UNWINDING FROM CONIC PACKAGES

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Abstract: *Mathematical modeling can be used to simulate the unwinding of yarn from packages of different shapes. This method can be applied to design packages that can sustain high unwinding velocities at low and steady tension in the yarn. In the case of conic packages the angular velocity of unwinding depends not only on the winding angle as is the case for cylindric packages, but also on the apex angle. We will show that the dimensionless angular velocity depends very little on the apex angle. The apex angle, however, also determines the effective radius of the package at the lift-off point, therefore the angular velocity can be proportionally higher. We will compare unwinding from a cylindrical and a conic package with equal smallest radius and show that unwinding from the conic package is faster due to higher average radius of the package at the lift-off point.*

Key words: *yarn unwinding, conic packages, angular velocity, winding angle, apex angle.*

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This Publication has to be referred as: Pracek, S[tanislav]; Sluga, F[ranci] & franken, G[regor] (2011). Unwinding from Conic Packages, Chapter 36 in DAAAM International Scientific Book 2011, pp. 445-452, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-901509-84-1, ISSN 1726-9687, Vienna, Austria
DOI: 10.2507/daaam.scibook.2011.36

1. Introduction

When the yarn is being unwound from a cylindrical package, the angular velocity of the yarn forming the balloon depends on three parameters: the package radius, the unwinding velocity, and the winding angle. Particularly important is the last parameter, since it is closely related to the oscillations of the tension in the yarn (Kong & Rahn, & Goswami 1999; Fraser & Ghosh & Batra, 1992). In this paper we will investigate the unwinding from conic packages and the role of another parameter: the apex angle. We will also devise a simple mathematical model which can be applied for simulating the unwinding process.

2. Theoretical part

We study the unwinding from conic packages (Fig. 1). We denote the apex angle by α and the winding angle by φ. In the present context, the winding angle is defined as the angle between the tangential line on the package surface which is perpendicular to the package axis and the line tangential to the yarn at the lift-off point. The smallest radius of the package is c_0 and angle of conic α.

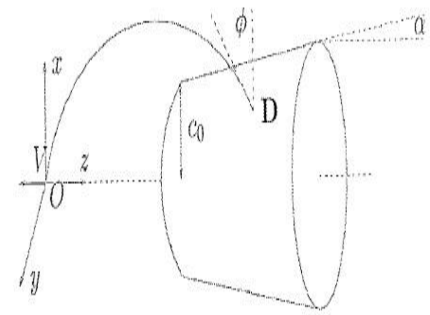

Fig. 1. Yarn unwinding from a conical package

The derivation of the formula which relates the angular velocity ω, the angles α and φ, as well as the unwinding velocity V and the effective radius of the conic package at the lift-off point c, is more involved as in the case of cylindrical package, since one cannot use a simple geometrical argument in the present case.

We introduce a new auxiliary quantity V_1, the velocity of the yarn at the lift-off point. This velocity is not necessarily equal to the unwinding velocity V. Instead we have(Padfield, 1956;Padfield, 1958) :

$$V_1 = V + \frac{ds}{dt} \qquad (1)$$

where s is the length of the yarn forming the balloon (i.e., between the lift-off point and the eyelet). V_1 is composed of the unwinding velocity and the time derivative of the length s: this is the “velocity” of the extension of the yarn in the balloon. This corresponds to the fact that in a given time interval, some of the yarn unwinds through the eyelet, while some yarn remains in the balloon and makes it longer (or, conversely, there may be a deficit of the yarn in the balloon if more yarn unwinds through the eyelet than there is yarn which leaves the package surface).

