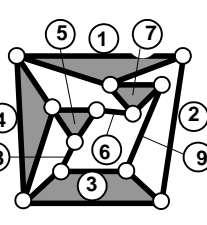
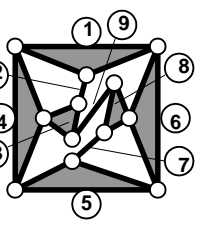
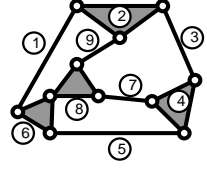
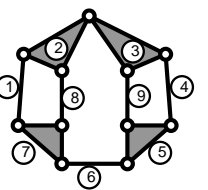
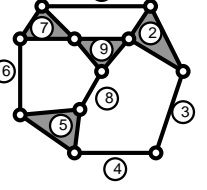
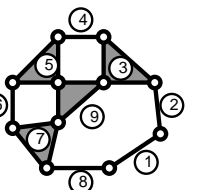
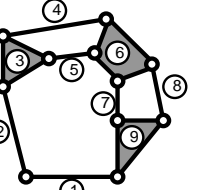
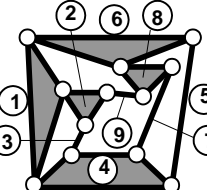
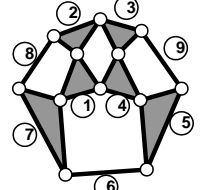
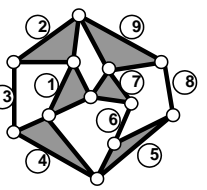
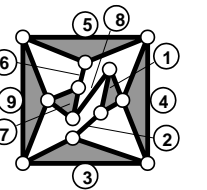
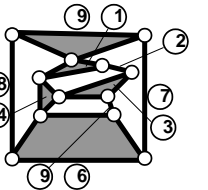
$$A = \begin{bmatrix}
 0 & A[1,2] & A[1,3] & A[1,4] & A[1,5] & A[1,6] & A[1,7] & A[1,8] & A[1,9] \\
 A[2,1] & 0 & A[2,3] & A[2,4] & A[2,5] & A[2,6] & A[2,7] & A[2,8] & A[2,9] \\
 A[3,1] & A[3,2] & 0 & A[3,4] & A[3,5] & A[3,6] & A[3,7] & A[3,8] & A[3,9] \\
 A[4,1] & A[4,2] & A[4,3] & 0 & A[4,5] & A[4,6] & A[4,7] & A[4,8] & A[4,9] \\
 A[5,1] & A[5,2] & A[5,3] & A[5,4] & 0 & A[5,6] & A[5,7] & A[5,8] & A[5,9] \\
 A[6,1] & A[6,2] & A[6,3] & A[6,4] & A[6,5] & 0 & A[6,7] & A[6,8] & A[6,9] \\
 A[7,1] & A[7,2] & A[7,3] & A[7,4] & A[7,5] & A[7,6] & 0 & A[7,8] & A[7,9] \\
 A[8,1] & A[8,2] & A[8,3] & A[8,4] & A[8,5] & A[8,6] & A[8,7] & 0 & A[8,9] \\
 A[9,1] & A[9,2] & A[9,3] & A[9,4] & A[9,5] & A[9,6] & A[9,7] & A[9,8] & 0
 \end{bmatrix} \tag{1}$$

The matrix is a symmetrical one, so that $A[i,j] = A[j,i]$ and $A[i,i] = 0$.

$A[i,j]$ includes the i basis and the j effector adopted for the linkage.

There are also verified all solutions for each linkage bases and effectors through the inverse structural model characterized by a zero instantaneous degree of mobility.

In the next tables at the top $A[i,j]$ is given. The linkage with the i basis and the j effector is mentioned at the top and below of each linkage the Baranov truss corresponding for its inverse model is also included. One may note that the Baranov truss is the same when the basis is changed with the effector.

A[1,4] or A[4,1]				
<p>L12</p> 	<p>L14</p> 	<p>L15</p> 	<p>L23</p> 	<p>L25</p> 
<p>BT 24</p> 	<p>BT 14</p> 	<p>BT 24</p> 	<p>BT 26</p> 	<p>BT 13</p> 
<p>L27</p> 	<p>L29</p> 	<p>L30</p> 	<p>L32</p> 	<p>L33</p> 
<p>BT 26</p> 	<p>BT 9</p> 	<p>BT 14</p> 	<p>BT 13</p> 	<p>BT 17</p> 

Tab. 3. The inverse models for linkages with 1 basis and 4 effector or 4 basis and 1 effector