The length of the yarn which unwinds within one period of the balloon motion t=2π/ω is equal to L=2πc/cos φ. In order to obtain this expression we had to assume that the effective radius at the lift-off point does not change appreciably on the length scale of one yarn loop. This approximation holds well when both the apex angle and the winding angle are small, i.e., for most packages used in practice. Since by definition V_1=L/t we obtain

$$V_1 = \frac{c\omega}{\cos\phi}\,. \tag{2}$$

The quantity ds/dt can be computed by the chain rule

$$\frac{ds}{dt} = \frac{ds}{dz_1}\frac{dz_1}{dt}, \tag{3}$$

where z_1 is the coordinate z of the lift-off point, which moves up and down the package during the unwinding. For cylindrical packages one has

$$\frac{dz_1}{dt} = \frac{M}{t} = \frac{2\pi\, c \tan\phi}{\frac{2\pi}{\omega}} = c\omega \tan\phi \tag{4}$$

as demonstrated in Fig. 2. For conical packages the movements along the z-axis are reduced by a factor of cosα: as the lift-off point by M on the package surface, its coordinate z changes by Mcosα. It is easy to see that this is indeed the case if one considers a section of the package along the z axis. We therefore have

$$\frac{dz_1}{dt} = c\omega \tan\phi \cos\alpha \tag{5}$$

When the unwinding is quasistationary, it holds approximately that the length of the balloon s changes for the amount equal to the displacement of the lift-off point along the z axis. In mathematical terms this approximation is

$$\frac{ds}{dz_1} = 1 \tag{6}$$

It is not possible to obtain a better approximation without performing a full numerical solution of the yarn unwinding, as described in previous publications (Praček & Jakšić, 2002) .

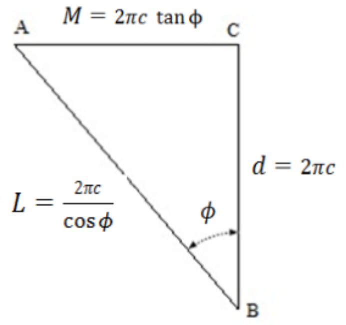

Fig. 2. Cross section of a package along a line parallel to the package axis

The winding angle is positive if during unwinding the lift-off point moves in the direction of larger values of the coordinate z (this corresponds to the unwinding in the backward direction). The winding angle is negative if during unwinding the lift-off point moves in the direction of smaller values of z (unwinding in the forward direction).

We now insert expressions (2) to (5) in Eq. (1) to obtain

$$\frac{c\omega}{\cos\phi} = V + c\omega\tan\phi\cos\alpha \tag{7}$$

which can be expressed in the final form as

$$\omega = \frac{V\cos\phi}{c\,(1-\cos\alpha\sin\phi)} \tag{8}$$

The quantity c in this equation is the effective radius of the conic package at the current lift-off point. If we denote the smallest radius by c_0 and the current position of the lift-off point along the z-axis by z, where z=0 corresponds to the package edge closest to the eyelet, then one has

$$c = c_0 + z\tan\alpha \tag{9}$$

The dimensionless angular velocity can be defined as before through Ω=cω/V, where one uses the current radius c. We obtain

$$\Omega = \frac{\cos\phi}{1-\cos\alpha\sin\phi} \tag{10}$$

The effect of the apex angle of the cone on the dimensionless velocity is shown in Fig. 3. It is clear that the effect of the typical apex angle on the dimensionless angular velocity is negligible for all practical purposes. Nevertheless, in calculations it is necessary to take into account that the angular velocity ω depends on the current winding angle φ and the current effective radius c. The next section is devoted to the time variation of these quantities.

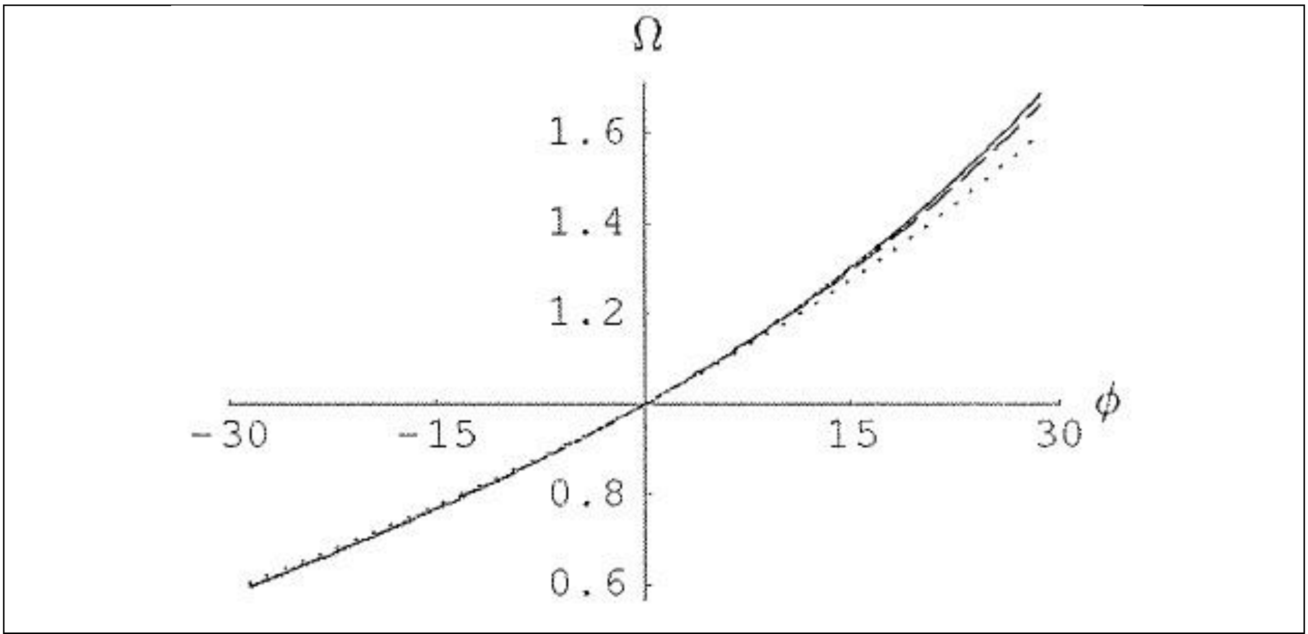

Fig.3. The effect of the apex angle on the dimensionless angular velocity.

The plot shows the relation between the apex angle α, the winding angle ϕ, and the dimensionless angular velocity Ω. The full line corresponds to α ~ 0°, the dashed line to α = 10°, and the dotted line to α = 20°. The apex angles of 10° and 20° are exaggerated for more clarity in the plot. The plot makes it clear that the dependence of the dimensionless angular velocity in the case of conical packages differs very little from the dependence in the case of cylindrical packages even if the apex angle becomes very large.

3. Practical part

During the unwinding the lift-off point moves up and down the package. Let us consider now the displacement of the point in an infinitesimally short time dt. If the motion is analyzed in the cylindrical coordinate system (rθz), where r is the distance of the point from the package axis, θ is the polar angle, and z the "height", then the infinitesimal change of the polar angle is dθ=ωdt. The current angular velocity ω is computed using Eq. (1) for cylindrical package or using Eq. (8) for conical package.

The displacement of the lift-off point along the perimeter of the package within the time dt is equal to dl=cdθ. On cylindrical packages c is the package radius which is constant, while on conical packages we need to use the current effective radius which is determined by Eq. (9).

By definition the winding angle φ is determined as the angle between the tangential line on the package surface, perpendicular to the package axis, and the tangential line to the yarn at the lift-off point. In mathematical terms, tan φ=dz/dl. The infinitesimal displacement of the point along the z-axis is thus equal to

$$dz = c\omega \tan\phi\, dt \tag{11}$$

In this equation we insert the value for the winding angle at the current position of the lift-off point. This value is approximately constant as the lift-off point moves up or down the package, but its sign changes abruptly at the package edges. If the bottom edge is given by z=0 and the upper one by z=Z, then the angle φ as a function of the coordinate z is a function which is equal to zero at z=0 and at z=Z (since it has to change sign at the edges), and approximately equal to φ_0 for other values of z. These requirements are met, for example, by the function