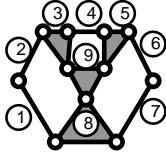
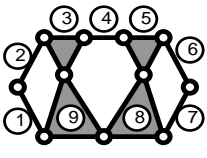
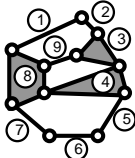
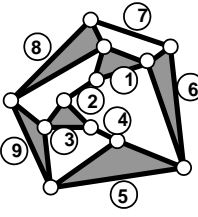
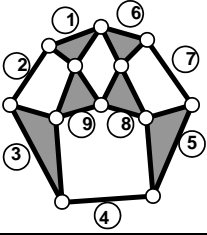
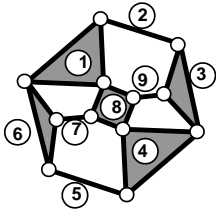
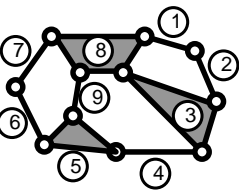
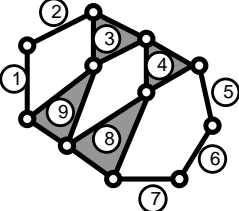
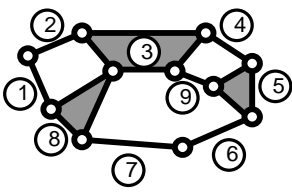
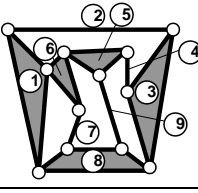
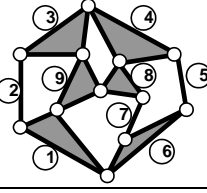
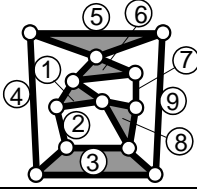
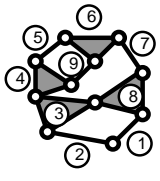
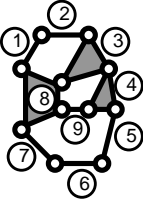
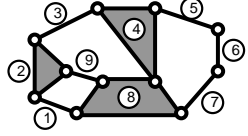
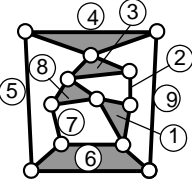
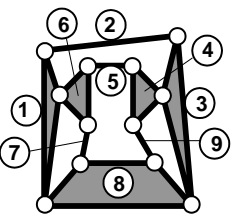
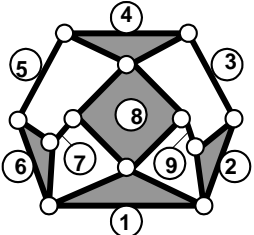
In the Tab. 3 there are mentioned 19 solutions due to the fact that the L29 linkage has a symmetrical structure relative to the 1 and 4 links.

When the basis is the link 1 and the effector is the link 5 or vice versa the solutions of the inverse models of the linkages are presented in the Tab. 4. There are found 17 distinct solutions, because the L12 linkage has a symmetrical structure.

A[1,5] or A[5,1]		
L12 	L20 	L22
BT 27 	BT 10 	BT 30
L23 	L28 	L29
BT 30 	BT 8 	BT 17
L32 	L34 	L37
BT 19 	BT 31 	BT 16

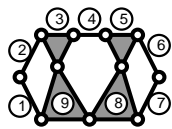
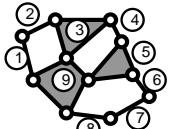
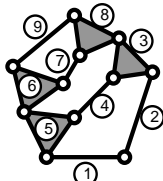
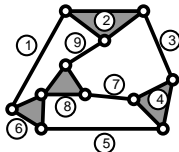
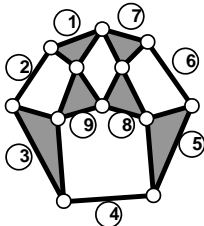
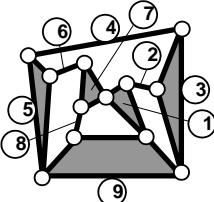
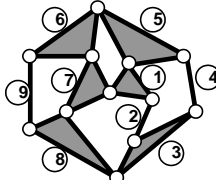
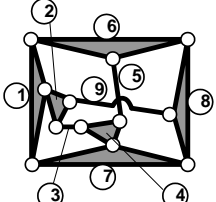
Tab. 4. The inverse models for linkages with the 1 basis and the 5 effector or the 5 basis and the 1 effector

In the Tab. 5 there are given the inverse models for the linkages (Tab. 1) when the basis is the link 1 and the link 6 is the effector or vice versa. The total distinct solutions are 18 ones.

A[1,6] or A[6,1]		
L2 	L6 	L8 
BT 10 	BT 9 	BT 22 
L10 	L11 	L17 
BT 23 	BT 14 	BT 25 
L20 	L21 	L26 
BT 25 	BT 21 	BT 19 

Tab. 5. The inverse models for linkages with the 1 basis and the 6 effector or the 6 basis and the 1 effector

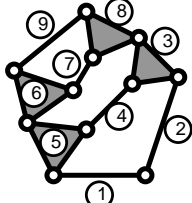
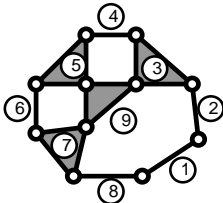
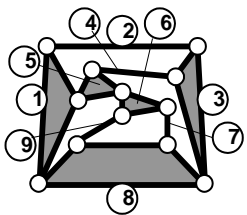
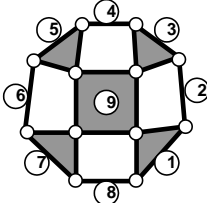
When the 1 basis and the 7 effector and vice versa are adopted, the results are presented in the Tab. 6.

A[1,7] or A[7,1]			
L6 	L7 	L19 	L27 
BT 9 	BT 18 	BT 14 	BT 7 

Tab. 6. The inverse models for the 1 basis and the 7 basis or vice versa

Due to the symmetry of the L6 and L27 linkages in the Tab. 6 one may only find 6 distinct solutions.

The solutions for the 1 basis and the 8 or 9 effectors are respectively presented in the Tab. 7 and 8, which of them having two distinct solutions for inverse models.

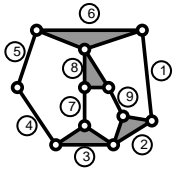
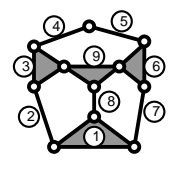
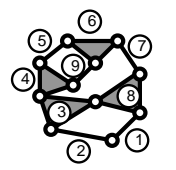
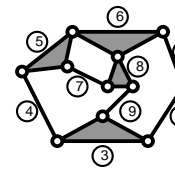
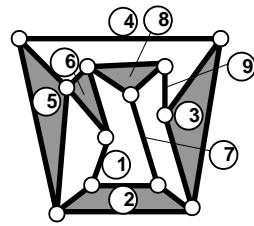
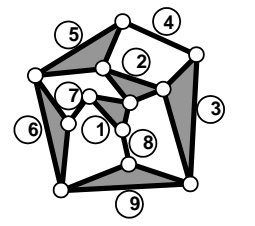
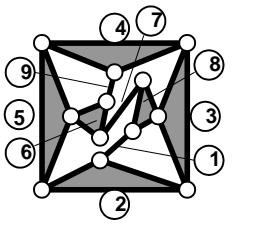
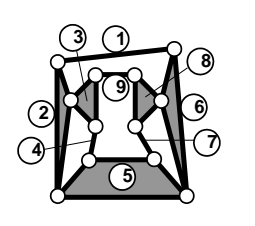
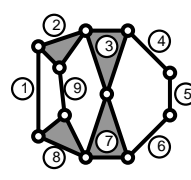
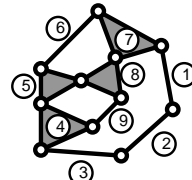
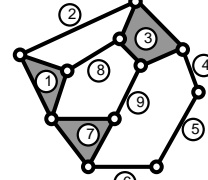
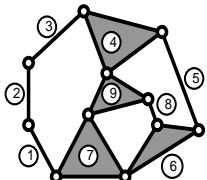
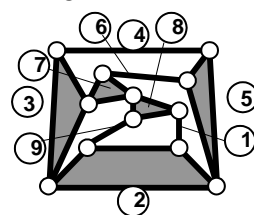
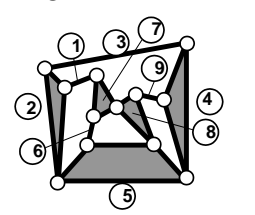
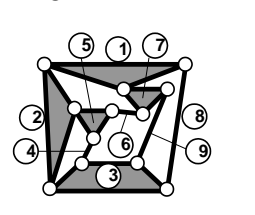
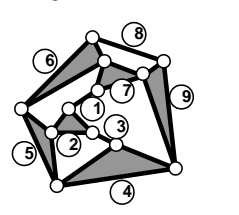
<p>A[1,8] or A[8,1]</p> <p>L19</p> 	<p>A[1,9] or A[9,1]</p> <p>L32</p> 
<p>BT 16</p> 	<p>BT 15</p> 

Tab. 7. Solutions for the 1 or 8 basis and the 8 or 1 effector

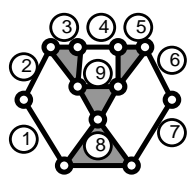
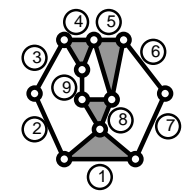
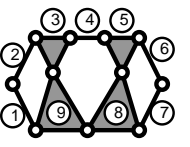
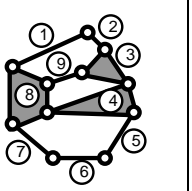
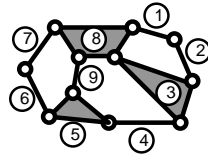
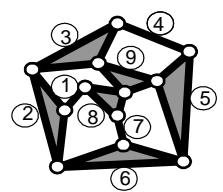
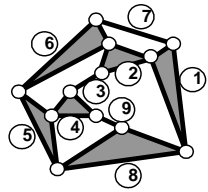
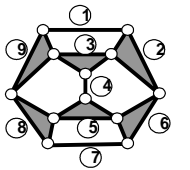
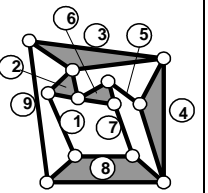
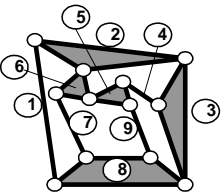
Tab. 8. Solutions for the 1 or 9 basis and the 9 or 1 effector