$$\phi(z) = \phi_0 \sin\left(\pi \frac{z}{Z}\right)^{\frac{1}{20}} \tag{12}$$

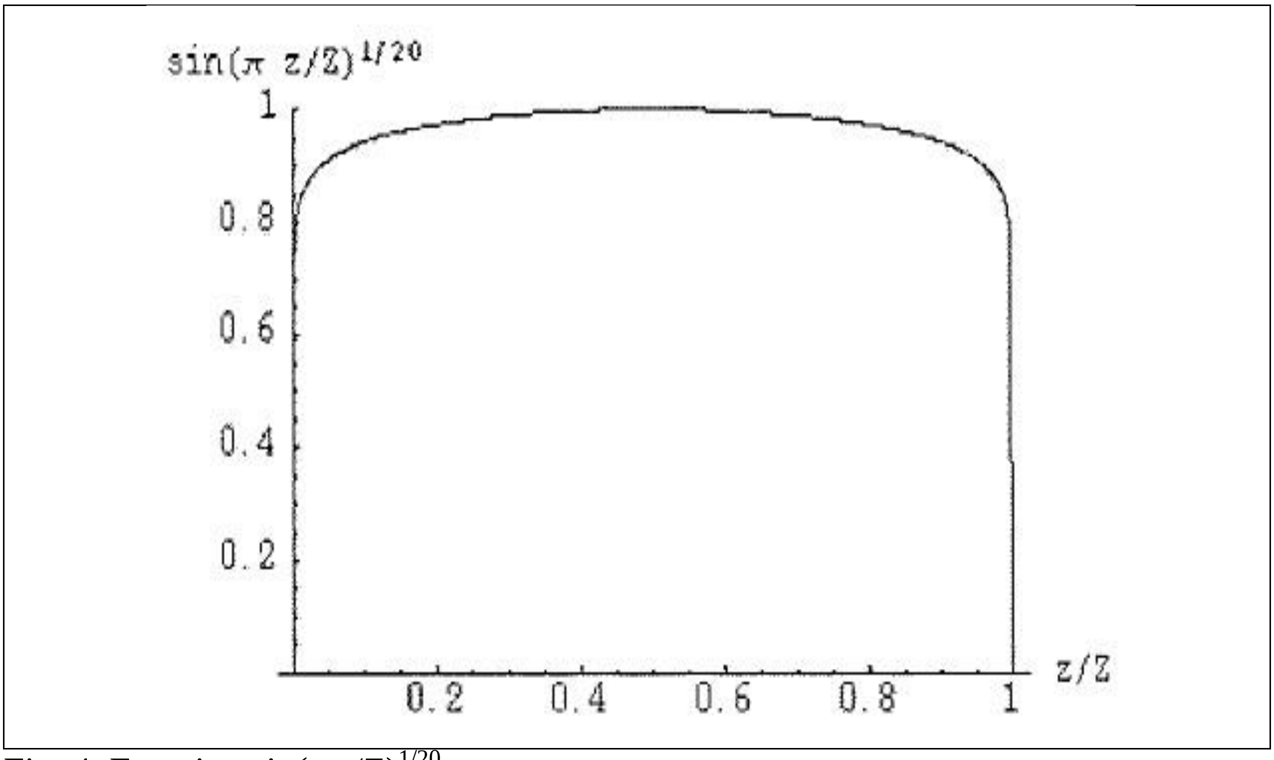

Fig. 4. Function $\sin(\pi z/Z)^{1/20}$

which is plotted in Fig. 4. In this equation one has to insert the winding angle in the layer which is currently being unwound. During unwinding in the forward direction this angle is negative, during unwinding in the backward direction it is positive.

The motion of the point on the package surface may therefore be obtained by solving the ordinary differential equation of the first order, Eq. (11), which one can express in the following form:

$$\frac{dz}{dt} = c(z)\,\omega(c(z), \phi(z)) \tan \phi(z) \qquad (13)$$

We have already explained how the quantities on the right-hand side of the equation depend on the value of z.