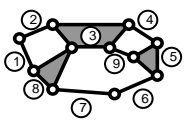
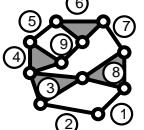
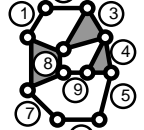
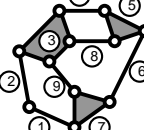
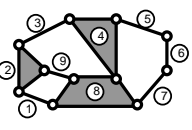
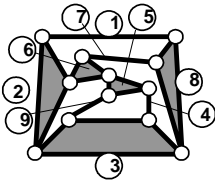
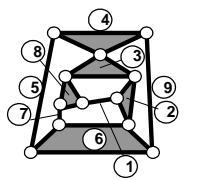
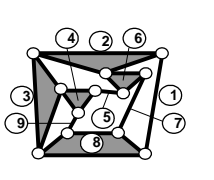
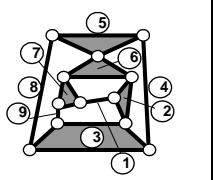
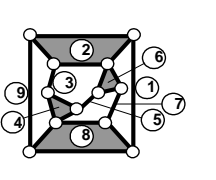
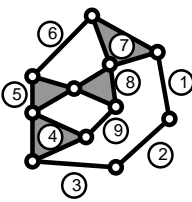
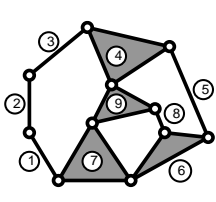
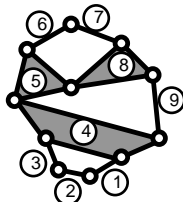
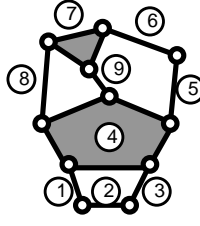
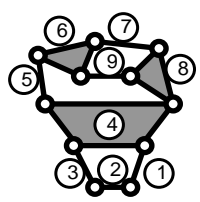
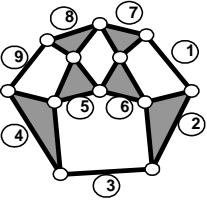
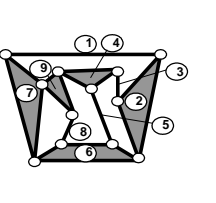
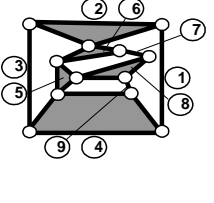
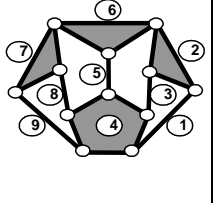
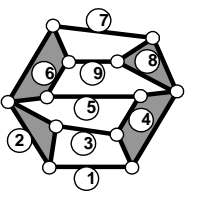
The solutions for the 2 or 5 and 6 links as a basis and the 5 or 6 and 2 links as an effector are presented in the Tab. 9 and Tab. 10.

In the Tab. 9 there are given 16 distinct solutions and similarly in the Tab. 10 there are found 29 distinct solutions the L6 linkage being a symmetrical one.

A[2,5] or A[5,2]			
L14 	L15 	L20 	L25 
BT 23 	BT 8 	BT 13 	BT 21 
L28 	L31 	L34 	L35 
BT 16 	BT 18 	BT 26 	BT 10 

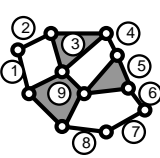
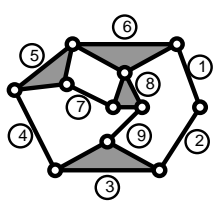
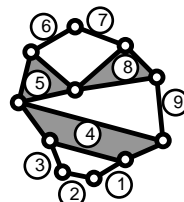
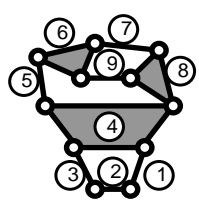
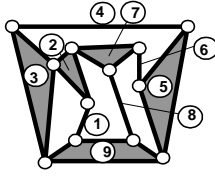
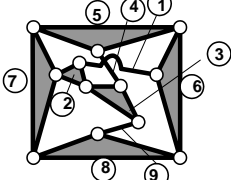
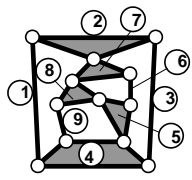
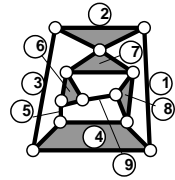
Tab. 9. Solutions for the 2 or 5 basis and the 5 or 2 effector

A[2,6] or A[6,2]				
L2 	L3 	L6 	L8 	L10 
BT 8 	BT 10 	BT 6 	BT 24 	BT 24 

L17 	L20 	L21 	L23 	L26 
BT 16 	BT 20 	BT 26 	BT 20 	BT 29 
L31 	L35 	L36 	L39 	L40 
BT 9 	BT 23 	BT 17 	BT 33 	BT 28 

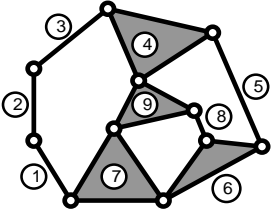
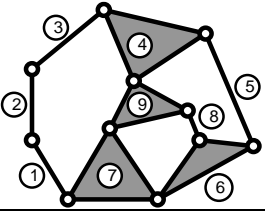
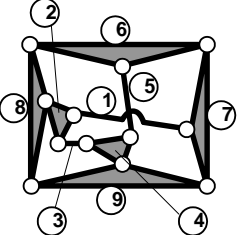
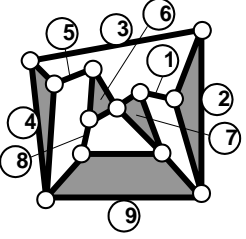
Tab. 10. The solutions for the 2 or 6 basis and the 6 or 2 effector

The inverse models for linkages when the basis is the 2 link and the effector is the 7 link or vice versa is also presented in the Tab. 11. There are shown eight distinct solutions.

A[2,7] or A[7,2]			
L7 	L25 	L36 	L40 
BT 23 	BT 12 	BT 25 	BT 20 

Tab. 11. The solutions for the 2 or 7 basis and the 7 or 2 effector

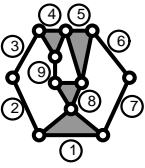
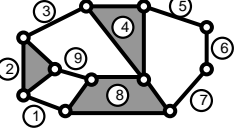
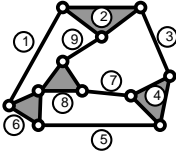
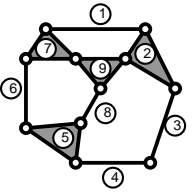
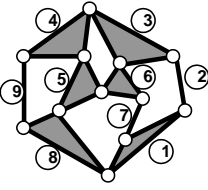
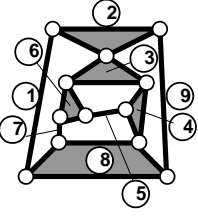
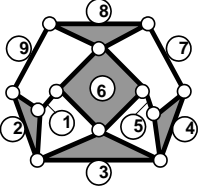
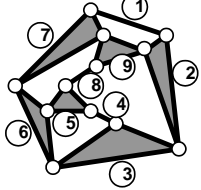
In the Tables 12, 13 there are given similar solutions, that is two for each table.

<p>A[2,8] or A[8,2]</p> <p>L35</p> 	<p>A[2,9] or A[9,2]</p> <p>L35</p> 
<p>BT 7</p> 	<p>BT 18</p> 

Tab. 12. Solutions for the 2 or 8 basis and the 8 or 2 effector

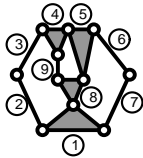
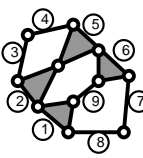
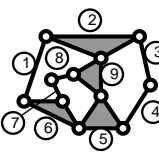
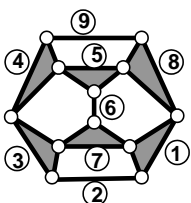
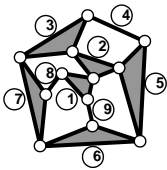
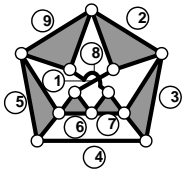
Tab. 13. Solutions for the 2 or 9 basis and the 9 or 2 effector

By changing the basis with the 3 link and the effector with the 6 link the new solutions are given in the Tab. 14. Totally one may find eight solutions.

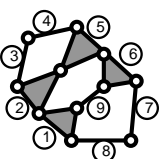
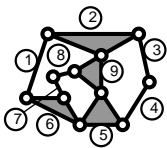
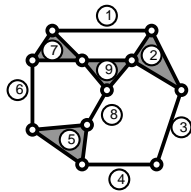
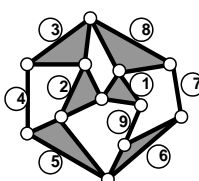
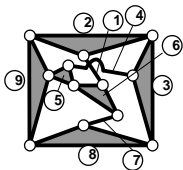
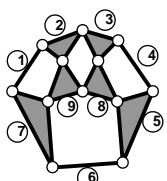
A[3,6] or A[6,3]			
<p>L3</p> 	<p>L26</p> 	<p>L27</p> 	<p>L30</p> 
<p>BT 14</p> 	<p>BT 20</p> 	<p>BT 19</p> 	<p>BT 10</p> 

Tab. 14. Solutions for the 3 or 6 basis and the 6 or 3 effector

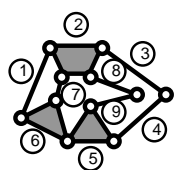
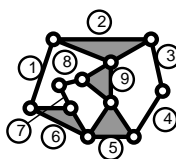
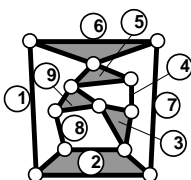
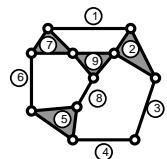
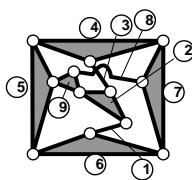
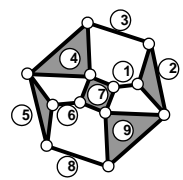
In the next tables there are given the other inverse models solutions for linkages with three independent loops according to the basis – effector matrix (1).

A[3,7] or A[7,3]		
L3 	L5 	L13 
BT 6 	BT 8 	BT 11 

Tab. 15. Solutions for the 3 or 7 basis and the 7 or 3 effector

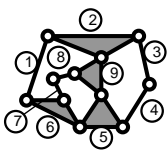
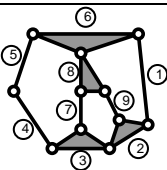
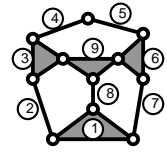
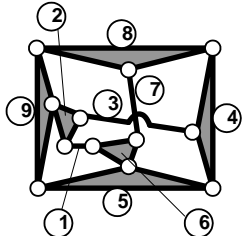
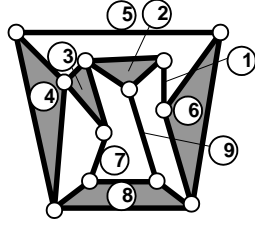
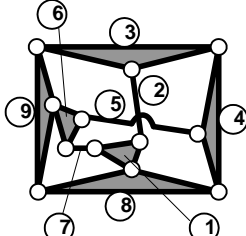
A[3,8] or A[8,3]		
L5 	L13 	L30 
BT 14 	BT 12 	BT 9 