The equation can be solved numerically, for example using the Runge-Kutta method, to obtain the time dependence z(t) of the lift-off point during the unwinding (Press & Teukolsky & Vetterling & Flannery, 1992). As a side product of the calculation we also obtain the time dependence of the current effective radius of the package c(t), the time dependence of the winding angle φ(t) and the current angular velocity ω(t). These time dependencies are plotted in Fig. 5 for the examples of cylindrical and conic package.

In both cases the unwinding in the backward direction (when z is increasing) is faster than the unwinding in the forward direction (when z is decreasing), because for the unwinding in the backward direction the angular velocity ω (as well as the dimensionless angular velocity Ω) is higher, see Fig. 5(a). It can also be observed that the conic package unwinds faster than the cylindrical, which is due to the higher average effective radius c, see Fig. 5(b).

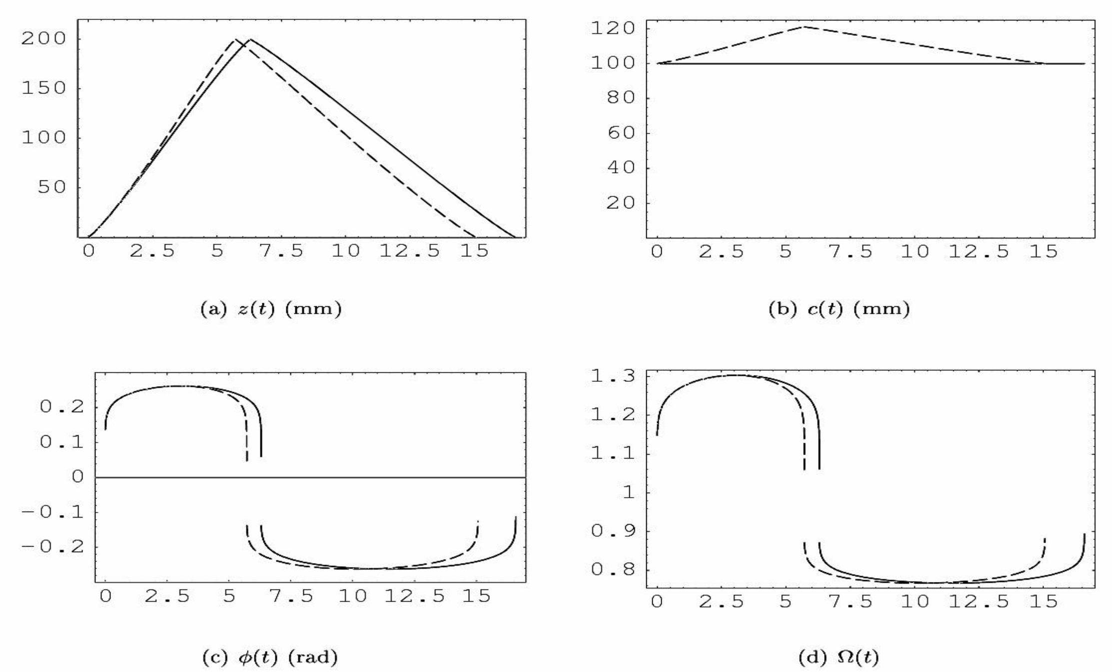

Fig. 5. Time dependencies of the quantities z (coordinate z of the lift-off point), c (the effective package radius at the lift-off point), φ (the winding angle at the lift-off point), and Ω (the current dimensionless angular velocity) during the unwinding from a cylindrical package (full line) and a conical package (dashed line). Both packages have the (minimal) radius of 100 mm, their length is 200 mm, the maximal winding angle is 15°, and the apex angle of the conical package is 6°. The time is expressed in units of seconds

4. Conclusions

We have derived the expressions for the time dependence of the angular velocity of the balloon rotation around the package axis, of the unwinding velocity and the winding angle during the yarn unwinding from cylindrical and conic packages. The expressions allow to study the motion of the lift-off point on the package surface as we have shown. This method makes it possible to consider more general package designs where c(z) and φ(z) are arbitrary functions which describe the package form and the type of winding of yarn in individual layers. This procedure allows to simulate yarn unwinding from general packages and to determine the package construction with the desired properties.

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