Tab. 16. Solutions for the 3 or 8 basis and the 8 or 3 effector

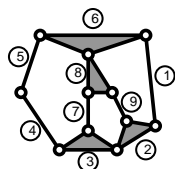
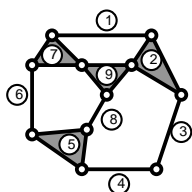
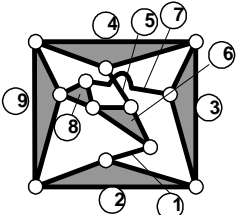
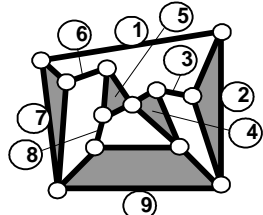
A[3,9] or A[9,3]	A[4,7] or A[7,4]
L18 	L13 
BT 25 	L30 
	BT 12 
	BT 22 

Tab. 17. Solutions for the 3 or 9 basis and the 9 or 3 effector

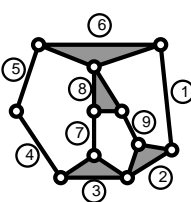
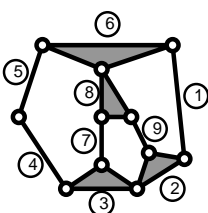
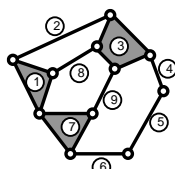
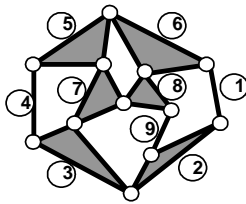
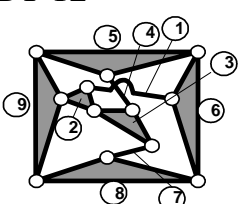
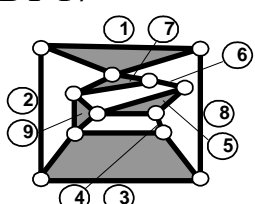
Tab. 18. Solutions for the 4 or 7 basis and the 7 or 4 effector

A[4,8] or A[8,4]		
L13 	L14 	L15 
BT 7 	BT 23 	BT 7 

Tab. 19. Solutions for the 4 or 8 basis and the 8 or 4 effector

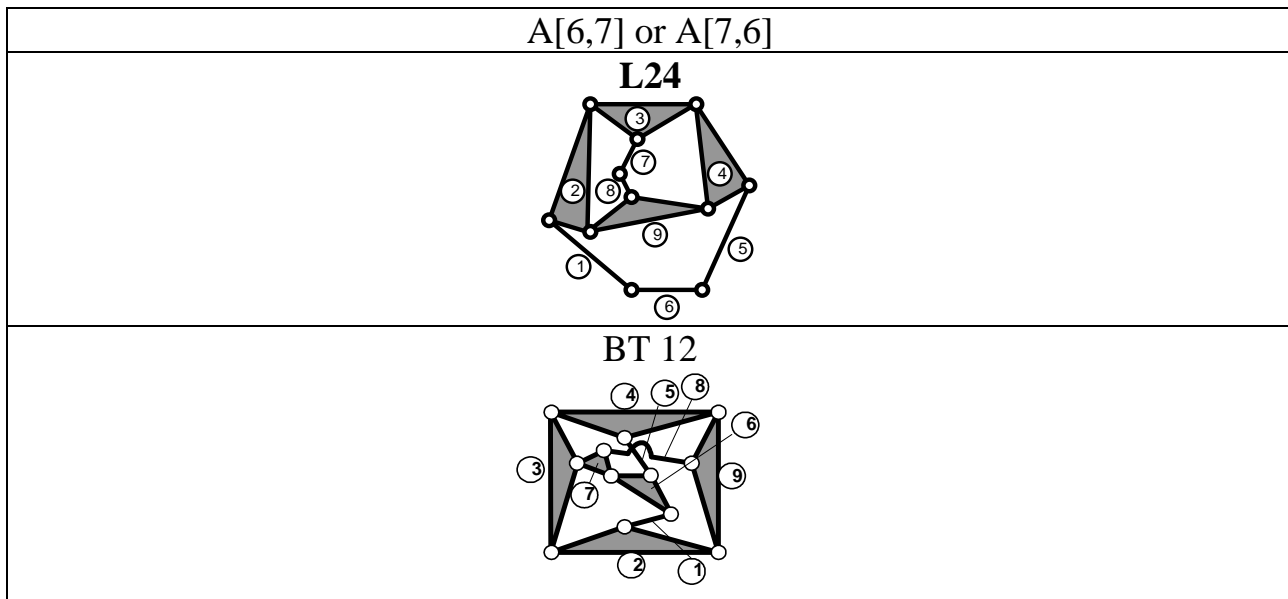
A[4,9] or A[9,4]	
L14 	L30 
BT 12 	BT 18 

Tab. 20. Solutions for the 4 or 9 basis and the 9 or 4 effector

A[5,7] or A[7,5]	A[5,9] or A[9,5]	
L14 	L14 	L34 
BT 14 	BT 12 	BT 17 

Tab. 21. Solutions for the 5 or 7 basis and the 7 or 5 effector

Tab. 22. Solutions for the 5 or 9 basis and the 9 or 5 effector



Tab. 23. Solutions for the 6 or 7 basis and the 7 or 6 effector

Having in view the results presented in Tables 3-23 there are found for all linkages (Tab. 1) with three independent loops and two degrees of mobility a total number of *165 solutions* for which their inverse models are optimal being a single passive group with eight links.

4. Optimal structural design of a bi-mobile pedipulator

At the beginning of this chapter there are mentioned the steps necessary to design a bi-mobile system [Comanescu et al., 2013].

The basis and the effector must satisfy the following requirements [Comanescu et al., 2010]:

- the effector must have a planar motion depending on two independent parameters;
- the effector can not be adjacent to the basis;
- the basis and the effector can not belong to the same four bars linkage, Watt linkage or Stephenson linkage.

For an example, in order to design a new pedipulator from the Table 1 the L33 linkage is selected.

Having in view the previous circumstances the basis-effector matrix is defined.

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

The L33 linkage (Fig. 1) has two mono-mobile structures respectively the 3,4,5,6 links and the 6,7,8,9 links and the following symmetrical links $4 \equiv 5 \equiv 8 \equiv 7$, $3 \equiv 9$ and $1 \equiv 2$. $A[i,j]$ has the i basis and the j effector and there are consequently obtained the following 12 distinct solutions:

$$\begin{aligned}
 & A(1,3) \equiv A(2,9) & & A(3,1) \equiv A(9,2) \\
 & A(1,4) \equiv A(1,5) \equiv A(2,7) \equiv A(2,8) & & A(4,1) \equiv A(5,1) \equiv A(7,2) \equiv A(8,2) \\
 & A(1,6) \equiv A(2,6) & & A(6,1) \equiv A(6,2) \\
 & A(1,7) \equiv A(1,8) \equiv A(2,4) \equiv A(2,5) & & A(7,1) \equiv A(8,1) \equiv A(4,2) \equiv A(5,2) \\
 & A(3,7) \equiv A(3,8) \equiv A(9,4) \equiv A(9,5) & & A(7,3) \equiv A(8,3) \equiv A(4,9) \equiv A(5,9) \\
 & & & A(3,9) \equiv A(9,3) \\
 & A(4,7) \equiv A(4,8) \equiv A(7,4) \equiv A(8,4) \equiv A(5,7) \equiv A(5,8) \equiv A(7,5) \equiv A(8,5)
 \end{aligned}
 \tag{3}$$

From the Tab. 3 one may find the optimal inverse models for the 1 or 4 link as a basis and respectively the 4 or 1 link as an effector (Fig. 2, Fig.3).

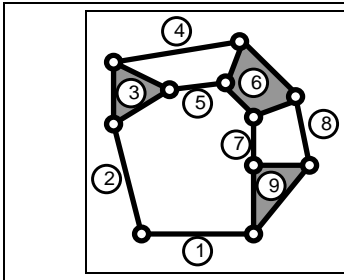


Fig.1. The L33 linkage from the Tab. 1

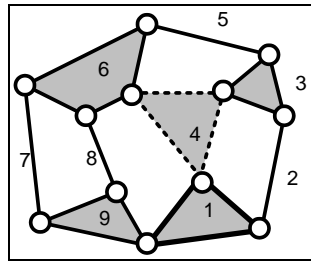


Fig.2. The inverse model with the 1 basis and the 4 effector

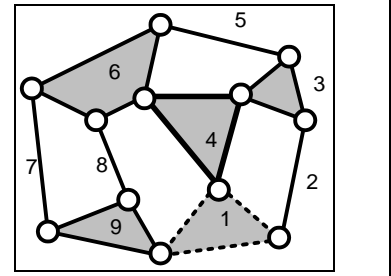


Fig.3. The inverse model with the 4 basis and the 1 effector

Both inverse models correspond to the BT 17 Baranov truss mentioned in the Tab. 1.

Each inverse model is analyzed in the next figures.

If one adopts the basis 1 and the effector 4 (Fig. 4) it is identified from Tab. 2 the BT 17 truss (Fig. 5) from which by eliminating the 1 link a new passive group with 8 elements is obtained (Fig. 6).

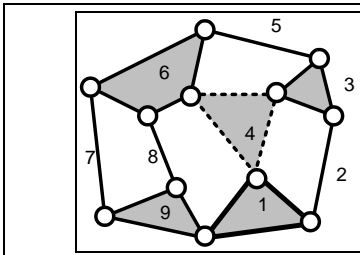


Fig. 4. The inverse model for the 1 basis and the 4 effector

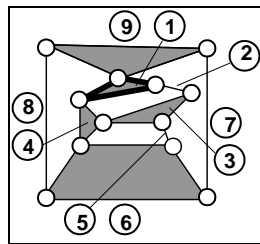


Fig. 5. The BT 17 truss from Table 2

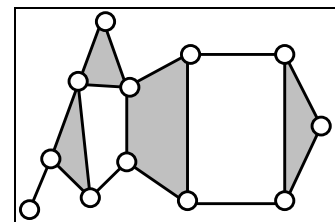


Fig. 6. The new passive group with 8 elements

Similarly if the 4 basis and the 1 effector are adopted (Fig. 7) the BT 17 truss from Tab. 2 is identified (Fig. 8) and another new passive group with 8 elements (Fig.9).

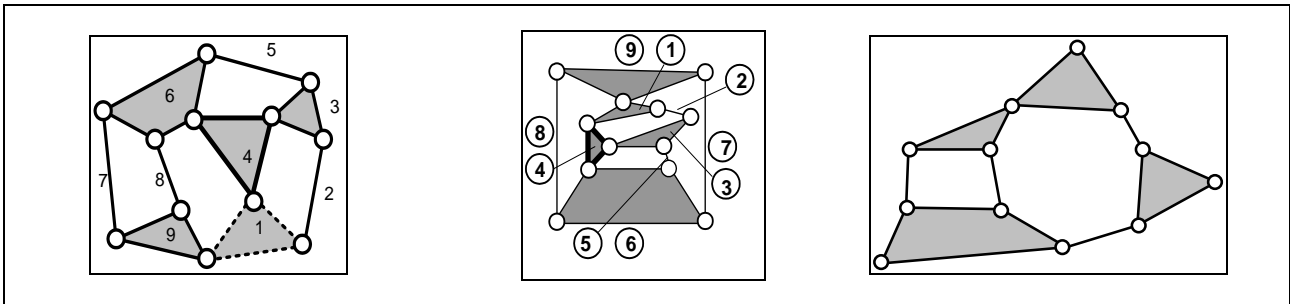


Fig. 7. The inverse model for the 4 basis and the 1 effector

Fig. 8. The BT 17 truss from Tab. 2

Fig. 9. The new passive group with 8 elements

When the solution with the 4 basis and the 1 effector is adopted the pedipulator (the robot leg) is achieved (Fig. 10) in an adequate software.

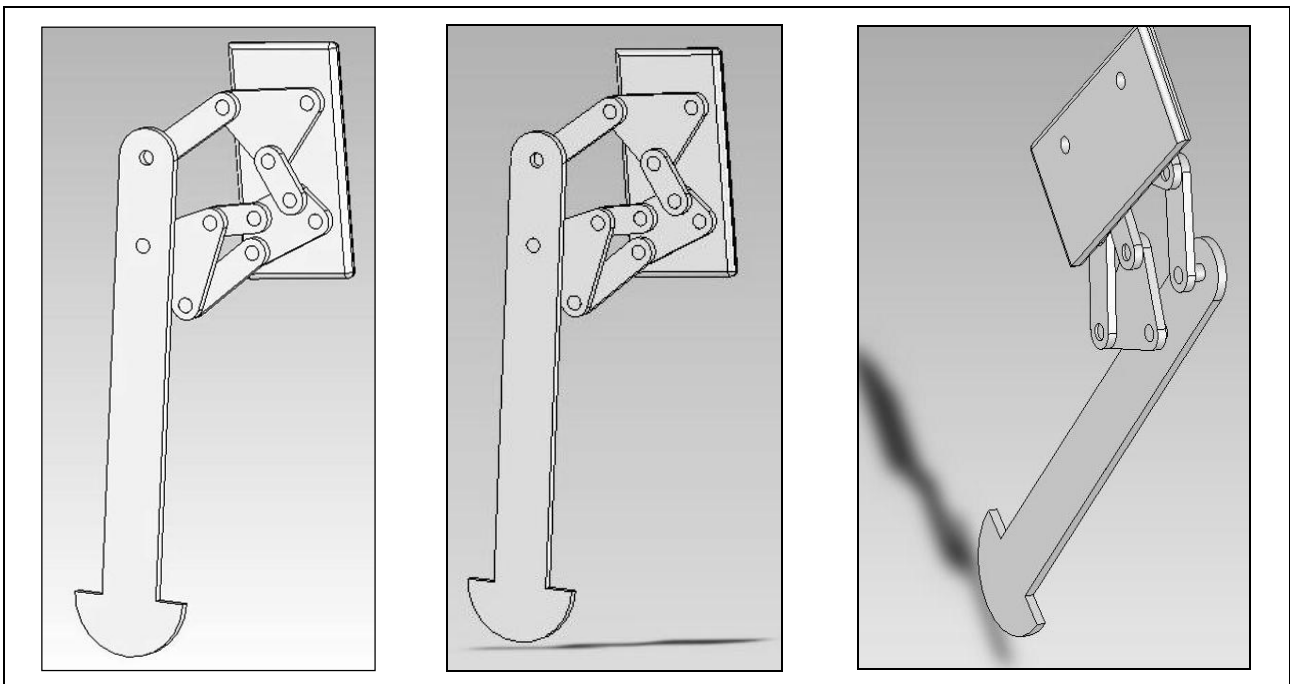


Fig.10. The pedipulator designed by using the inverse model structure

5. Conclusion

The paper is relevant in the following aspects:

- a general and unitary method for the structural design of the planar bi-mobile mechanisms especially applied in robotics is elaborated;
- the determination of the optimal structure based on the inverse models for this class of mechanisms firstly presented in the speciality literature.

After some years of applied research there are selected *165 new optimal structural solutions for bi-mobile planar mechanisms.*

Any qualified person may design or verify a new solution for a bi mobile mechanism nevertheless its destination by selecting the linkages from the Table 1 and by choosing the basis and the effector for Tables 3-23, where the respective linkage is found. In this manner it is not necessary to know the theoretical approaches.

The *References* include the classical titles from the literature dedicated to the Theory of Mechanisms, which are the basis of design for the systems with one degree of mobility. New equipments many of them belonging to robotics are multi-mobile ones and new theoretical methods and techniques are necessary for their optimal design. The research results presented in this chapter fulfil this demand as to the optimal structure of bi mobile mechanisms.

Due to the rich graphical illustration in the next future all these solutions will be included in a date base useful to researchers and designers and to appreciate the patents.

6. References

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